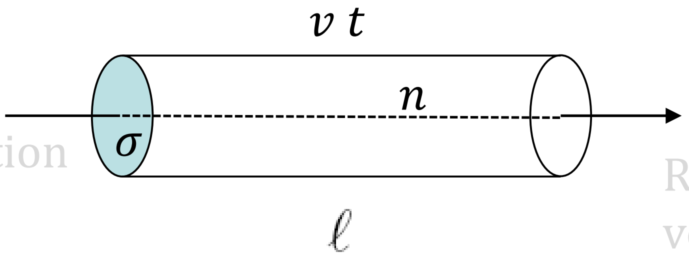
Collisional Processes

- Long range interaction
  --- between charges (ions, electrons); Coulomb $1/r^2$
- Intermediate range interaction
  --- between charges and neutrals (atoms/molecules);
  induced dipole, $1/r^4$
- Short range interaction
  --- between neutrals, $1/r^6$

Collision



A two-body encounter,

[# of collisions] = [total # of particles in the (moving) volume]

so $N = n\,(\sigma v t)$

σv: collision rate

- ✓ # of collisions per unit time $= {}^{N}/_{t} = n\,\sigma v$
- ✓ Time (interval) between 2 consecutive collisions, <u>mean free time</u> ($N = 1$), $t_{\mathrm{col}} = 1/\,(n\,\sigma v)$
- ✓ <u>Mean free path</u> $\ell = v t_{\mathrm{col}} = 1/(n\,\sigma)$

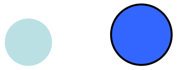
Thermal Motion

Gas (mostly H atoms), the root-mean-squared speed

$$\frac{1}{2} m_H \sqrt{\langle v^2 \rangle} = \frac{3}{2}\, k_B\, T$$

In H I regions, $T \sim 100$ K, $v_{\mathrm{rms,HI}} \sim 1\ \mathrm{km\ s^{-1}}$, $v_{\mathrm{rms},e^-} \sim 50\ \mathrm{km\ s^{-1}}$

Cross Section

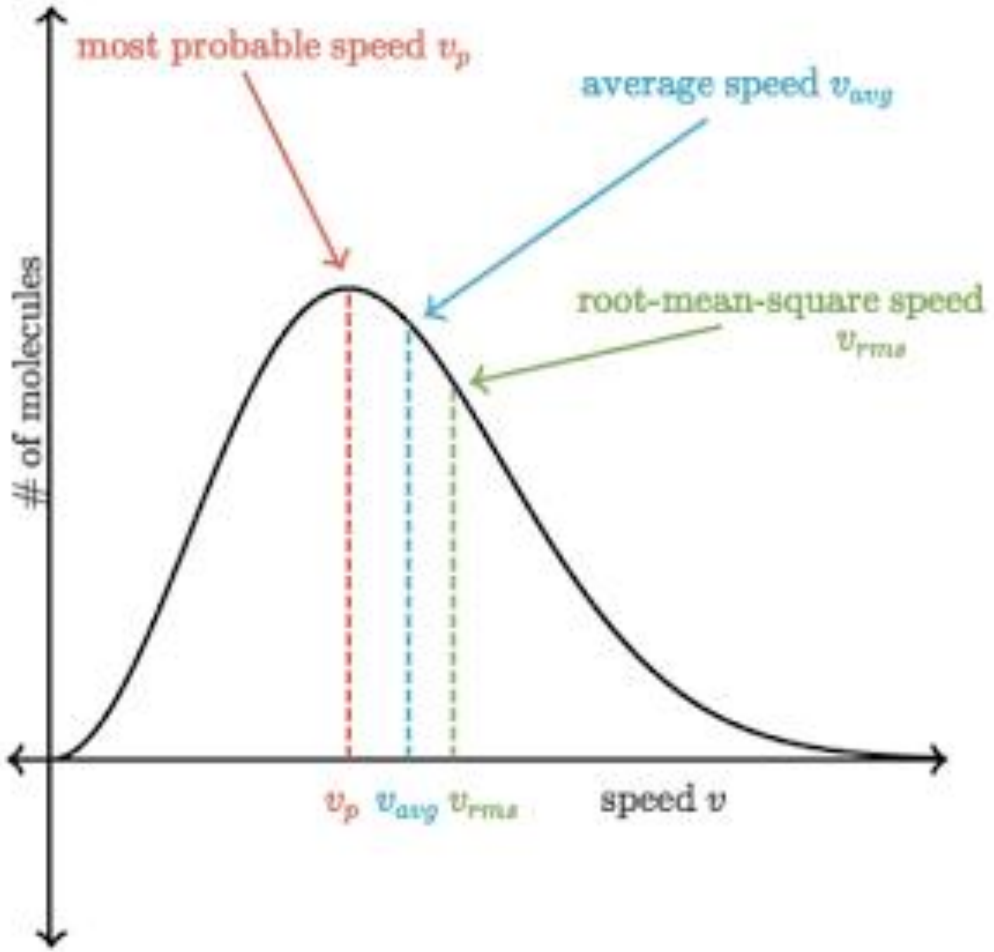


$$\sigma = \pi (a_1 + a_2)^2$$

For neutrals, hard spheres (physical cross section) OK,

$$\sigma_{\mathrm{HI,HI}} \leftarrow a \sim 5.6 \times 10^{-9}\ \mathrm{cm}$$

This is to be compared with the Bohr radius of the first orbit of $a_0 = 5.3 \times 10^{-9}$ cm



$$v_{\mathrm{mp}} = \sqrt{\frac{2\,k_B\,T}{m}} \qquad \langle v \rangle = \sqrt{\frac{8\,k_B\,T}{\pi\,m}} \qquad v_{\mathrm{rms}} = \sqrt{\frac{3\,k_B\,T}{m}}$$

In an HI cloud, $n_{HI} \sim 10\ \mathrm{cm}^{-3}$; $v_{HI} \sim 1\ \mathrm{km\ s}^{-1}$; $\sigma_{HI,HI} \sim 10^{-16}\ \mathrm{cm}^2$

$$t_{HI,HI} \sim 10^{10}\ \mathrm{s} \sim 300\ \mathrm{years};\ \ell \sim 10^{15}\ \mathrm{cm} \sim 100\ \mathrm{au}$$

∴ Collisions are indeed very rare.

$$\sigma_{HI,\,e^-} \sim 10^{-15}\ \mathrm{cm}^2\ \text{(polarization)}$$

$$t_{HI,\,e^-} \sim (10 \times 10^{-15} \times 10^5)^{-1} \sim 10^{10}\ \mathrm{s} \sim 30\ \mathrm{years}$$

$$\sigma_{e^-,\,e^-} \sim 10^{-12}\ \mathrm{cm}^2;\ n_e \sim 0.2\ \mathrm{cm}^{-3}$$

$$t_{HI,\,e^-} \sim 10^{10}\ \mathrm{s} \sim 10\ \mathrm{days}$$

Cross Section (*cont.*)

For free e^- and p^+, $\sigma \gg \sigma_{\text{physical},}$ because of Coulomb force

Need QM, $a \sim 2.5 \times 10^{-2}/v_{\text{km/s}}^2$ [cm]

If $v_{e^-} \sim 50$ km s^{-1}, $a \sim 10^{-5}$ cm for e^--e^- encounters

If $T = 3 \times 10^4$ K, $\langle v \rangle \sim 10^3$ km s^{-1} $\longrightarrow a \sim 2.5 \times 10^{-8}$ [cm]

c.f., the classical electron radius $r_e = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13}$ [cm]

$r_{\text{proton}} \approx ?\, 0.8$ fm $\approx 0.8 \times 10^{-13}$ cm

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Conventional unit: 1 barn $= 10^{-24}$ [cm^2]

$\sigma_{HI,HI} \sim 10^{-16}$ cm$^2 \sim 10^8$ barns

In general, for a two-body collision,

$A + B \rightarrow$ Products

[reaction rate per unit volume] $= n_A n_B \langle \sigma v \rangle_{AB}$,
where the **rate coefficient** is

$$\langle \sigma v \rangle_{AB} \equiv \int_0^\infty \sigma_{AB} \, v \, f(v) \, dv \ \ [\mathrm{cm^3 \ s^{-1}}]$$

and

v: relative velocity between A and B
$\sigma_{AB}(v)$: reaction cross section; vel. dependent
$f(v)$: velocity distribution function

In thermal equilibrium, the Maxwellian velocity distribution applies

$$f_v \, dv = 4\pi \left(\frac{\mu}{2\pi \, kT}\right)^{3/2} e^{-\mu v^2/2kT} \, v^2 \, dv$$

$\mu \equiv m_A \, m_B/(m_A + m_B)$ is the reduced mass

In terms of energy,

$$\langle \sigma v \rangle_{AB} = \left(\frac{8 \, kT}{\pi \mu}\right)^{1/2} \int_0^\infty \sigma_{AB}(E) \, \frac{E}{kT} e^{-E/kT} \, \frac{dE}{kT}$$

If the density is high, e.g., in the Earth's atmosphere, three-body collisions may become important,

$$A + B + C \rightarrow \text{Products},$$

The reaction rate per unit volume is then $\kappa_{ABC} \, n_A n_B n_C$, where κ_{ABC} is the three-body collisional rate coefficient [$\text{cm}^6 \ \text{s}^{-1}$]

Elastic scattering by an **inverse-square force**, e.g., Rutherford scattering

Exact solutions complicated; so use the "**impact approximation**", i.e., motion in a straight line

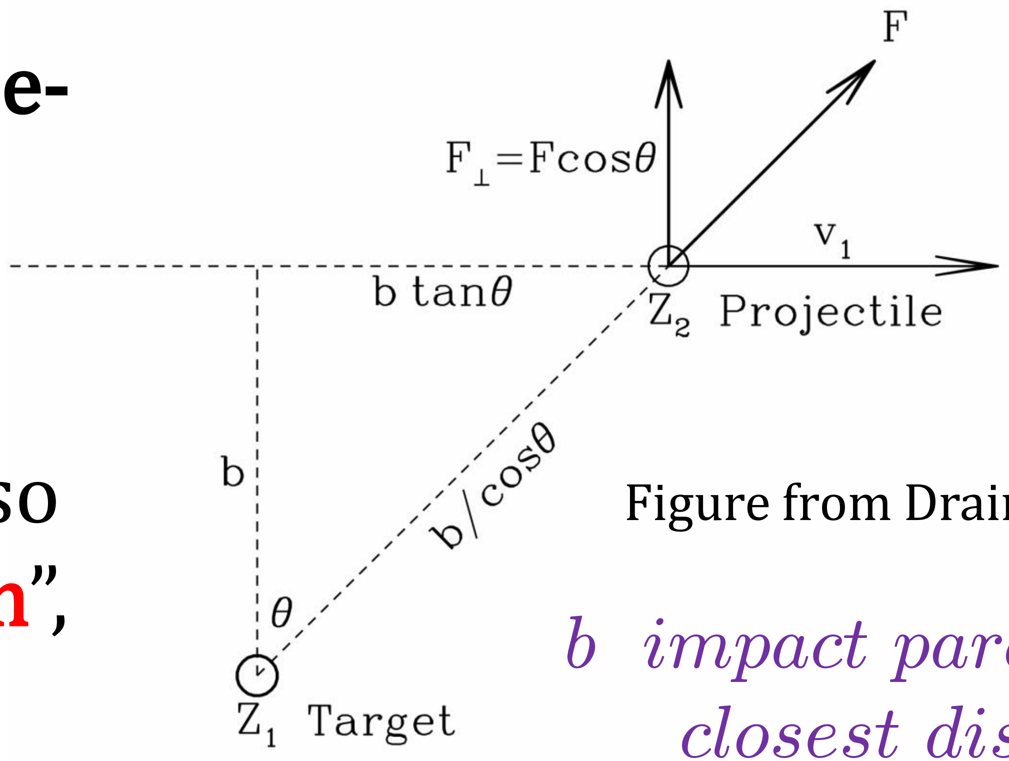


Figure from Draine's book

b impact parameter, closest distance

v_1 relative velocity

Assumption: constant velocity during the encounter between the target and the projectile

Question: How much momentum is transferred (⊥ direction)?

Impact Approximation

Coulomb force

$$F_{\perp} = \frac{Z_1 e Z_2 e}{(b/\cos\theta)^2}\cos\theta = \frac{Z_1 Z_2 e^2}{b^2}\cos^3\theta$$

Interaction time scale $dt = \frac{d(b\tan\theta)}{v_1} = \frac{b}{v_1}\frac{d\theta}{\cos^2\theta}$

Total momentum transfer is

$$\Delta p_{\perp} = \int_{-\infty}^{\infty} F_{\perp}\,dt = \frac{Z_1 Z_2 e^2}{b v_1}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$= \frac{2 Z_1 Z_2 e^2}{b v_1} \approx \underbrace{\frac{Z_1 Z_2 e^2}{b^2}}_{\text{Force at closest distance}}\ \underbrace{\frac{b}{v_1}}_{\text{Time scale}}$$

Collisional ionization: an electron energetic/fast enough ($E = p^2/2m$) to ionize an atom or ion of ionization energy E_I

Fast moving
→ $(1/2)\, m_e v^2 >> E_I$

⊕ e^-

$$(\Delta P_{\perp})^2 > 2mE_I \Rightarrow (\frac{2Z_1 Z_2 e^2}{b v_1})^2 > 2mE_I$$

So,

$$b^2 < b_{\text{max}}^2(v) = \frac{(2Z_1 Z_2 e^2)^2}{v_1^2 \cdot 2mE_I} = \frac{2Z_p^2 e^4}{m_e v^2 E_I}$$

and the ionization cross section becomes

$$\sigma(v) \approx \pi b_{\text{max}}^2 = \frac{2\pi Z_p^2 e^4}{m_e v^2 E_I}$$ This is ok if v↑↑.

For minimum velocity, $v_{min} = (2I/m_e)^{1/2}$

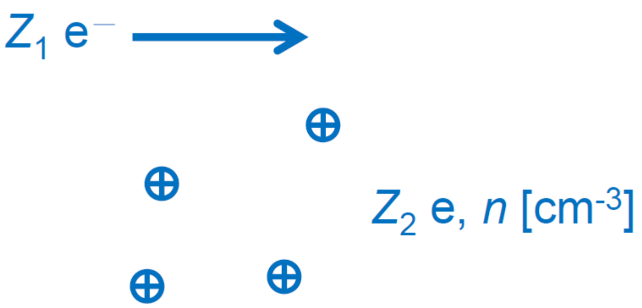
$$
\begin{aligned}
< \sigma v > &= \int \sigma(v) v f(v)\, dv \\
&= \int_{v_{\rm min}}^{\infty} \frac{2\pi Z_p^2 e^4}{m_e v^2 E_I} v\, 4\pi \left(\frac{m_e}{2\pi kT}\right)^{3/2} v^2\, e^{-m_e v^2/2kT}\, dv \\
&= Z_p^2 \left(\frac{8\pi}{m_e kT}\right)^{1/2} \frac{e^4}{E_I} e^{-E_I/kT}
\end{aligned}
$$

For an H atom at level n, $E_{\rm I} = 13.6\ [\rm eV]/n^2$, so for a large n, e.g., n~100, and T~10^4 K, $E_{\rm I}$ ↓↓ (<< kT)
→ in radio frequencies.

$< \sigma v > \propto \frac{1}{E_I} \propto n^2$, so is very large.

For large n (highly excited), the collisional ionization rate is high (i.e., easy to happen)

Deflection Timescale



Net momentum transfer

$$< \frac{d}{dt}(\Delta P_\perp)^2 >$$

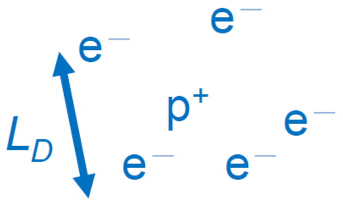
There must be a range of distance, for which

$b_{\rm min} = Z_1 Z_2 e^2/$Energy, and

$b_{\rm max} \approx L_D$ (Debye length) The effective range of the $\vec{E}$ field of a charge

In plasma, the distributions of ions and electrons are correlated because of charge neutrality.

Near a proton → more electrons than protons
→ the proton is "shielded"



Average charge within a region $\langle Q(L_D) \rangle = -e$

$$L_D = \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = 690\ T_4^{1/2} \left[\frac{n_e}{\text{cm}^{-3}}\right]^{-1/2} \text{[cm]}$$

$$< \frac{d}{dt}(\Delta P_\perp)^2 > \propto \frac{n_2}{v_1} \ln \Lambda$$

$\Lambda \equiv b_{max}/b_{min}$ = relative importance of distant encounters to close encounters

$$\ln \Lambda = 22.1 + \ln \left[E_{\mathrm{kT}}\, T_4^{3/2} n_{\mathrm{e}}^{-1} \right]$$

generally very large; in ISM, $\ln \Lambda \approx 20 - 35$

$\Rightarrow$ For elastic scattering of electrons by ions, weak distant encounters ($>>$ atomic scales) more important than close encounters.

So impact approximation OK

Electron-Ion Inelastic Scattering

An ion originally in state 1, with degeneracy g_1, is deexcited to state 0.

When an electron comes in about the atomic dimensions, the atom is suddenly perturbed
→ transition → deexcitation → line radiation

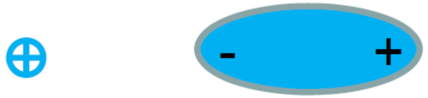
$$\langle \sigma v \rangle_{1\to 0} \equiv \gamma_{10} = \frac{8.629 \times 10^{-8}}{\sqrt{T_4}} \frac{\Omega_{10}(T)}{g_1} \ [\mathrm{cm^3 s^{-1}}]$$

where $\Omega_{10}(T)$ is **collision strength**.

- ☐ Ω_{ji} is dimensionless; almost independent of T for $T \lesssim 10^4$ K
- ☐ Typically $1 \lesssim \Omega_{ji}(T) \lesssim 10$.

Ion-Neutral Collisions

Neutral is polarized.



Dipole moment $\vec{P} = \alpha_N \vec{E}$

Interaction potential

$$U(r) = -\frac{1}{2}\alpha_N \frac{Z^2 e^2}{r^4} \propto r^{-4}$$

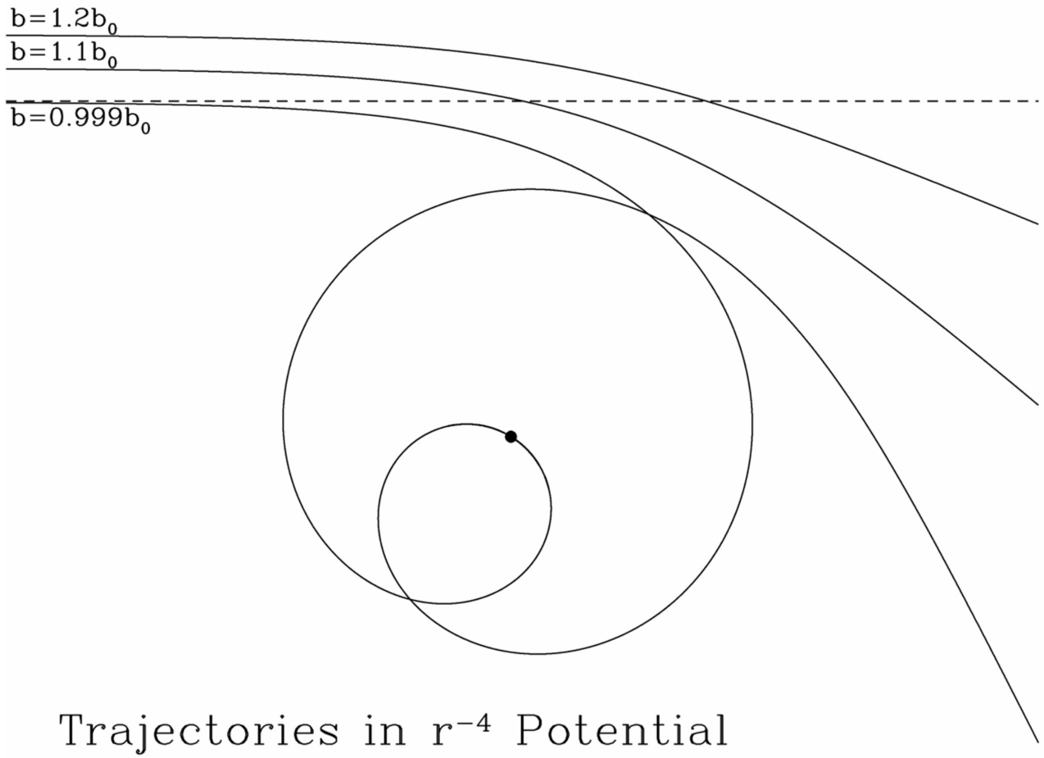
where α_N is **polarizability** $\approx$ a few a_0.

a_0 = Bohr radius $\equiv \frac{\hbar^2}{m_e e^2} = 5.292 \times 10^{-9}$ [cm]

For such a potential, if $b < b_0$, the deflection cross section is large.

$\sigma = \pi\, b_0^2 \propto 1/v$, and the rate coefficient $\langle \sigma\, v \rangle \not\leftrightarrow T$

Usually if T ↑, σ ↓



Trajectories in r^{-4} Potential

The ion-neutral reactions are important in <u>cool</u> ISM.

Electron-Neutral Collisions

In low-ionization ISM (e.g., protoplanetary disks) ions are rare. e^--neutral (H_2, He) scattering is important.

e^--H_2 scattering

(1) If $E < 0.044$ eV → pure elastic scattering

(2) If $E > 0.044$ eV → rotational excitation possible

(3) If $E > 0.5$ eV → vibrational excitation possible

(4) If $E > 11$ eV → electronic excitation

By experiment $\sigma \simeq 7.3 \times 10^{-6} \left(\frac{E}{0.01\mathrm{eV}}\right)$ [cm^2]

and $< \sigma v > \simeq 4.8 \times 10^{-9} \left(\frac{T}{10^2\mathrm{K}}\right)^{0.68}$ [cm^3 s^{-1}]

Neutral-Neutral Collisions

Repulsive at short distances, and weakly attractive at longer distances due to van der Waals interaction (mutually induced electron dipole moment)

Hard sphere OK; each with a radius $R_\mathrm{i} \approx 1\text{Å}$; impact parameter $b < R_1 + R_2, \sigma = \pi(R_1 + R_2)^2 \approx 1.2 \times 10^{-15}\ \mathrm{cm}^2$

$$\langle \sigma \mathrm{v} \rangle = 1.81 \times 10^{-10} \left(\frac{\mathrm{T}}{10^2\ \mathrm{K}}\right)^{1/2} \left(\frac{\mathrm{m_H}}{\mu}\right)^{1/2} \left(\frac{R_1 + R_2}{2\text{Å}}\right)^2\ [\mathrm{cm}^3\ \mathrm{s}^{-1}]$$

For $T \lesssim 100$ K, the rate coefficient for neutral-neutral scattering is smaller by more than an order than that for ion-neutral scattering.