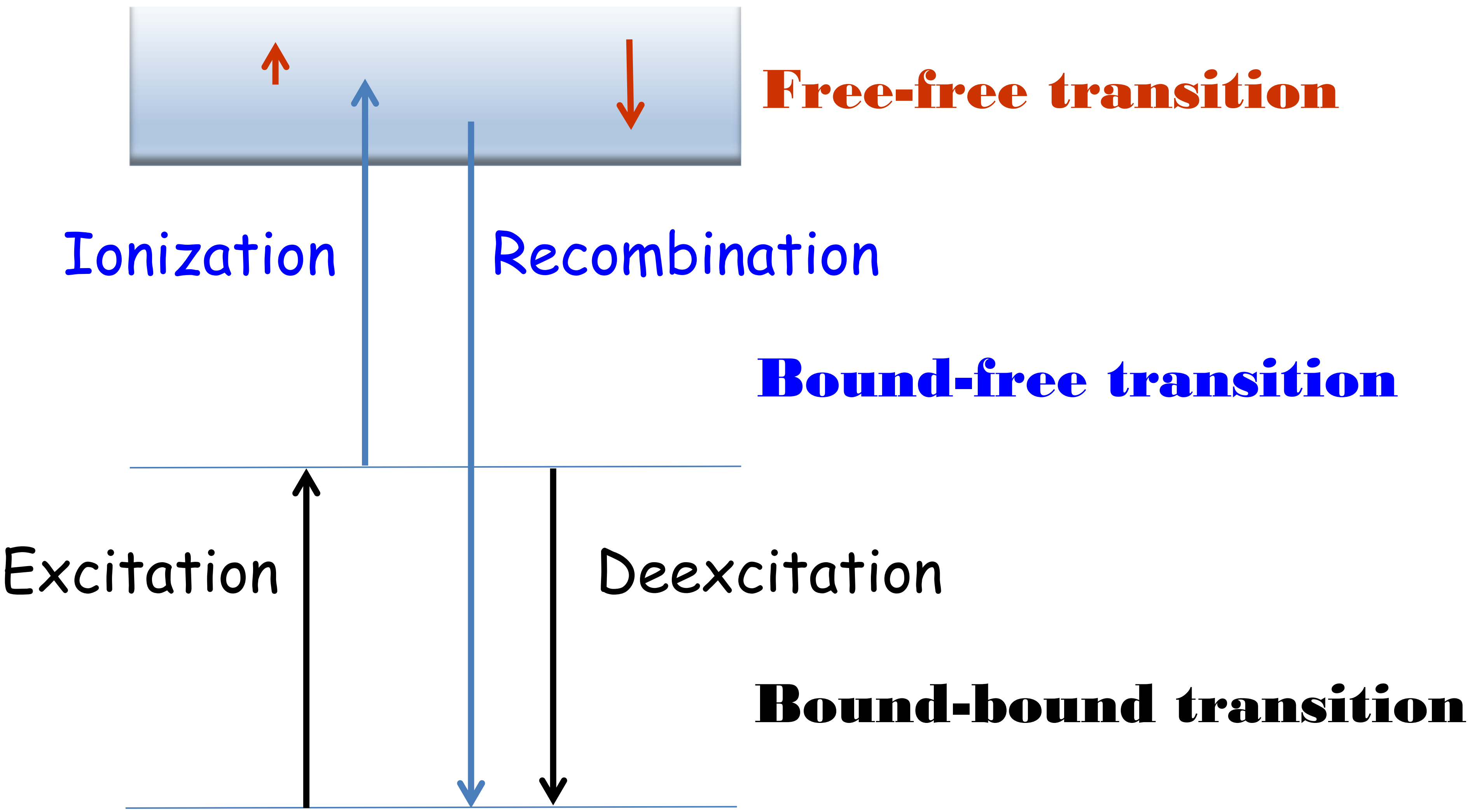
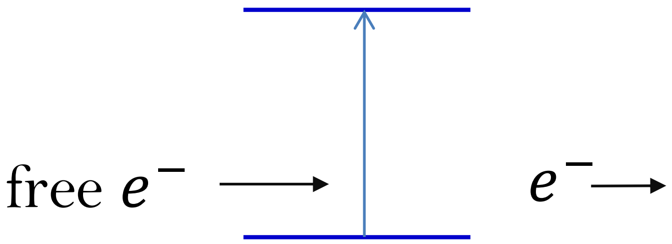
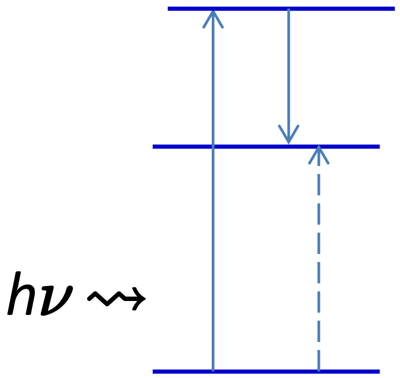
Electronic transitions Matter ↔ matter; matter ↔ photons



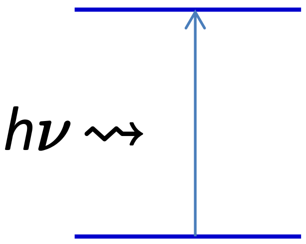
Excitation



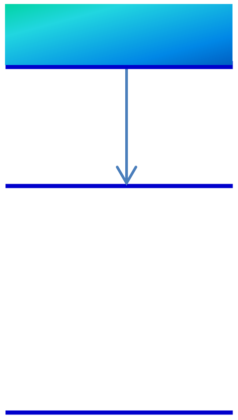
Collision



Photon Pumping

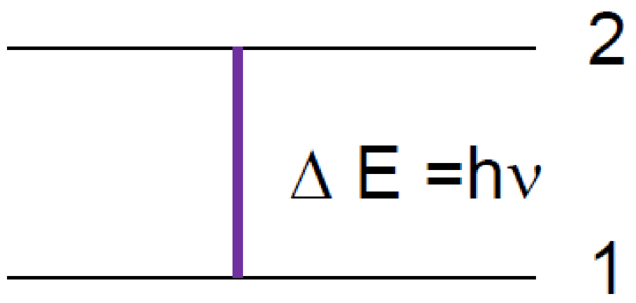


Radiation



Recombination

Emission and Absorption



Two ways to decay down from an excited state

- **Spontaneous emission**

$$X_2 \rightarrow X_1 \ + \ h\nu$$

occurrence rate ↔ atomic properties

- **Stimulated emission**

$$X_2 \ + \ h\nu \rightarrow X_1 \ + \ 2\, h\nu$$

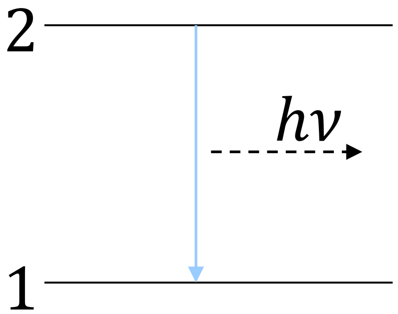
occurrence rate ↔ density of incoming photons of the same ν, polarization, and direction of propagation

- **Collisional deexcitation → no emission of photons**

Einstein Coefficients

Einstein (1917)

Spontaneous emission



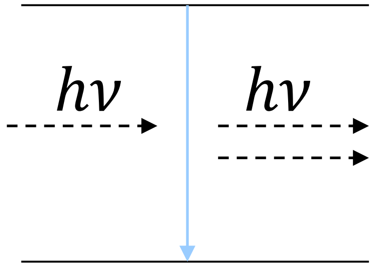
$$X_2 \longrightarrow X_1 + h\nu$$

$$\nu = (E_2 - E_1)/h$$

$\boldsymbol{A_{21}}$ --- probability [s^{-1}]

$n_2\, A_{21} dt$: # of spontaneous radiative transitions during dt

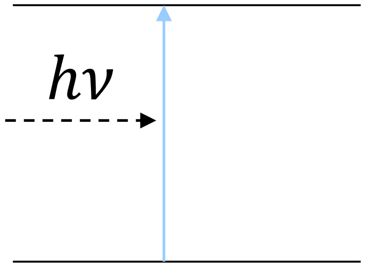
Stimulated (induced) emission



$$X_2 + h\nu \longrightarrow X_1 + 2\, h\nu$$

$$\boldsymbol{B_{21}}$$

(Stimulated) absorption



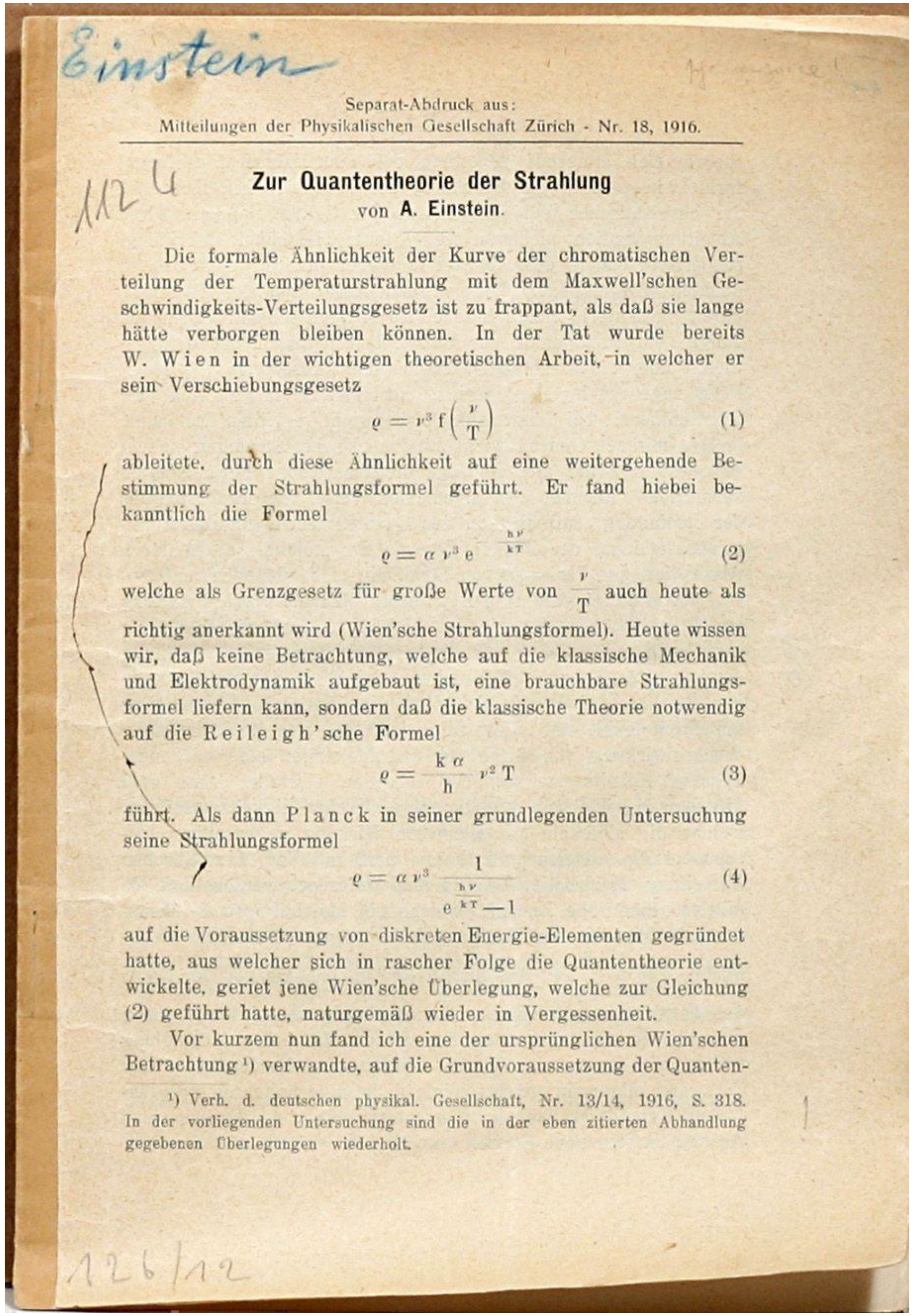
$$X_1 + h\nu \longrightarrow X_2$$

$$\boldsymbol{B_{12}}$$

$\boldsymbol{B\, I_\nu}$ --- probability

or $\boldsymbol{B\, u_\nu}$ then unit different

$n_2\, B_{21} I_\nu\, dt$ or $n_1\, B_{12}\, I_\nu\, dt$: # of (stimulated) or radiative transitions during dt when irradiated with $\boldsymbol{I_\nu}$



Einstein

Separat-Abdruck aus:
Mitteilungen der Physikalischen Gesellschaft Zürich - Nr. 18, 1916.

Zur Quantentheorie der Strahlung

von A. Einstein.

Die formale Ähnlichkeit der Kurve der chromatischen Verteilung der Temperaturstrahlung mit dem Maxwell'schen Geschwindigkeits-Verteilungsgesetz ist zu frappant, als daß sie lange hätte verborgen bleiben können. In der Tat wurde bereits W. Wien in der wichtigen theoretischen Arbeit, in welcher er sein Verschiebungsgesetz

$$\varrho = \nu^3 f\left(\frac{\nu}{T}\right) \qquad (1)$$

ableitete, durch diese Ähnlichkeit auf eine weitergehende Bestimmung der Strahlungsformel geführt. Er fand hiebei bekanntlich die Formel

$$\varrho = \alpha \nu^3 e^{-\frac{h\nu}{kT}} \qquad (2)$$

welche als Grenzgesetz für große Werte von $\frac{\nu}{T}$ auch heute als richtig anerkannt wird (Wien'sche Strahlungsformel). Heute wissen wir, daß keine Betrachtung, welche auf die klassische Mechanik und Elektrodynamik aufgebaut ist, eine brauchbare Strahlungsformel liefern kann, sondern daß die klassische Theorie notwendig auf die Reileigh'sche Formel

$$\varrho = \frac{k\alpha}{h} \nu^2 T \qquad (3)$$

führt. Als dann Planck in seiner grundlegenden Untersuchung seine Strahlungsformel

$$\varrho = \alpha \nu^3 \frac{1}{e^{\frac{h\nu}{kT}} - 1} \qquad (4)$$

auf die Voraussetzung von diskreten Energie-Elementen gegründet hatte, aus welcher sich in rascher Folge die Quantentheorie entwickelte, geriet jene Wien'sche Überlegung, welche zur Gleichung (2) geführt hatte, naturgemäß wieder in Vergessenheit.

Vor kurzem nun fand ich eine der ursprünglichen Wien'schen Betrachtung [1]) verwandte, auf die Grundvoraussetzung der Quanten-

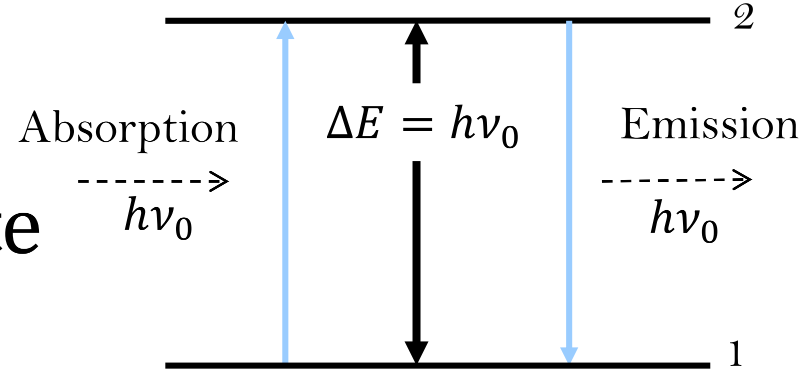
[1]) Verh. d. deutschen physikal. Gesellschaft, Nr. 13/14, 1916, S. 318. In der vorliegenden Untersuchung sind die in der eben zitierten Abhandlung gegebenen Überlegungen wiederholt.

"On the Quantum Theory of Radiation" from A. Einstein

https://einstein.manhattanrarebooks.com/pages/books/17/albert-einstein/zur-quantentheorie-der-strahlung-on-the-quantum-theory-of-radiation

Transition Probability

Considering a 2-level system, we calculate the emission arising from the transition.

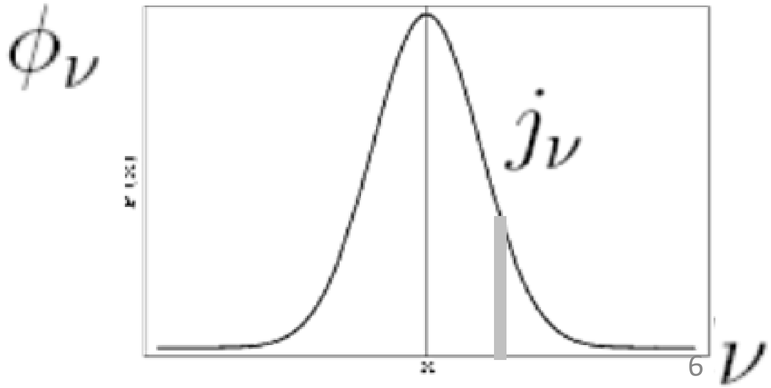


$$j_\nu \ [\mathrm{erg\ s^{-1}\ cm^{-3}\ ster^{-1}\ Hz^{-1}}]$$

$$j = \int j_\nu \, d\nu \ [\mathrm{erg\ s^{-1} cm^{-3}\ ster^{-1}}]$$ **volume emissivity**

For a line emission, assuming $j_\nu \not\Leftrightarrow \theta, \varphi$, j_ν is governed by a distribution function $\phi(\nu)$ (**line profile**),

$$\int_0^\infty \Phi_\nu \, d\nu = 1$$



Once an atom is excited, there is a finite **probability** within dt of $A\,(2,1)\,dt$ to jump spontaneously from level 2 to level 1 (deexcitation), emitting a photon. The total number of downward transitions $2 \rightarrow 1$ is $n_2\,A\,(2,1)$, where n_2 is the number of atoms (population) in level 2 per unit volume.

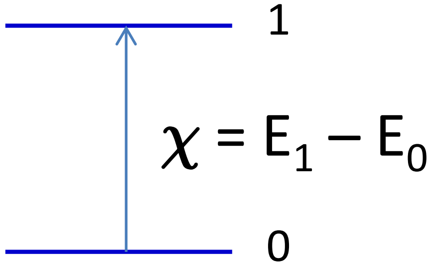
A_{21} [s^{-1}]: Einstein A coefficient for spontaneous transition
= probability per unit time.

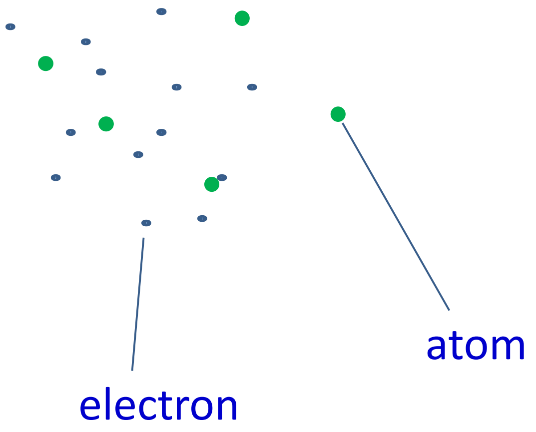
$1/A_{21}$ [s]: lifetime staying at level 2 (remaining excited)

$$j_\nu = \frac{h\nu_0}{4\pi} n_2\, A_{21} \phi(\nu)$$

Principle of detailed balancing

Consider a 2-level system, excitation occurs if the incoming free electrons have kinetic energy $\frac{1}{2}\ mv^2 > \ \chi$





Define the **excitation rate coefficient** γ_{01}, so that

$$\#\text{ of excitation s}^{-1}\text{ cm}^{-3}\ (= n_e n_0\ v\sigma) \equiv n_e n_0 \gamma_{01},$$

where both n_e and n_0 have units of [cm^{-3}]

$$\gamma_{01} \equiv \langle \sigma v \rangle = \int_{\chi=\frac{1}{2}mv^2}^{\infty} \sigma_{01}(v) v \, f(\vec{v}) \, d^3\vec{v}$$

Here σ_{01} is the excitation cross section, and $f(\vec{v})$ is the Maxwellian distribution function,

$$f(\vec{v}) \, dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \, e^{-\frac{mv^2}{2kT}} \, dv$$

So

$$\gamma_{01} = \frac{4}{\sqrt{\pi}} \left(\frac{1}{2kT}\right)^{1/2} \int_{\chi=\frac{1}{2}mv^2}^{\infty} v^3 \, \sigma_{01}(v) \, e^{-\frac{mv^2}{2kT}} \, dv \quad \ldots \text{(A)}$$

This is upward $0 \rightarrow 1$ transition.

For downward 1 →0 transition,
the spontaneous emission rate $= n_1 \, A_{10}$,
and the deexcitation rate by collisions $= n_1 n_e \, \gamma_{10}$,
where $\gamma_{10} = \int_0^\infty v \, \sigma_{10}(v) \, f(\vec{v}) \, d^3\vec{v} = \gamma_{10}(T)$

In steady state, [upwards rate]=[downwards rate],
i.e., **detailed balancing**,

$$\boxed{n_0 n_e \, \gamma_{01}(T) = n_1 [A_{10} + n_e \, \gamma_{10}(T)]}\text{, so}$$

$$\frac{n_1}{n_0} = \frac{n_e \, \gamma_{01}}{A_{10} + n_e \, \gamma_{10}} = \frac{\gamma_{01}}{\gamma_{10}} \frac{1}{1 + \frac{A_{10}}{n_e \, \gamma_{10}}} \quad \ldots \text{(B)}$$

$$\frac{n_1}{n_0} = \frac{n_e\,\gamma_{01}}{A_{10} + n_e\,\gamma_{10}} = \frac{\gamma_{01}}{\gamma_{10}}\,\frac{1}{1 + \frac{A_{10}}{n_e\,\gamma_{10}}}$$

(i) At **high** densities, i.e., $n_e \to \infty$

(i.e., collisional excitation and deexcitation dominate → in TE)

$$\frac{n_1}{n_0} \approx \frac{\gamma_{01}}{\gamma_{10}}$$

but because $\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\chi/kT}$

$$\frac{\gamma_{01}}{\gamma_{10}} = \frac{g_1}{g_0} e^{-\chi/kT} \quad \text{for } n_e \gg 1$$

So when collision dominates, c.f. (A)

$$n_e n_0 v_0^3 \sigma_{01}(v_0)\exp(-\mu v_0^2/(2kT))\,dv_0$$
$$= n_e n_1 v_1^3 \sigma_{10}(v_1)\exp(-\mu v_1^2/(2kT))\,dv_1$$

where μ: reduced mass, v_0 and v_1 are speed of colliding particles.

At **high** densities (*cont.*)

Energy conservation, $(1/2)\,\mu v_0^2 = (1/2)\,\mu v_1^2 + \chi$,
so $v_0\,dv_0 = v_1\,dv_1$. Plugging this back, we get

$$n_0 v_0^2 \sigma_{01} \exp\left(-\frac{\mu v_0^2}{2kT}\right) = n_1 v_1^2 \sigma_{10} \exp\left(-\frac{\mu v_1^2}{2kT}\right)$$

$$= n_0 \frac{g_1}{g_0} e^{-\chi/kT}\, v_1^2 \sigma_{10} \exp\left(-\frac{\mu v_1^2}{2kT}\right)$$

The exponential parts are eliminated from energy conservation, so

$$\boxed{g_0 v_0^2 \sigma_{01} = g_1 v_1^2 \sigma_{10}}$$

$$\frac{n_1}{n_0} = \frac{n_e\,\gamma_{01}}{A_{10} + n_e\,\gamma_{10}} = \frac{\gamma_{01}}{\gamma_{10}} \frac{1}{1 + \frac{A_{10}}{n_e\,\gamma_{10}}}$$

(i) At **low** densities, i.e., $n_e \to 0$

$$\frac{n_1}{n_0} \approx \frac{\gamma_{01}}{\gamma_{10}} \frac{n_e\,\gamma_{10}}{A_{10}} = \frac{n_e \gamma_{01}}{A_{10}} = \frac{\text{[upward by collisions]}}{\text{[downward by radiation only]}}$$

This means every collisional excitation is followed by emission of a photon.

The cooling rate [erg s^{-1} cm^{-3}] in this case then, is

$$n_1 A_{10}\, h\nu_{10} = n_\mathrm{e} n_0 \gamma_{01}\, h\nu_{10}$$

$$n_0 n_e\, \gamma_{01}(T) = n_1 [A_{10} + n_e\, \gamma_{10}(T)]$$

The competition for downward transition between the two terms in the bracket → the critical density

$$n_{\text{crit}} = \frac{A_{10}}{\gamma_{10}}$$

When $n_e > n_{\text{crit}}$, collisions dominate deexcitation process → LTE, populations governed by Boltmann equation.

Consider the radiative transition $1 \rightarrow 0$, the rate of emission of line photons $[\mathrm{s}^{-1}\mathrm{atom}^{-1}]$ ... cf. eq. (B)

$$\frac{n_1}{n_0} = \frac{n_e\,\gamma_{01}}{A_{10} + n_e\,\gamma_{10}} = \frac{\gamma_{01}}{\gamma_{10}} \frac{1}{1 + \frac{A_{10}}{n_e\,\gamma_{10}}} \quad \text{... (B)}$$

$$\frac{n_1}{n_0} A_{10} = A_{10} \frac{\gamma_{01}}{\gamma_{10}} \frac{1}{1 + \frac{A_{10}}{n_e\,\gamma_{10}}}$$

(i) At **high** densities, TE

$$\frac{n_1}{n_0} A_{10} = A_{10} \frac{\gamma_{01}}{\gamma_{10}} = A_{10} \frac{g_1}{g_2}\, e^{-\chi/kT} \not\leftrightarrow n_e$$

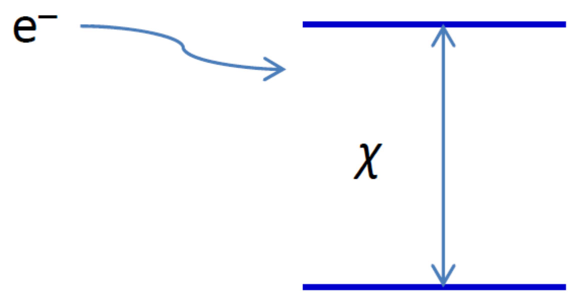
(ii) At **low** densities,

$$\frac{n_1}{n_0} A_{10} = A_{10} \frac{\gamma_{01}}{\gamma_{10}} \frac{n_e\,\gamma_{10}}{A_{10}} = n_e\,\gamma_{01} \not\leftrightarrow T$$

Every collisional excitation → emission of a line photon.

Consider a 2-level system, for which the electron collides with an ion in the lower level. cross section, $\sigma_{01} = \sigma_{01}(v)$.

Collisions between electrons and ions in a lower level



Consider electron v only; ions are neglected.

$$\begin{cases} \sigma_{01} = 0, \text{ if } (1/2)\, mv^2 < \chi \\ \sigma_{01} \propto 1/v^2, \text{ if } (1/2)\, mv^2 > \chi \end{cases}$$

Usually σ is expressed in terms of **collision strength** $\Omega(0,1)$,

$$\sigma_{01}(v) = \frac{\pi\hbar^2}{m_e^2\, v_0^2} \frac{\Omega(0,1)}{g_0} = \frac{4.21}{v^2} \frac{\Omega(0,1)}{g_0} \text{ [cm}^2\text{]}$$

Recall that $g_0\, v_0^2\, \sigma_{01} = g_1\, v_1^2\, \sigma_{10}$

The deexcitation rate coefficient is

$$\gamma_{10} = \int_0^\infty v\sigma_{10}(v)\, f(v)\, dv$$

$$= \sqrt{\frac{2\pi}{kT}}\,\frac{\hbar^2}{m^{3/2}}\frac{\Omega(0,1)}{g_1} = \;8.629 \times 10^{-6}\,\frac{\Omega(0,1)}{g_1 T^{1/2}}$$

Excitation per volume per time is $n_e n_0 \gamma_{01}$, where $\gamma_{01} = (g_1/g_0)\gamma_{10}\,\exp(-\chi/kT)$

- Ω must be calculated quantum mechanically;
- tabulation available with specific temperature values;
- typically on the order of unity.

The collisional deexcitation rate is then

$$n_e n_1\,\gamma_{10} = n_1 \int_0^\infty n_e\, v\sigma_{10}(v)\, f(v)\, dv$$

$$= n_e n_1 \sqrt{\frac{2\pi}{kT}}\,\frac{\hbar^2}{m^{3/2}}\frac{\Omega(1,0)}{g_1}$$

$$= \; 8.629 \times 10^{-6}\,\frac{n_e n_1}{g_1 T^{1/2}}\,\Omega(1,0) \quad [\mathrm{cm^{-3} s^{-1}}]$$

For typical nebular $T = 7000$ K, and abundances,
$\gamma_{10} \approx\; 10^{-7}\ \mathrm{cm^3\ s^{-1}}$

Table 8. Wavelengths, λ_{ij}, transition probabilities, A_{ij}, and collision strengths, $\Omega(i,j)$, for the forbidden transitions of the most abundant elements[1]

Element	λ_{21} (Å)	A_{21} (sec^{-1})	$\Omega(1,2)$	λ_{31} (Å)	A_{31} (sec^{-1})	$\Omega(1,3)$	λ_{32} (Å)	A_{32} (sec^{-1})	$\Omega(3,2)$
O II	3,728.8 +3,726.0	4.8×10^{-5} $+1.70 \times 10^{-4}$	1.43	2,470.4 +2,470.3	0.060 +0.0238	0.428	7,319.4 +7,330.7 +7,318.6 +7,329.9	0.115 +0.061 +0.061 +0.100	1.70
O III	5,006.8 (N_1) +4,958.9 (N_2)	0.021 +0.0071	2.39	2,321.1	0.23	0.335	4,363.2	1.60	0.310
N II	6,583.4 +6,548.1	0.003 +0.00103	3.14	3,063.0	0.034	0.342	5,754.6	1.08	0.376
Ne III	3,868.8 +3,967.5	0.17 +0.052	1.27	1,814.8	2.2	0.164	3,342.5	2.8	0.188
Ne IV	2,441.3 +2,438.6	5.9×10^{-4} $+5.6 \times 10^{-3}$	1.04	1,608.8 +1,609.0	1.33 +0.53	0.427	4,714.3 +4,724.2 +4,715.6 +4,725.6	0.40 +0.44 +0.11 +0.39	1.42
Ne V	3,425.9 +3,345.8	0.38 +0.138	1.38	1,575.2	4.2	0.218	2,972	2.60	0.185
S II	6,716.4 +6,730.8	4.7×10^{-5} $+3.0 \times 10^{-4}$	3.07	4,068.6 +4,076.4	0.34 +0.134	1.28	10,320.6 +10,287.1 +10,372.6 +10,338.8	0.21 +0.17 +0.087 +0.20	6.22
S III	9,532.1 +9,069.4	0.064 +0.025	4.97	3,721.7 +3,796.7	0.85 +0.016	1.07	6,312.1	2.54	0.961
Ar III	7,135.8 +7,751.0	0.32 +0.083	4.75	3,109.0 +3,005.1	4.0 +0.043	0.724	5,191.8	3.1	0.665
Ar IV	4,740.2 +4,711.3	0.028 0.0022	1.43	2,854.8 +2,869.1	2.55 +0.97	0.645	7,237.3 +7,170.6 +7,332.0 +7,262.8	0.67 +0.91 +0.122 +0.68	4.92
Ar V	7,005.7 +6,435.1	0.51 +0.22	1.19	2,691.4 +2,784.4	6.8 +0.081	0.141	4,625.5	3.78	0.945

[1] After GARSTANG (1968) and CZYZAK *et al.* (1968) by permission of the International Astronomical Union.

Lang

Spectroscopic Notation

Ionization State

I ---- neutral atom, e.g., H I → H^0

II --- singly ionized atom, e.g., H II → H^+

III – doubly ionized atom, e.g., O III → O^{++}

….. and so on….e.g., Fe XXIII

Peculiar Spectra

e (emission lines), p (peculiar, affected by magnetic fields), m (anomalous metal abundances), e.g., B5 Ve

Forbidden Lines

Allowed transitions (via an **electric dipole**) satisfying selection rules

1. Parity change
2. $\Delta L = 0, \pm 1, L = 0 \rightarrow 0$ forbidden
3. $\Delta J = 0, \pm 1, J = 0 \rightarrow 0$ forbidden
4. Only one electron with $\Delta \ell = \pm 1$
5. $\Delta S = 0$ (Spin not changed)

A **forbidden** transition is one that fails to fulfill at least one of the selection rules 1 to 4. It may arise from a **magnetic dipole** or an **electric quadrupole** transition.

Bowen (1936) Rev. Mod. Phys. 8, 55-81

The Origin of the Nebulium Spectrum.

The frequency of lines due to jumps between these

Another strong pair occurs at 6583·6 A.U. and 6548·1 Å.U., showing a separation of 82·3 frequency

The Origin of the Nebulium Spectrum.

In the spectra of the gaseous nebulæ several very strong lines are found which have not been duplicated in any terrestrial source. Many lines of evidence point to the fact that the lines are emitted by an element of low atomic weight. Since the spectra of the light elements, as excited in terrestrial sources, are well known, this leads to the conclusion that there must be some condition, presumably low density, which exists in the nebulæ, that causes additional lines to be emitted.

since any jump between them involves a zero change in the azimuthal quantum number. In a five-electron system such as O_{II}, the normal configuration of 2 (2s) and 3 (2p) electrons forms ^4S, ^2D, and ^2P terms. These are likewise metastable.

3726·16 O_{II} ^4S -$^2D_2$

I. S. Bowen.

Norman Bridge Laboratory of Physics,
California Institute of Technology,
Sept. 7.

Ira Sprague Bowen

REVIEWS OF

MODERN PHYSICS

VOLUME 8 APRIL, 1936 NUMBER 2

Forbidden Lines

I. S. BOWEN, *California Institute of Technology*

TABLE OF CONTENTS

- **Allowed (regular) Lines** (no bracket),
  $A \approx 10^{+8}\ \mathrm{s}^{-1}$, e.g., C IV

- **Semi-forbidden Lines** (a single bracket),
  $A \approx 10^{+2}\ \mathrm{s}^{-1}$, e.g., [OII

- **Forbidden Lines** (a pair of square brackets),
  $A \approx 10^{0}$ to $10^{-4}\ \mathrm{s}^{-1}$, e.g., [O III], [N II]

Some examples,

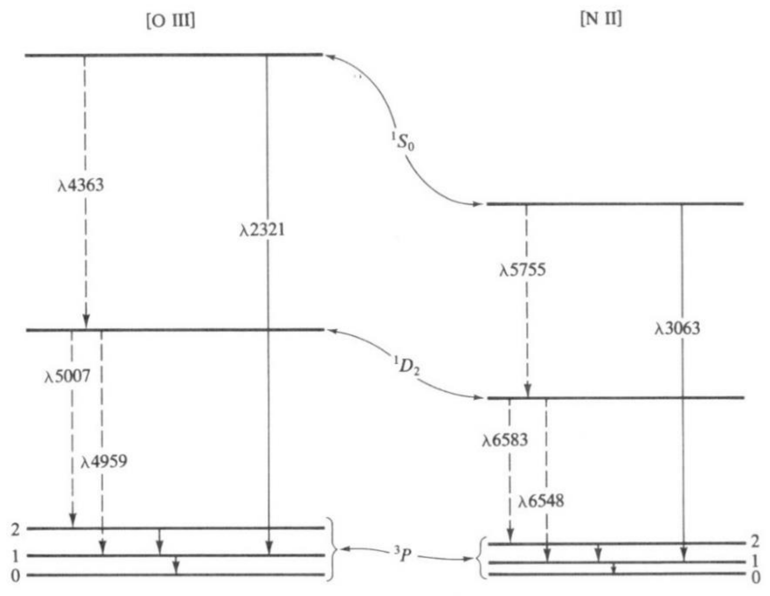
Lyman α, $A_{21} \approx 6.25 \times 10^{8}\ \mathrm{s}^{-1}$

[O III] $A_{21} = 0.021\ \mathrm{s}^{-1}, \lambda_{21} = 5007\ \text{Å}$

$A_{21} = 0.0281\ \mathrm{s}^{-1}, \lambda_{21} = 4959\ \text{Å}$

$A_{32} = 1.60\ \mathrm{s}^{-1}, \lambda_{32} = 4364\ \text{Å}$

[S II] $A_{21} = 4.7 \times 10^{-5}\ \mathrm{s}^{-1}, \lambda_{21} = 6716\ \text{Å}$

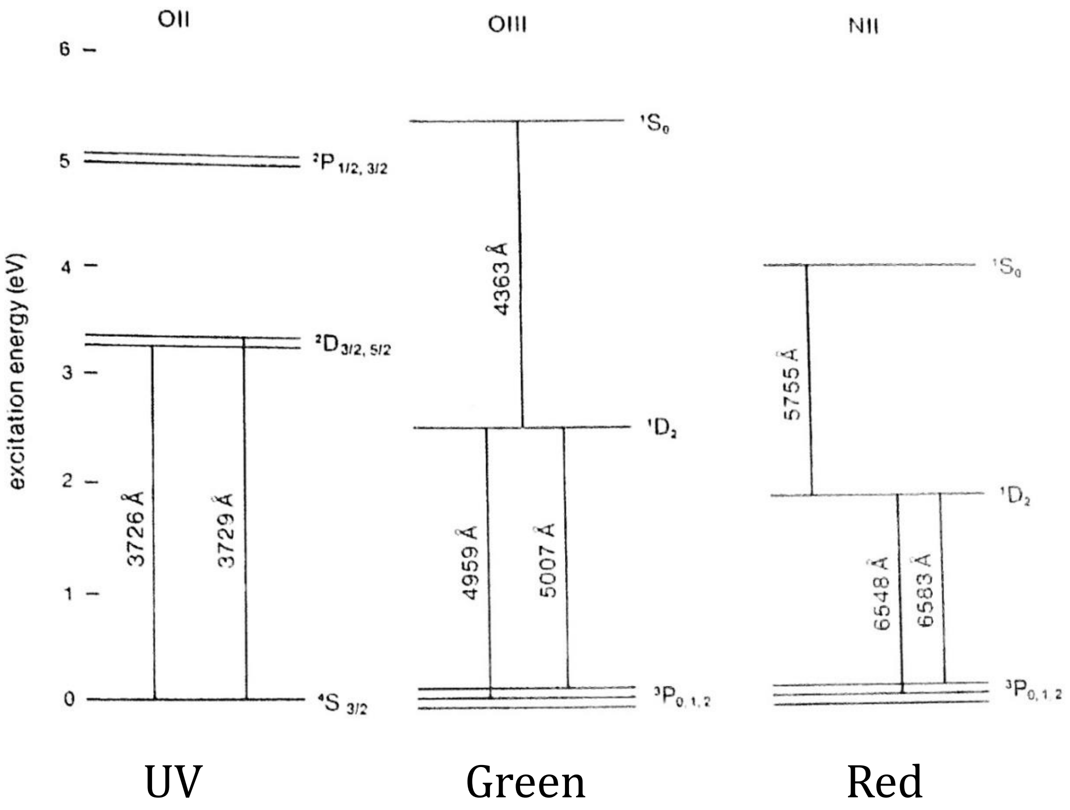


H I 21 cm hyperfine line $A_{21} \approx 2.88 \times 10^{-15}\ \mathrm{s}^{-1}$; probability extremely low

- Normally an atom stays in the excited state for 10^{-8} s.
- A forbidden transition occurs for excitation levels $<$ a few Ev; stays in the excited state for seconds or longer before returning to the ground state.
- In the lab n ↑↑, both excitation and de-excitation take place frequently, so radiative transition (emitting a photon) is unlikely.
- In ISM, the electrons are not energetic enough to excite the atoms to normal levels (10 to 20 eV) , but enough to excite to metastable levels. In hot, low-density environments, e.g., H II regions, PNe, solar corona, earth aurora
- Once (collisionally) excited → emission
  → photons escaped ➔ **efficient cooling**

Forbidden lines observed in space and terrestrial upper atmosphere, where densities are low so collisions are rare. The most efficient cooling mechanism in nebular gas: intermediate-mass ions excited by collision with electrons (kinetic energy about kT) → emission of forbidden line photons

Also the 21-cm line for cold atomic H gas



Compare to hydrogen,
$E_{1\to 2} = 10.2$ eV,
$E_{1\to\infty} = 13.6$ eV

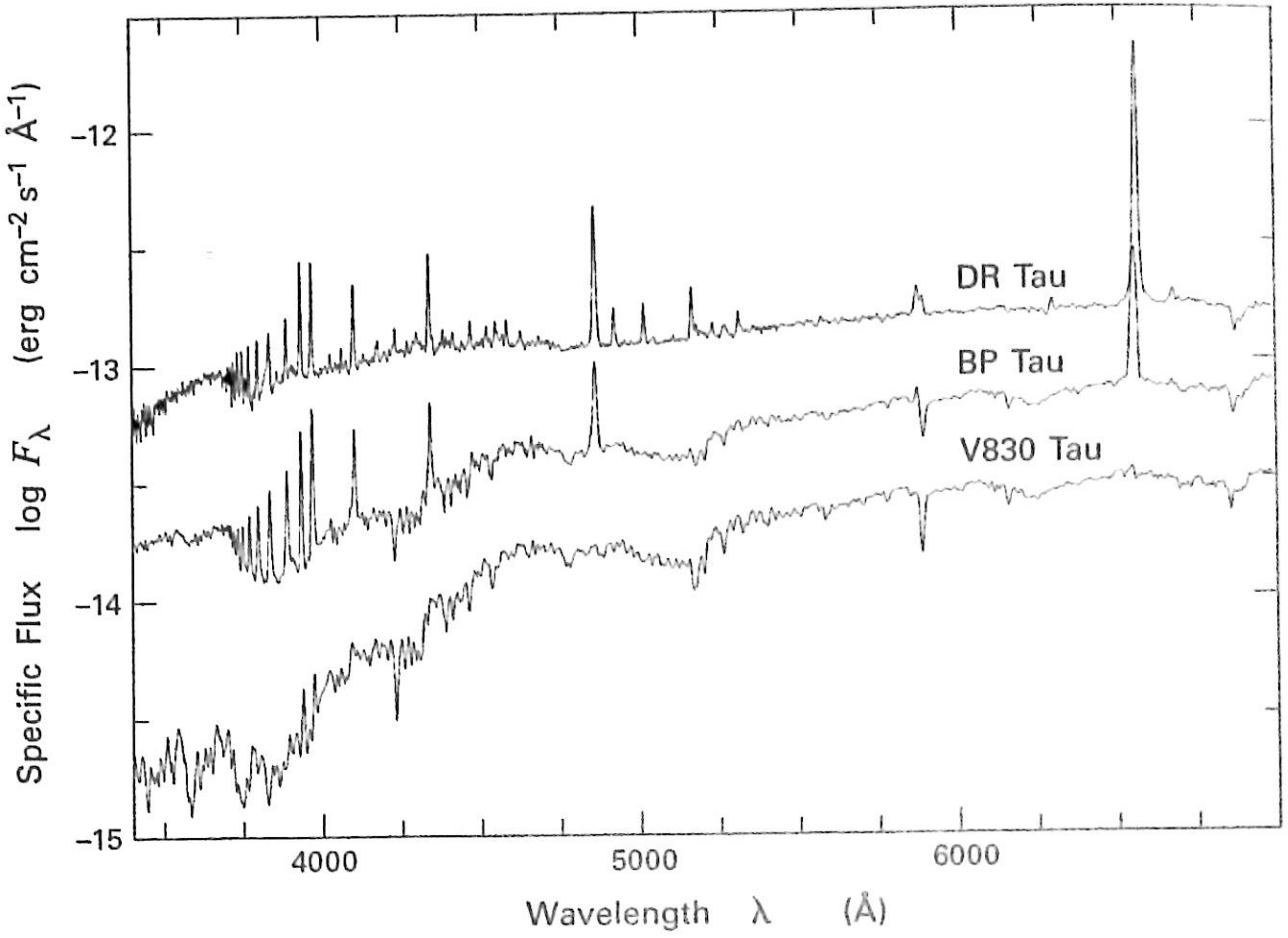


Table 17.1 Main Emission Lines in Classical T Tauri Stars

Line	Transition	Wavelength (Å)	A_{ul} (s^{-1})
Infrared			
Br γ	$n = 7 \rightarrow 4$	21661	3.0×10^5
Pa β	$n = 5 \rightarrow 3$	12822	2.2×10^6
Ca II	$^2P_{1/2} \rightarrow {}^2D_{3/2}$	8662	2.8×10^5
Ca II	$^2P_{3/2} \rightarrow {}^2D_{5/2}$	8542	1.2×10^6
Ca II	$^2P_{3/2} \rightarrow {}^2D_{3/2}$	8498	6.3×10^5
Optical			
[S II]	$^2D_{3/2} \rightarrow {}^4S_{3/2}$	6731	8.8×10^{-4}
[S II]	$^2D_{5/2} \rightarrow {}^4S_{3/2}$	6716	2.6×10^{-4}
Hα	$n = 3 \rightarrow 2$	6563	1.0×10^8
[O I]	$^1D_2 \rightarrow {}^3P_2$	6300	6.3×10^{-3}
Na I D_1	$^2P_{1/2} \rightarrow {}^2S_{1/2}$	5896	6.2×10^7
Na I D_2	$^2P_{3/2} \rightarrow {}^2S_{1/2}$	5890	6.2×10^7
He I	$^3D_3 \rightarrow {}^3P_2$	5876	7.1×10^7
Fe II	$^6P_{3/2} \rightarrow {}^6S_{5/2}$	4924	3.3×10^6
Hβ	$n = 4 \rightarrow 2$	4861	3.8×10^7
Hγ	$n = 5 \rightarrow 2$	4340	1.6×10^7
Fe I	$^3F_3 \rightarrow {}^3F_2$	4132	1.2×10^7
[S II]	$^2P_{1/2} \rightarrow {}^4S_{3/2}$	4076	9.1×10^{-2}
Ca II H	$^2P_{1/2} \rightarrow {}^2S_{1/2}$	3969	1.4×10^8
Ca II K	$^2P_{3/2} \rightarrow {}^2S_{1/2}$	3934	1.5×10^8
Ultraviolet			
Mg II h	$^2P_{1/2} \rightarrow {}^2S_{1/2}$	2803	2.6×10^8
Mg II k	$^2P_{3/2} \rightarrow {}^2S_{1/2}$	2796	2.6×10^8
C IV	$^2P_{3/2} \rightarrow {}^2S_{1/2}$	1548	2.7×10^8
Si IV	$^2P_{1/2} \rightarrow {}^2S_{1/2}$	1403	7.6×10^8
O I	$^3S_1 \rightarrow {}^3P_1$	1305	2.0×10^8
S I	$^3P_1 \rightarrow {}^3P_2$	1296	4.9×10^8
Lyα	$2p \rightarrow 1s$	1216	6.3×10^8

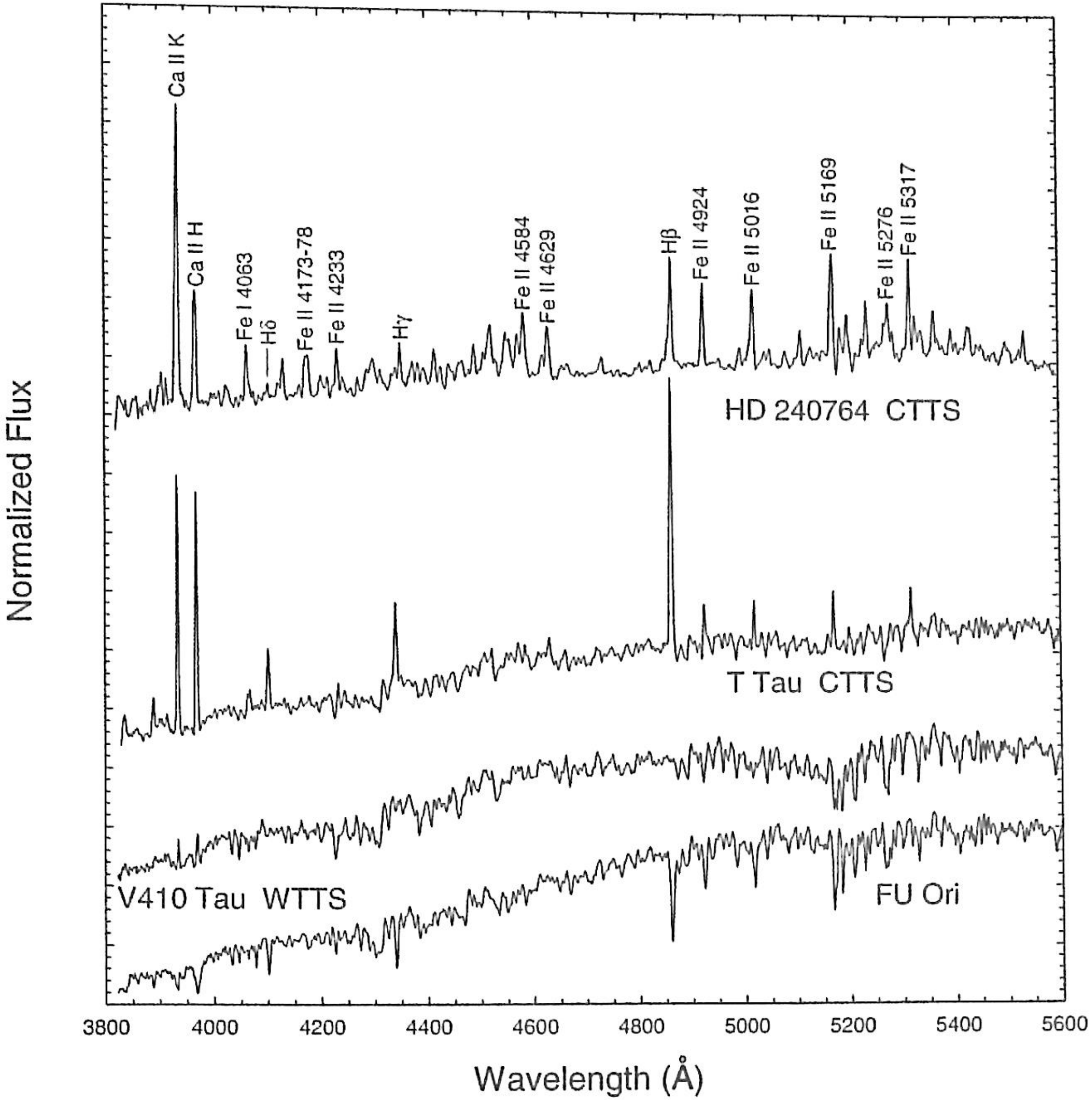


Figure 7.14 A montage of T Tauri stars and the Fuor prototype.

Gray & Corbally

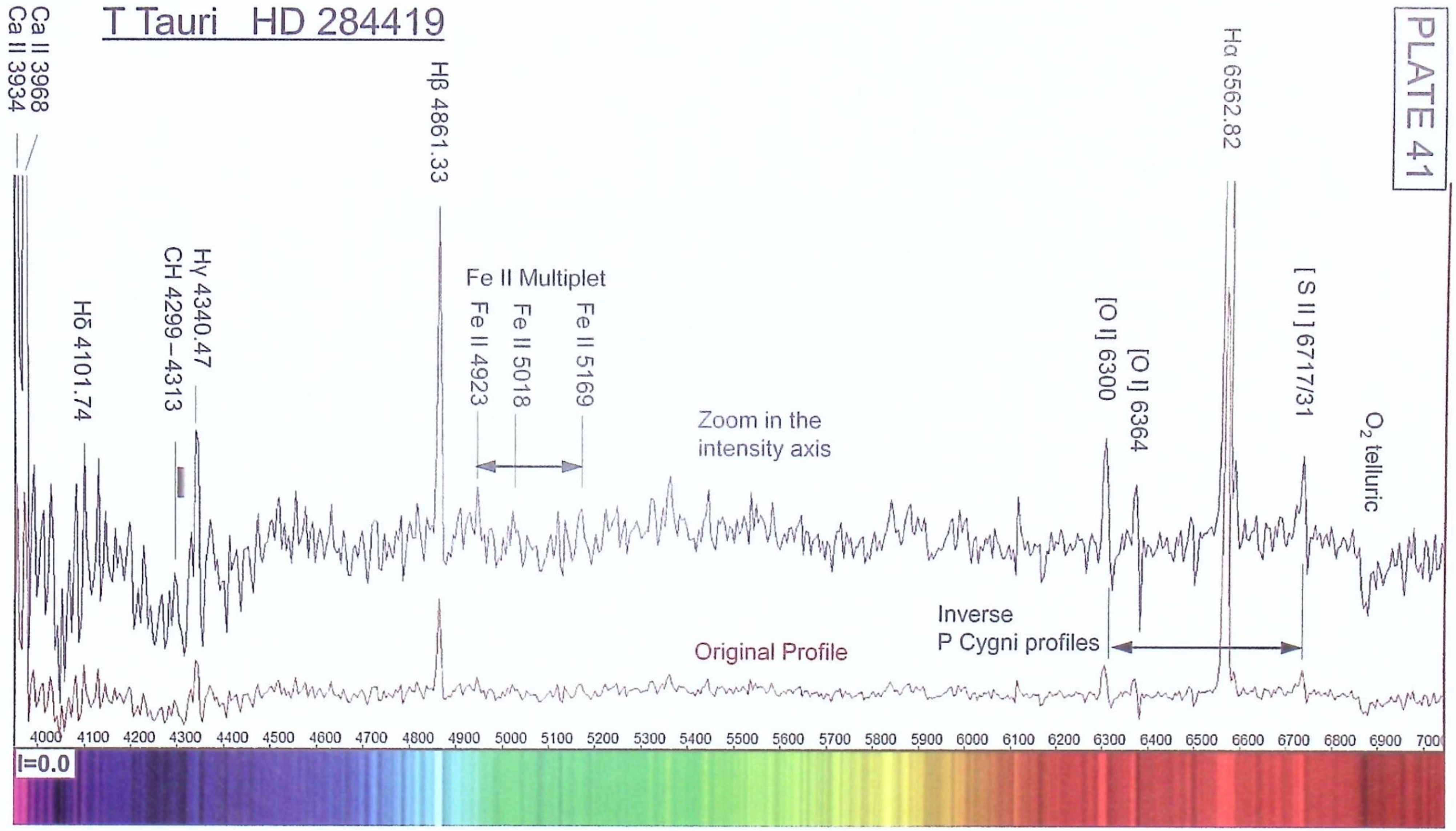
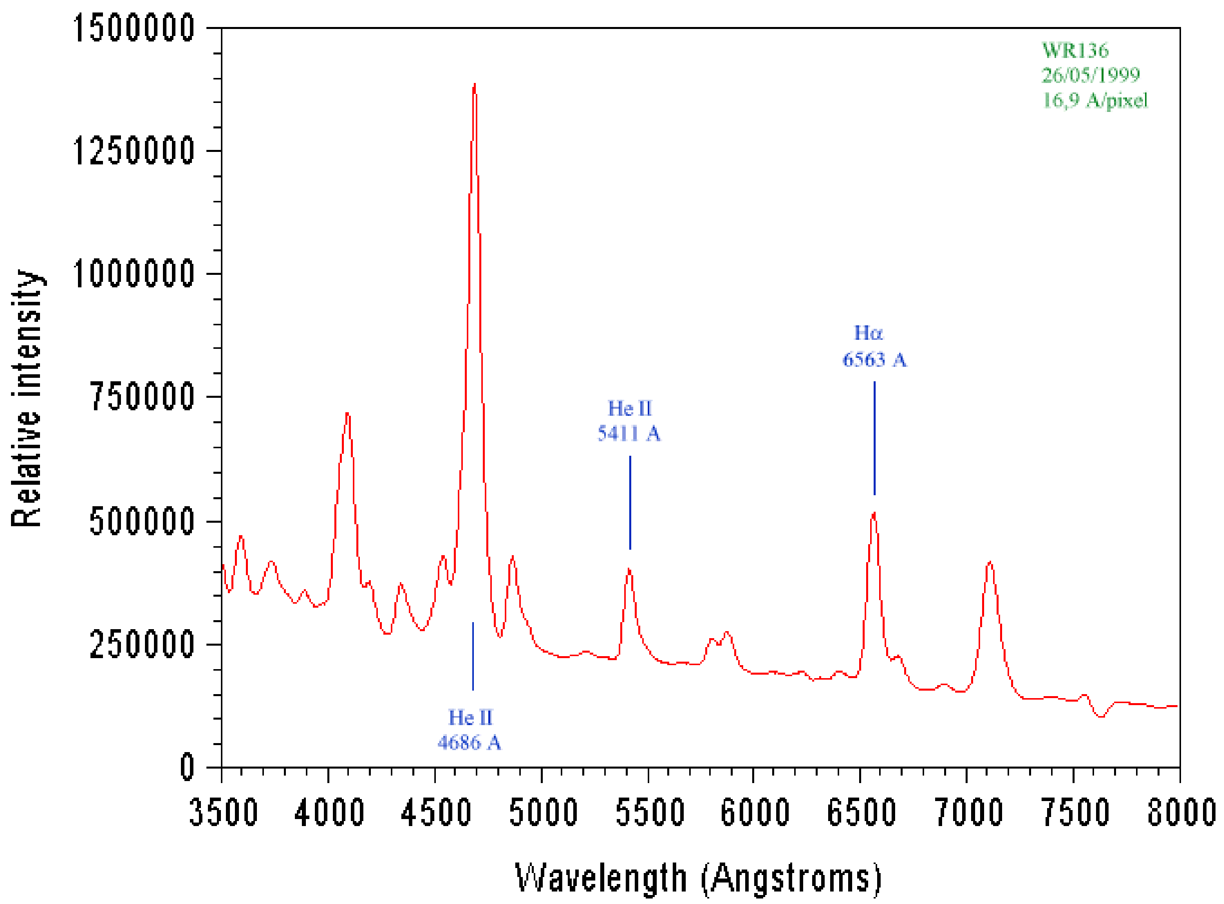


Plate 41 Prototype of the T Tauri Stars

Walker



Spectrum of a Wolf-Rayet star

WR stars

$T_{eff} \sim 25{,}000$ to $50{,}000$ K

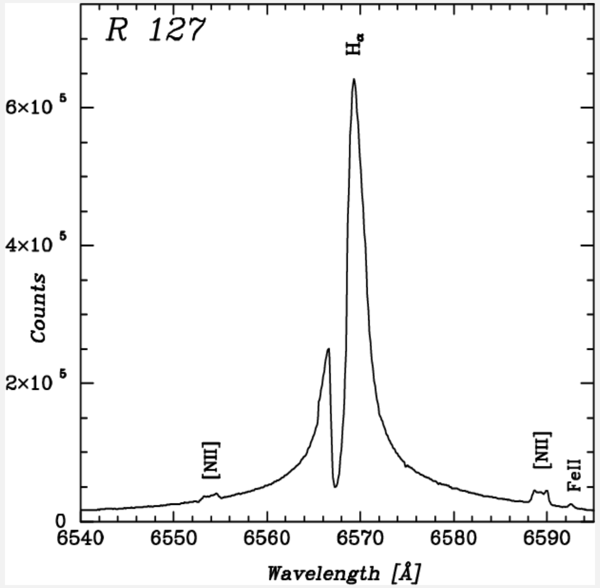
Evolved massive ($\gtrsim 20 M_\odot$) stars

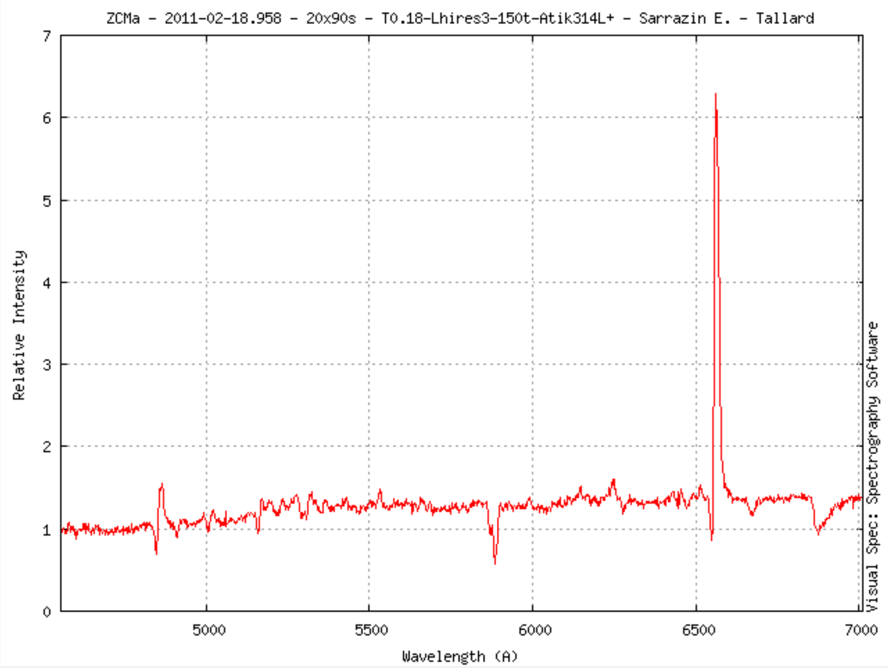
Wind ~ 2000 km s^{-1}

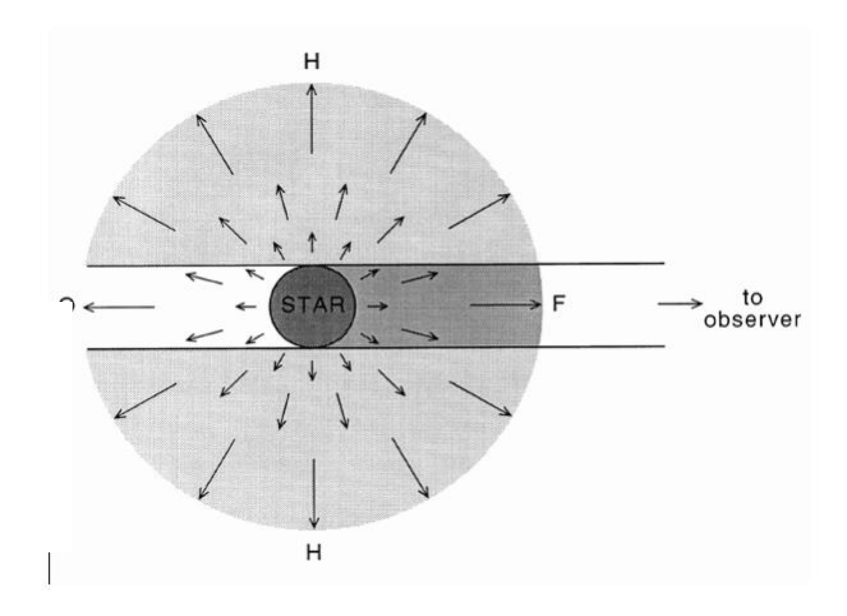
$\dot{m} \sim 10^{-5} M_\odot$ yr^{-1}

O ($\rightarrow$ WN) $\rightarrow$ LBV $\rightarrow$ WN / WC

$\rightarrow$ SN Ib,c







P Cygni profile of a spectral line --- a blue-shifted absorption superimposed on an emission line → **mass loss** (cool gas toward us)

Lamers and Cassinelli, *Introduction to Stellar Winds*, Cambridge, 1999

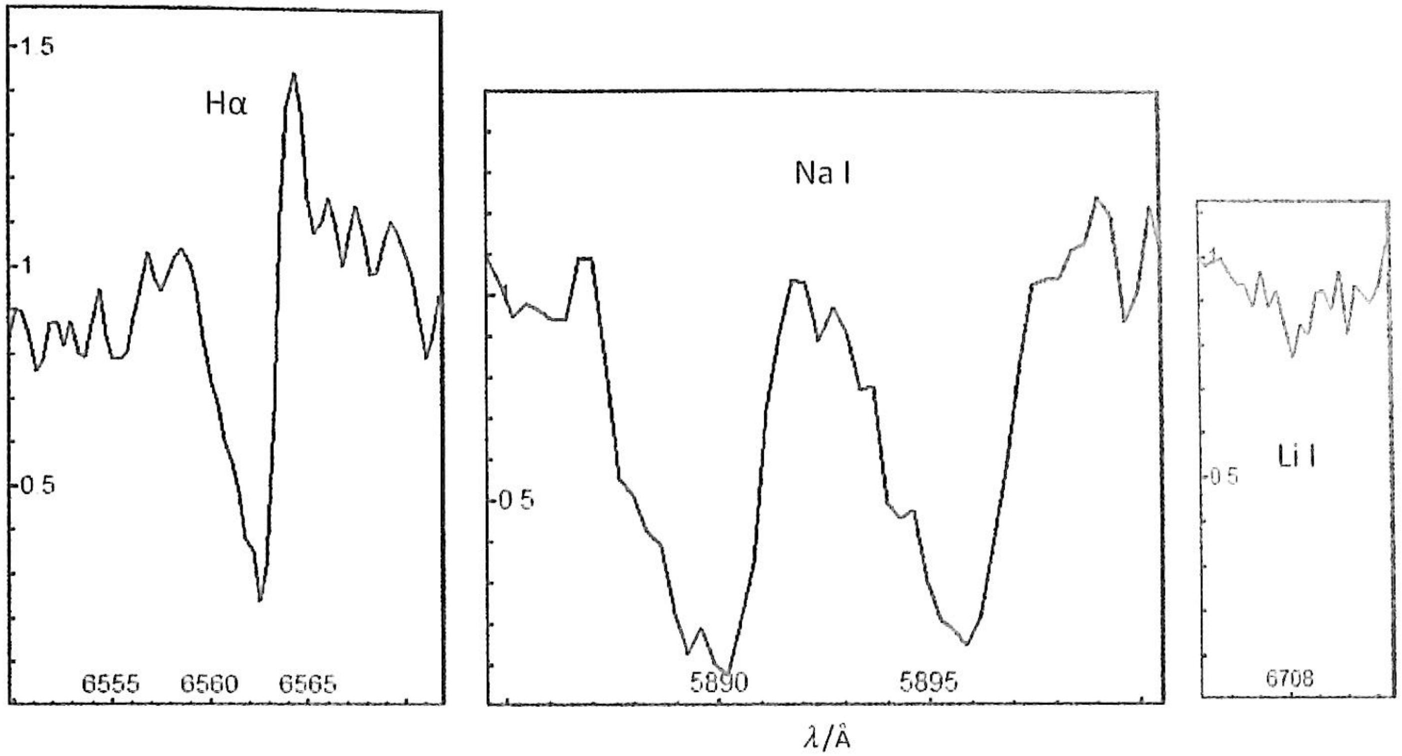


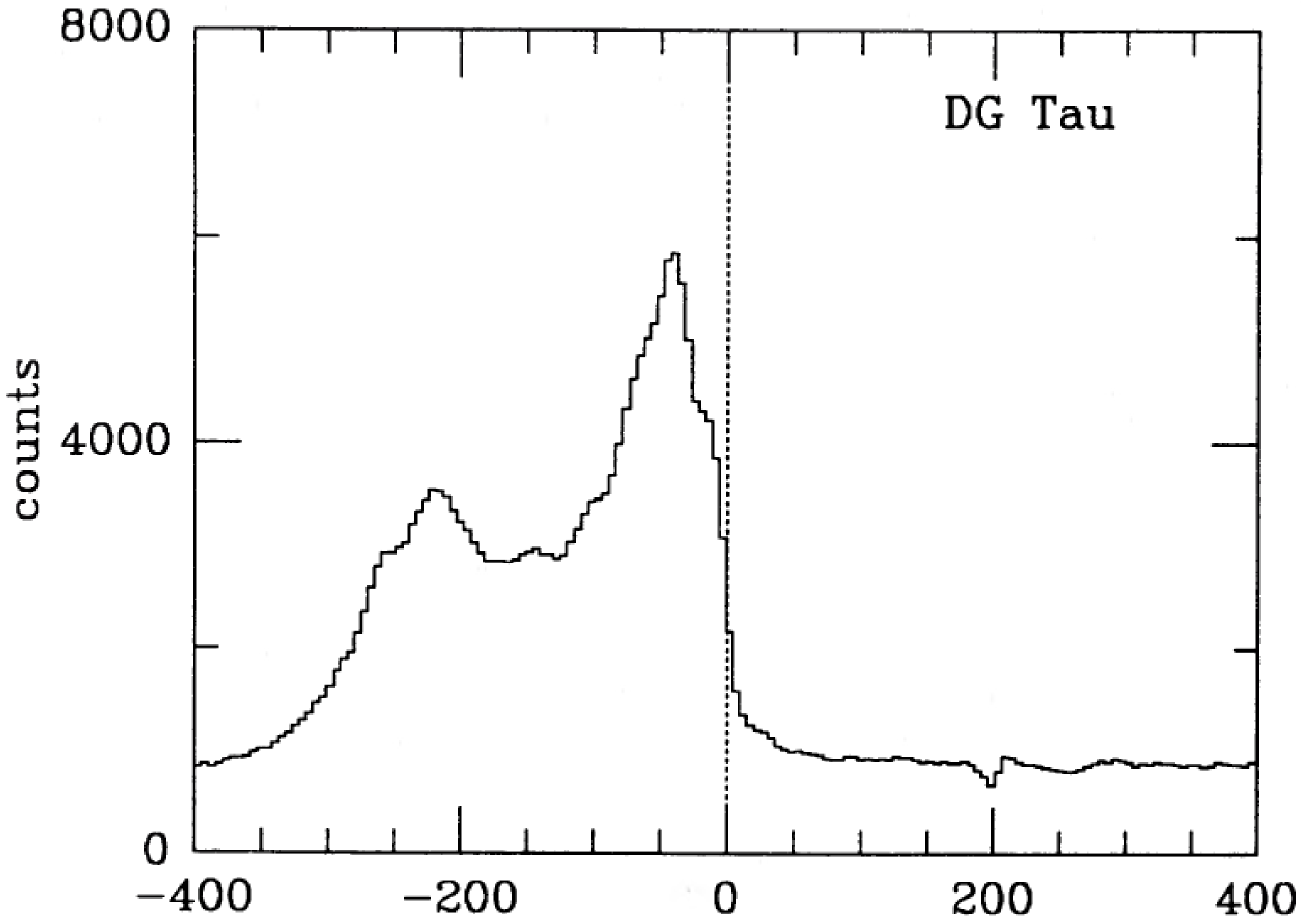
Figure 21.5 FU Orionis, Hα appearing as P Cygni profile and massively broadened, fully saturated Na I lines – clear evidences for a strong outflowing wind. Li I absorption is evidence for a very young object, SQUES echelle spectrograph. Hα and Na I lines, SQUES, slit width 70 μm, 2 × 3600 s, 2 × 2 binning. Li I line, SQUES, slit width 85 μm, 2 × 3600 s, 3 × 3 binning

P Cygni stars

- Higher mass-loss rate, $> \ 10^{-5}\ \mathrm{M_\odot\ yr^{-1}}$
- Lower terminal velocity, $v_\infty < 10^{2.5}\ \mathrm{km\ s^{-1}}$
- Higher wind density, $n_H > 10^{10}\ \mathrm{cm^{-3}}$ at $2\ R_*$

than normal stars (Lamers 1986).

The [O I]6300 profile of a T Tauri star; blueshifted wind



Inference:
the redshifted emission is blocked by an optically thick dusty disk

Hartmann & Raymond (1989)

Herbig-Haro objects: shocked excited nebulosity by young stars

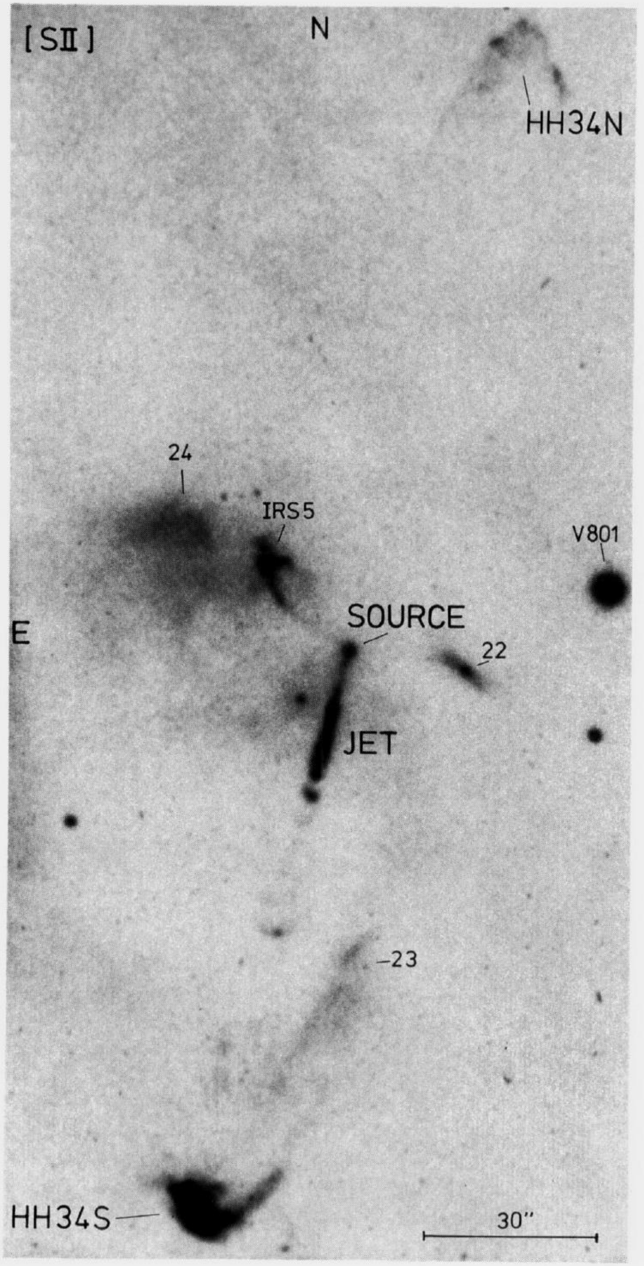
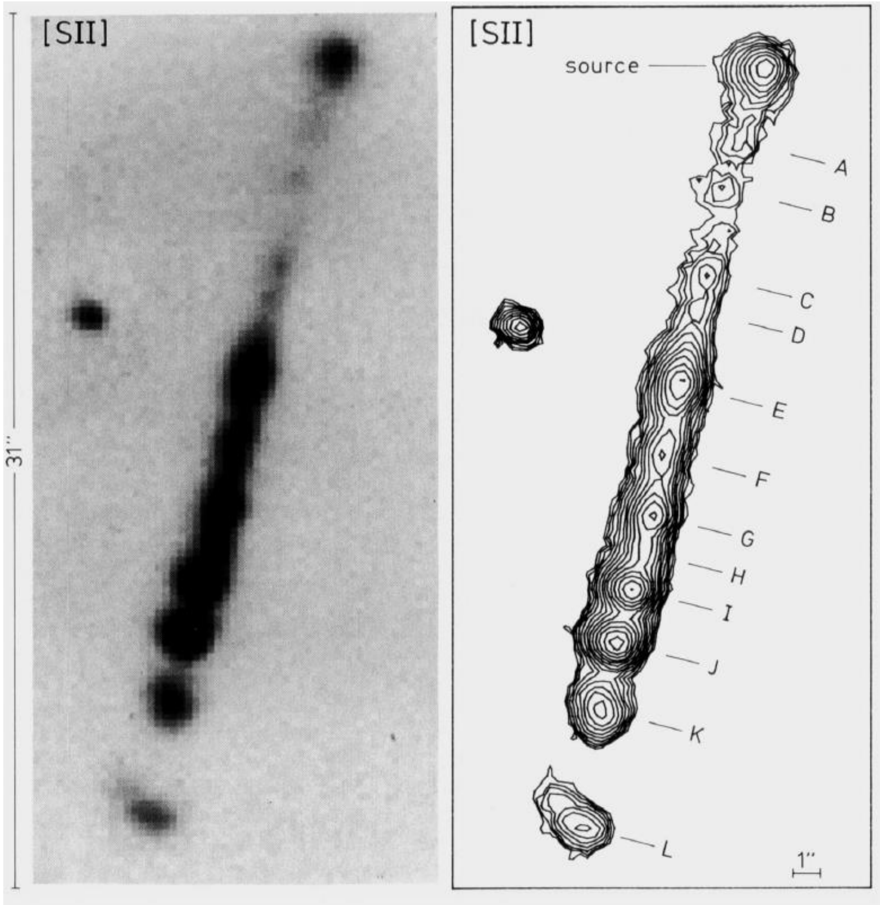


Fig. 1. A montage of two CCD-images of the HH 34-region taken through a [S II] λλ6716/6731 Å filter. The nebulous objects in this field are labeled according to Reipurth (1985b)



Bührke & Mundt (1988)

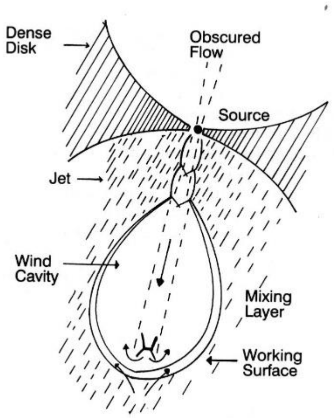


Figure 1. A diagram of a typical outflow from a young stellar object.

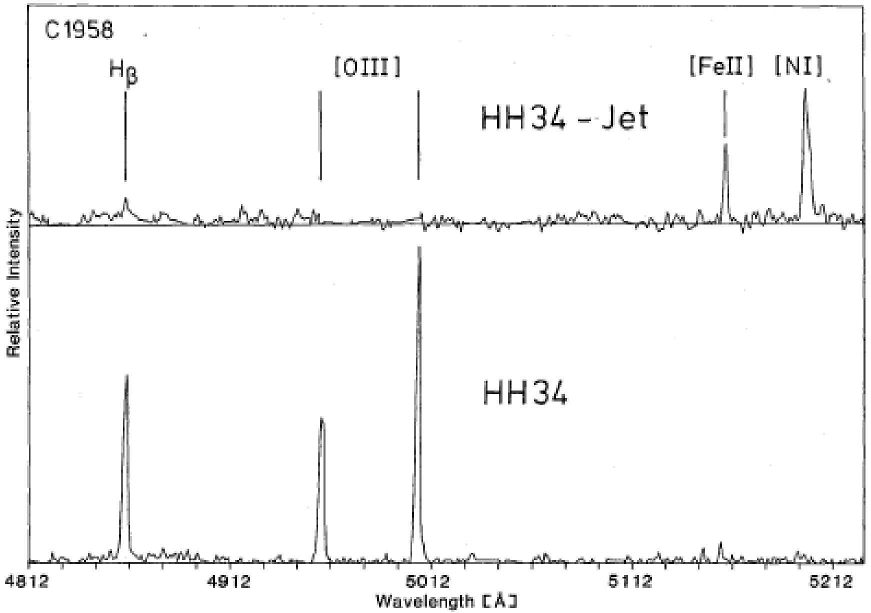
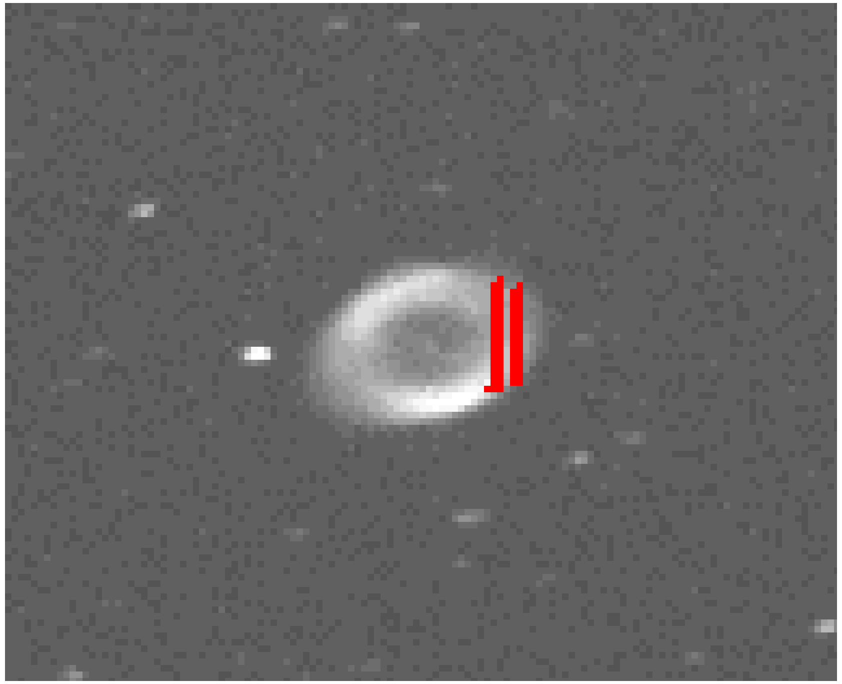


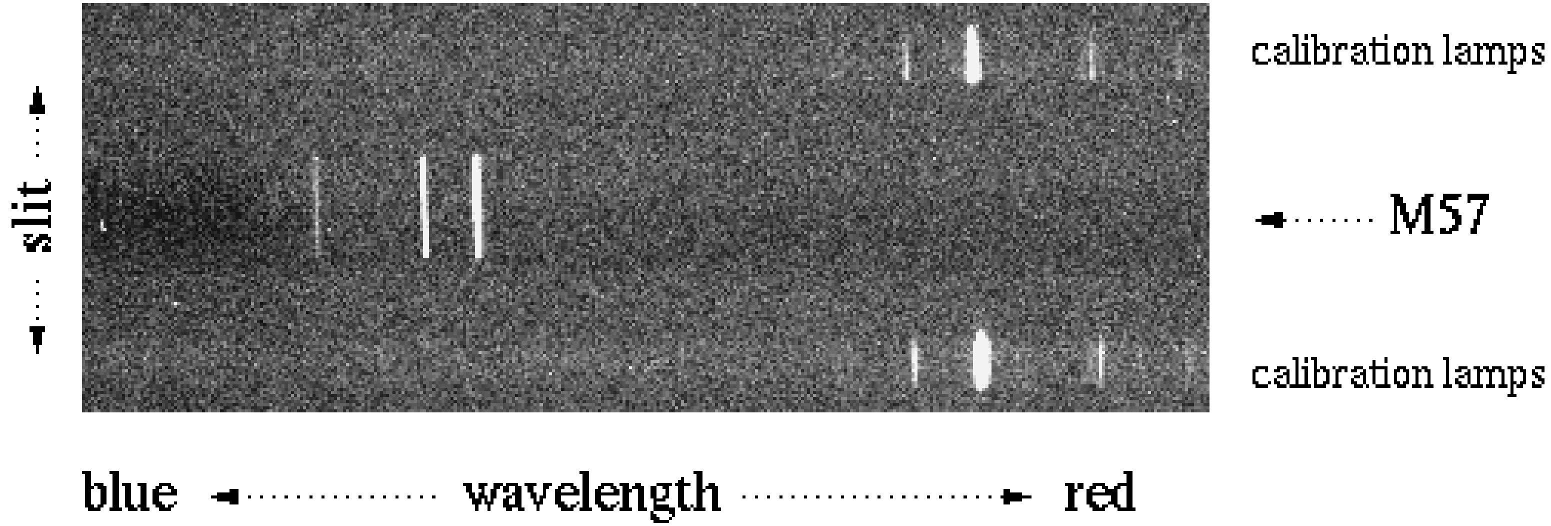
Fig. 5. Spectrum of the jet and HH 34S around the [O III]-lines, demonstrating the different excitation conditions

An example -----
Ring Nebula (M57), a planetary nebula

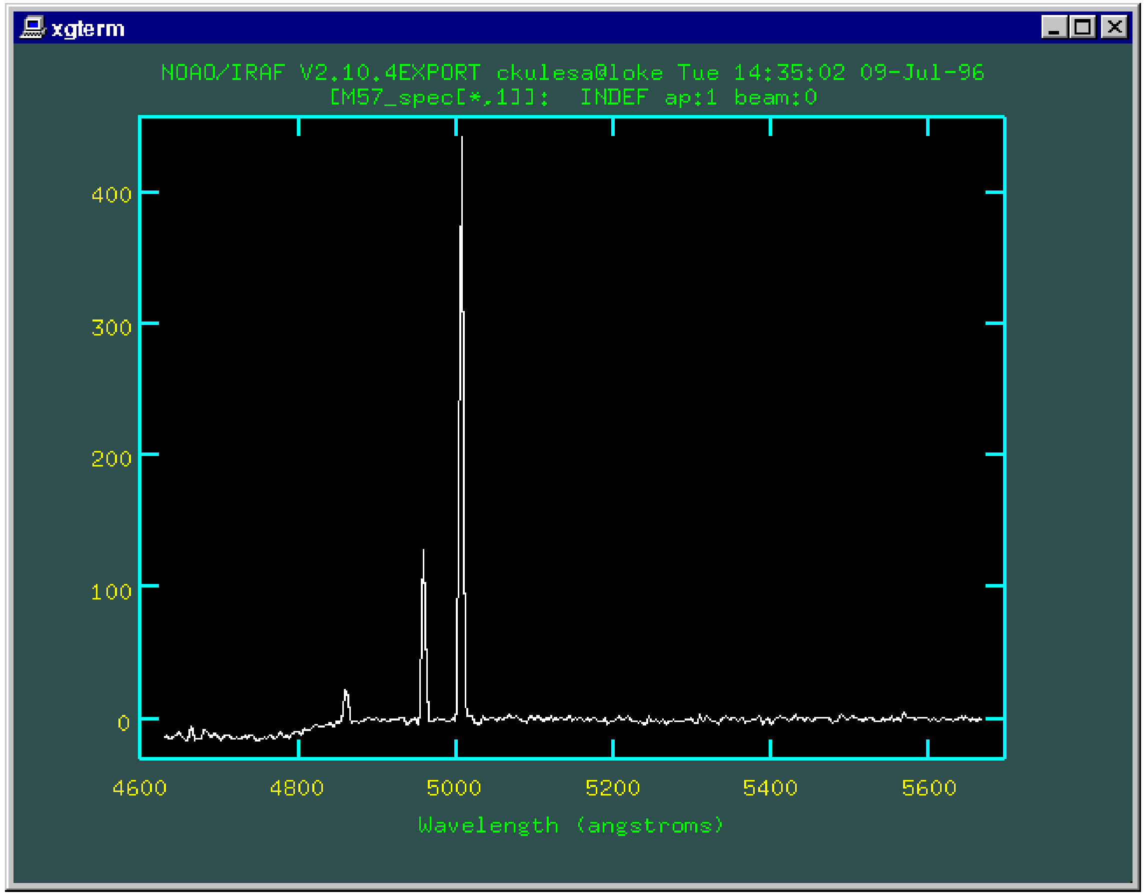


Slit = 8’ x 1”

Hg+He



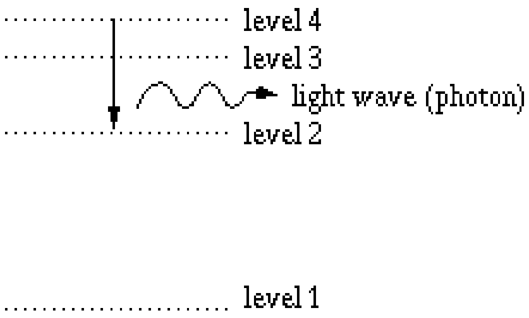
http://loke.as.arizona.edu/~ckulesa/camp/camp_spectroscopy.html



1-D spectrum shows little continuum, and a few emission lines

➔ **A line spectrum**

4959Å and 5007Å doublet from twice-ionized oxygen, O++, or OIII in spectroscopic notation
➔ (oxygen) gas is ionized, with T > a few thousand K and density < $100/cm^3$



4861Å line from hydrogen
$n = 4 \rightarrow 2$
(called H_β line)
➔ gas is highly excited

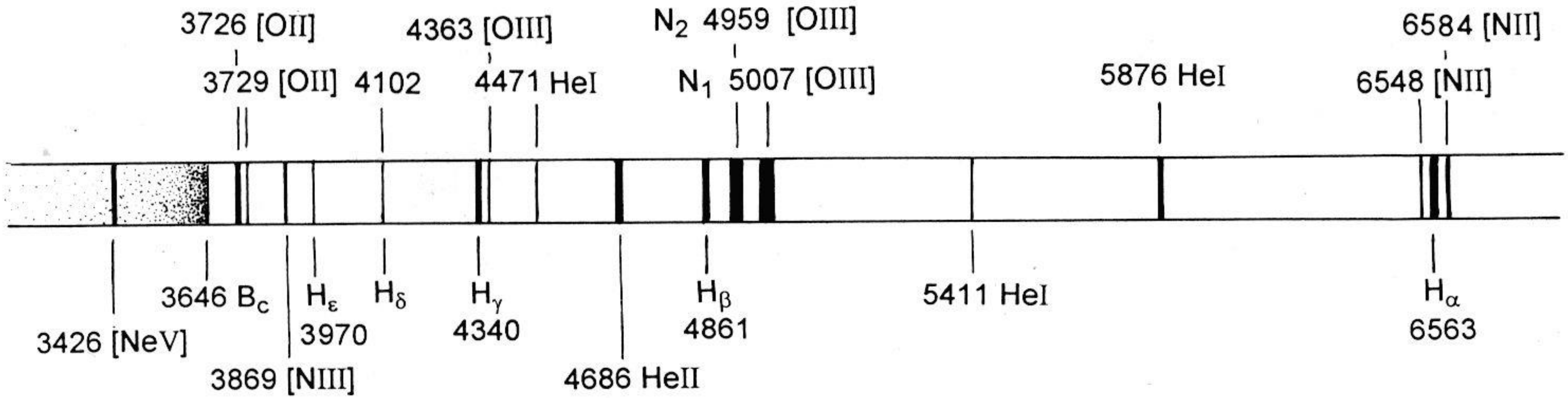
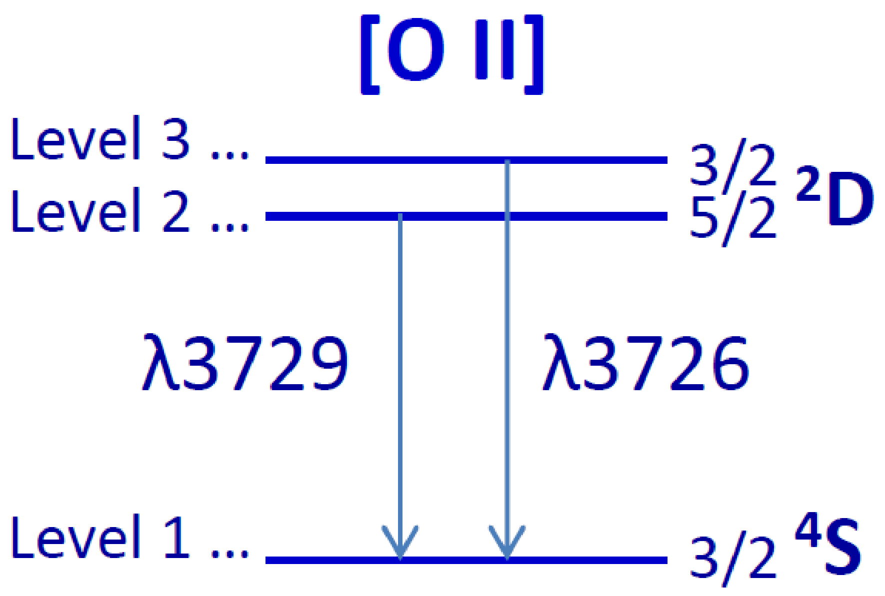


Fig. 1.1. General structure of the spectrum of a planetary nebula in the optical region, 3 300–7 000 Å. Only the most important emission lines, both permitted and forbidden, are shown. The shaded part from the left, beginning from $\lambda = 3\,646$ Å, is the Balmer continuum of hydrogen

Gurzadyan "PNe"

Excitation Theory --- Applications

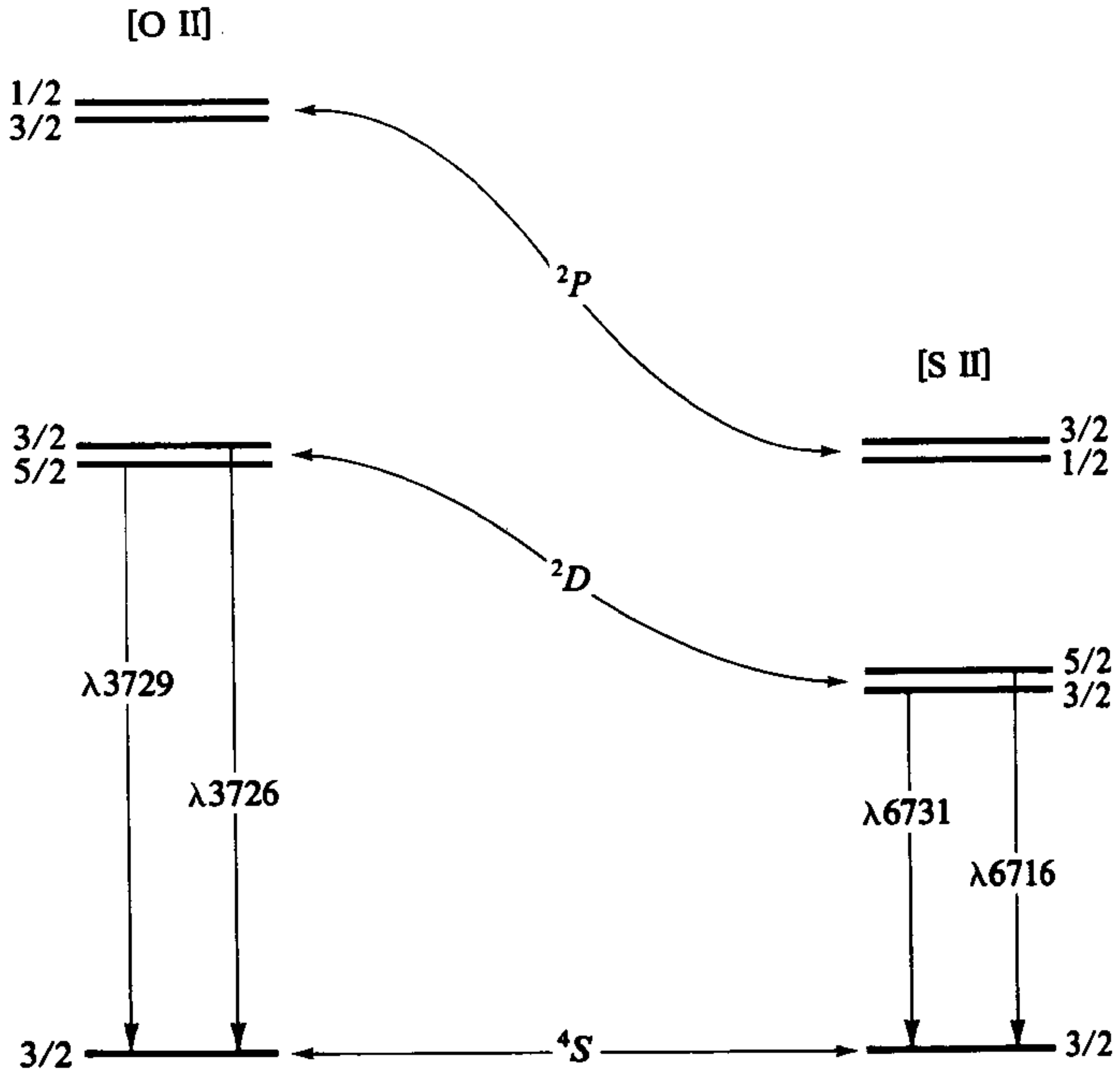


For [O II],

consider a 3-level system, with the two **upper** levels close together,

$$\frac{j_{\lambda 3729}}{j_{\lambda 3726}} = \frac{j_{21}}{j_{31}} = \frac{n_2 A_{21} h\nu_{21}}{n_3 A_{31} h\nu_{31}}$$

Note: $\Delta\lambda = 0.3$ nm → need high-dispersion spectroscopy


[O II]
1/2
3/2
3/2
5/2
λ3729
λ3726
3/2
^2P
^2D
^4S
[S II]
3/2
1/2
5/2
3/2
λ6731
λ6716
3/2

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$$\frac{j_{\lambda 3729}}{j_{\lambda 3726}} = \frac{j_{21}}{j_{31}} = \frac{n_2 A_{21} h\nu_{21}}{n_3 A_{31} h\nu_{31}}$$

✓ $n_e \to \infty$, collisional excitation and deexcitation dominate

$$\frac{j_{21}}{j_{31}} = \frac{g_2 A_{21} \nu_{21}}{g_3 A_{31} \nu_{31}}\, e^{-E_{23}/kT} \approx \frac{g_2 A_{21}}{g_3 A_{31}} = \frac{6}{4}\, \frac{3.6\times 10^{-5}}{1.8\times 10^{-4}} = 0.3$$

Note: statistical weight $g = 2J + 1$

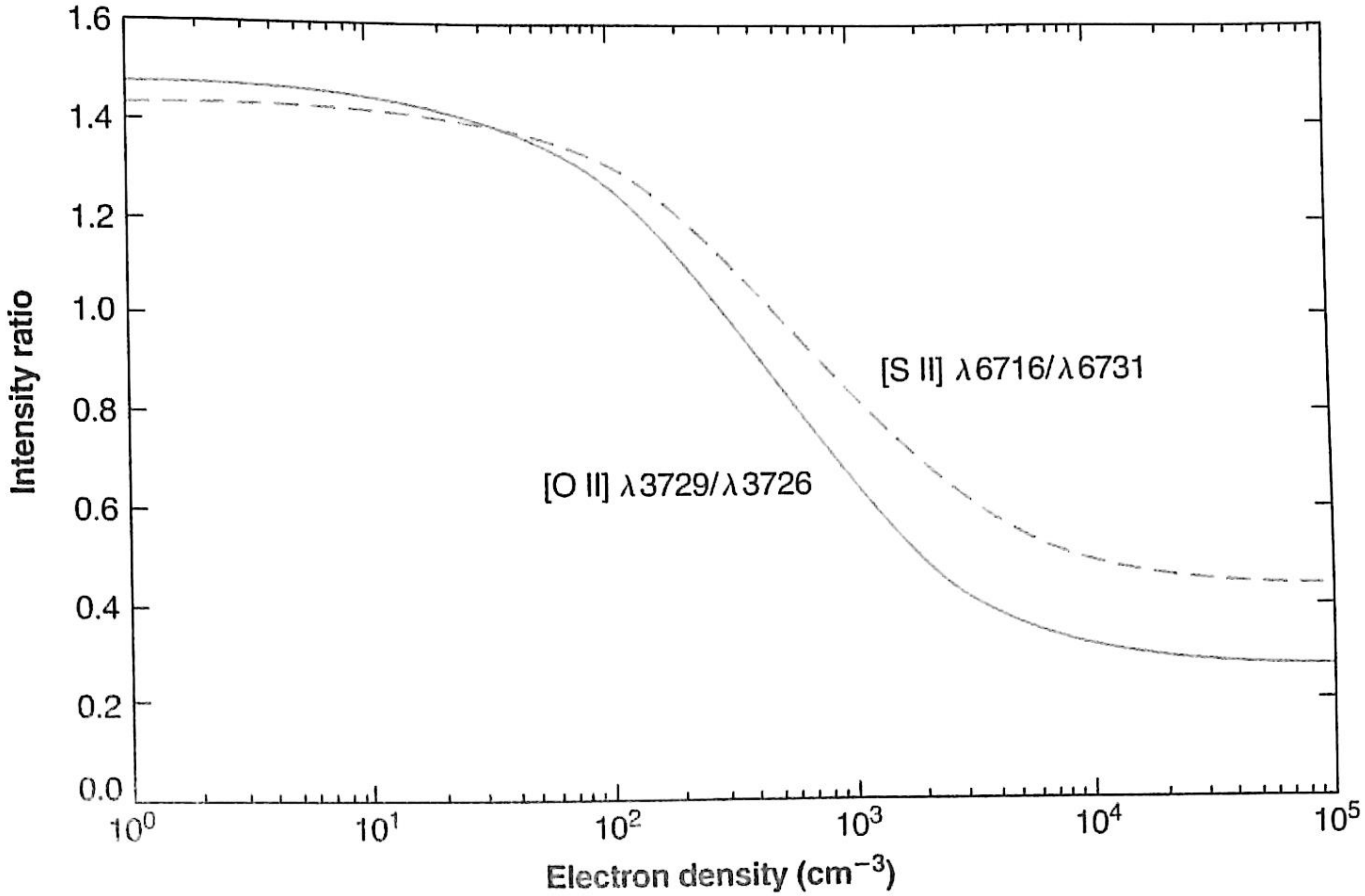
✓ $n_e \to 0$, every collisional excitation followed by emission

$$\frac{j_{21}}{j_{31}} = \frac{\gamma_{12}}{\gamma_{13}} = \frac{g_2}{g_3}\, e^{-E_{23}/kT} \approx \frac{g_2}{g_3} = \frac{6}{4} = 1.5$$

Because $\gamma_{21} \approx \gamma_{12}$, and $E_{23} \ll kT$

Transition of density limits occurs $n_{e,2} \approx 3 \times 10^3\ \mathrm{cm}^{-3}$; $n_{e,3} \approx 1.4 \times 10^4\ \mathrm{cm}^{-3}$

So this kind of level configuration (upper close), the line ratio is **sensitive to the electron number density**.



Similar pairs of lines

[O II]

[S II]

[N I]

[C III]

[Ar IV]

[K V]

[Ne IV] λ2422, 2424

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Some examples of density determinations for H II regions

TABLE 5.6
Electron densities in H II *regions*

Object	$\frac{I(\lambda3729)}{I(\lambda3726)}$	$N_e(\text{cm}^{-3})$
NGC 1976 A	0.50	3.0×10^3
NGC 1976 M	1.26	1.4×10^2
M 8 Hourglass	0.65	1.5×10^3
M 8 outer	1.26	1.5×10^2
NGC 281	1.37	7×10
NGC 7000	1.38	6×10

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For planetary nebulae

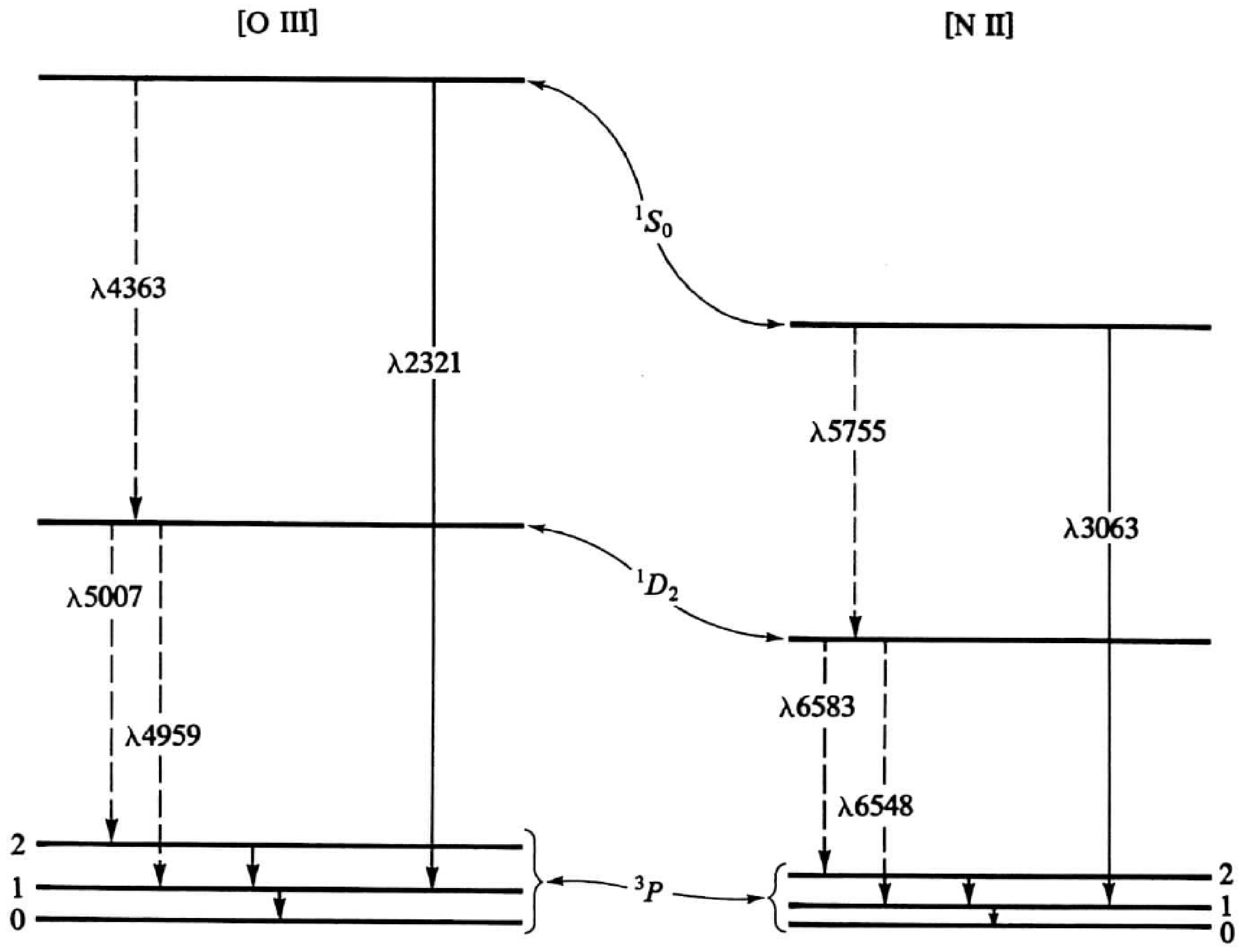
Osterbrock

TABLE 5.7
Electron densities in planetary nebulae

Nebula	[O II] $\lambda3729/\lambda3726$	[O II] N_e [a] (cm^{-3})	[S II] $\lambda6716/\lambda6731$	[S II] N_e [a] (cm^{-3})
NGC 40	0.78	1.1×10^3	0.69	2.1×10^3
NGC 650/1	1.23	2.1×10^2	1.08	4.0×10^2
NGC 2392	0.78	1.1×10^3	0.88	9.1×10^2
NGC 2440	0.64	1.9×10^3	0.62	3.2×10^3
NGC 3242	0.62	2.2×10^3	0.64	2.8×10^3
NGC 3587	1.30	1.4×10^2	1.25	1.8×10^2
NGC 6210	0.47	5.8×10^3	0.66	2.5×10^3
NGC 6543	0.44	7.9×10^3	0.54	5.9×10^3
NGC 6572	0.38	2.1×10^4	0.51	8.9×10^3
NGC 6720	1.04	4.7×10^2	1.14	3.2×10^2
NGC 6803	0.57	2.8×10^3	–	–
NGC 6853	1.16	2.9×10^2	–	–
NGC 7009	0.50	4.6×10^3	0.61	3.3×10^3
NGC 7027	0.48	5.2×10^3	0.59	4.0×10^3
NGC 7293	1.32	1.3×10^2	1.28	1.6×10^2
NGC 7662	0.56	3.0×10^3	0.64	2.8×10^3
IC 418	0.37	3.2×10^5	0.49	9.5×10^3
IC 2149	0.56	3.0×10^3	0.57	4.6×10^3
IC 4593	0.63	2.0×10^3	–	–
IC 4997	0.34	1.0×10^6	0.45	1.0×10^5

[a] N_e given for assumed $T = 10^4$ °K; for any other T divide listed value by $(T/10^4)^{1/2}$.

Now consider a different level configuration with [O III] or [N II], for which the two **lower** levels are close together.



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Rate of excitation to ^1S and ^1D levels $\Leftrightarrow T$

When $n \rightarrow 0$, i.e., collisional deexcitation is negligible

- Every excitation to $^1D \rightarrow \lambda 5007$ *or* $\lambda 4959$ (probability 3:1)
- Every excitation to $^1S \rightarrow \lambda 4363$ *or* $\lambda 2321$
  - ↳ $\lambda 5007$ or $\lambda 4959$

One can show that

$$I_{4959} \propto \gamma_{(^3P,^1D)} \frac{A_{(^1D,^3P_1)}}{A_{(^1D,^3P_2)} + A_{(^1D,^3P_1)}} h\nu_{4959}$$

$$I_{5007} \propto \gamma_{(^3P,^1D)} \frac{A_{(^1D,^3P_2)}}{A_{(^1D,^3P_2)} + A_{(^1D,^3P_1)}} h\nu_{5007}$$

$$I_{4363} \propto \gamma_{(^3P,^1S)} \frac{A_{(^1S,^1D)}}{A_{(^1S,^1D)} + A_{(^1S,^3P)}} h\nu_{4363}$$

So

$$\frac{j_{4959}+j_{5007}}{j_{4363}} = \frac{\Omega_{(^3P,^1D)}}{\Omega_{(^3P,^1S)}}\left[\frac{A_{(^1S,^1D)}+A_{(^1S,^3P)}}{A_{(^1S,^1D)}}\right]\frac{\bar{\nu}_{(^3P,^1D)}}{\nu_{4363}}\exp(\Delta E/kT)$$

$$\approx \frac{7.73\exp[(3.29\times 10^4)/T]}{1+4.5\times 10^{-4}(N_e/T^{1/2})} = \frac{7.15}{1+0.0028\,x}\,10^{14300/T_e}$$

where

$$\bar{\nu} = \frac{A_{(^1D,^3P_2)}\,\nu_{5007}+A_{(^1D,^3P_1)}\,\nu_{4959}}{A_{(^1D,^3P_2)}+A_{(^1D,^3P_1)}}$$

$$x = \frac{0.01\,n_e}{\sqrt{T_e}}$$

and ΔE is the energy difference between ^1D and ^1S.

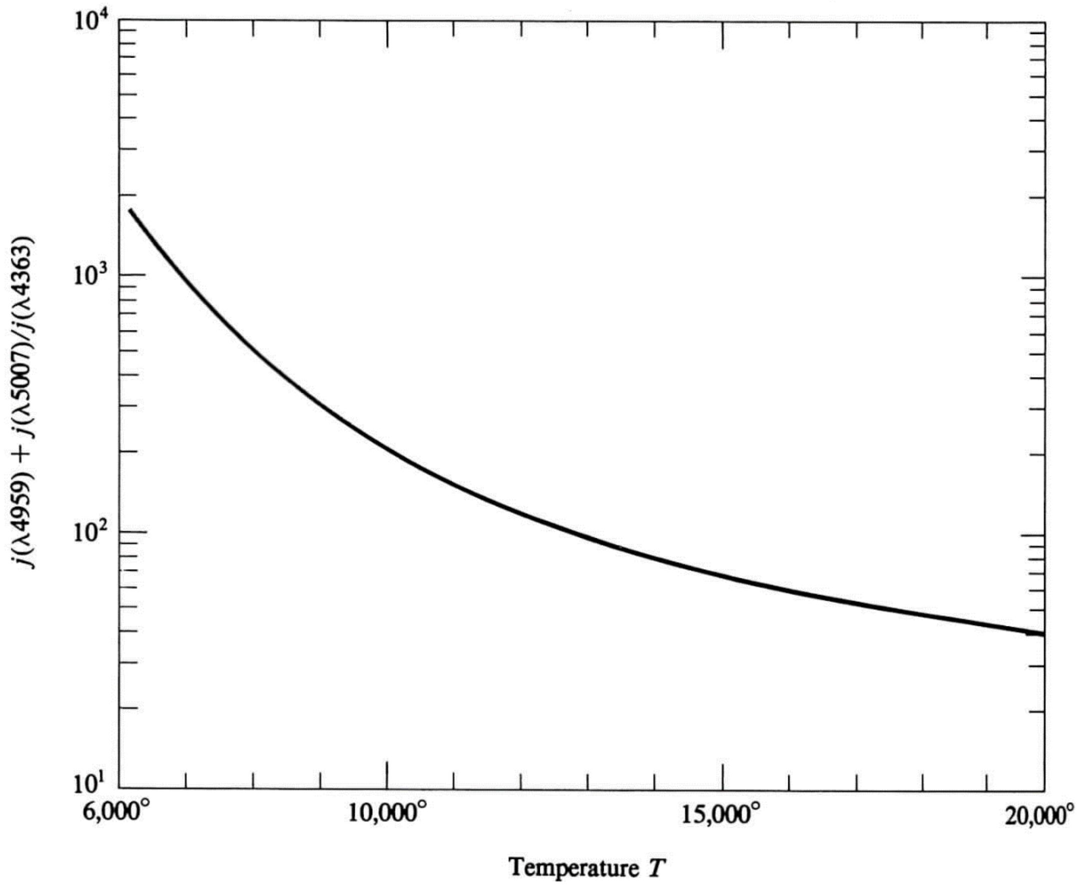
This holds up to $n_e \approx 10^5$ cm^{-3}.

At higher densities, collisionalde-excitation begins to play a role.

Similarly, for [N II],

$$\frac{j_{6548}+j_{6583}}{j_{5755}} \approx \frac{6.91\exp[(2.50\times 10^4)/T]}{1+2.5\times 10^{-3}(N_e/T^{1/2}).} = \frac{8.5}{1+0.29\,x}\,10^{10800/T_e}$$

So with this kind of level configuration (lower close; [O III] or [N II]), the line ratio is **sensitive to temperature**.



Difficulties:

1. I_{4959} and I_{5007} are strong but I_{4363} is weak
2. I_{4363} is close to Hg I λ4358 (sky!)

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Temperature determinations for H II regions

TABLE 5.1
Temperature determinations in H II regions

Nebula	[N II] $\frac{I(\lambda 6548)+I(\lambda 6583)}{I(\lambda 5755)}$	[N II] $T(°\ K)$	[N II] $N_e/T^{1/2}$	[O III] $\frac{I(\lambda 4959)+I(\lambda 5007)}{I(\lambda 4363)}$	[O III] $T(°\ K)$
NGC 1976 2b	81	10,000	51	338	8,700
NGC 1976 1a	102	9,100	68	371	8,500
NGC 1976 5b	111	8,900	21	310	8,900
NGC 1976 5a	189	7,500	12	263	9,300
M 8 I	162	7,900	(10)	445	8,100
M 17 I	257	6,900	(10)	330	8,700
NGC 2467 1a	46	13,000	(1)	129	11,600
NGC 2467 1b	53	12,200	(1)	137	11,400
NGC 2359 av	—	—	(1)	90	13,200

For PNe

TABLE 5.2
Temperature determinations for planetary nebulae

Nebula	T[N II] (° K)	T[O III] (° K)
NGC 650	9,500	10,700
NGC 4342	10,100	11,300
NGC 6210	10,700	9,700
NGC 6543	9,000	8,100
NGC 6572	—	10,300
NGC 6720	10,600	11,100
NGC 6853	10,000	11,000
NGC 7027	—	12,400
NGC 7293	9,300	11,000
NGC 7662	10,600	12,800
IC 418	—	9,700
IC 5217	—	11,600
BB 1	10,500	12,900
Haro 4-1	—	12,000
K 648	—	13,100

Typically T~10,000 K

THE ASTROPHYSICAL JOURNAL, **308**:322–336, 1986 September 1

ELECTRON TEMPERATURES IN PLANETARY NEBULAE

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Received 1985 November 4; accepted 1986 February 12

ABSTRACT

Electron temperatures for 107 planetary nebulae are calculated with the most recent atomic parameters from [O III] or [N II] line intensities or both taken from a variety of sources. The two temperatures exhibit quite different variations with respect to nebular ionization level, or excitation. Within somewhat broad limits, T_e[O III] can be taken as constant at 10,200 K for nebulae without He II λ4686; with the onset of that line, this temperature quickly climbs according to T_e[O III] = 9700 K + 58I(λ4686), where the line intensity is scaled as usual to $I(\mathrm{H}\beta) = 100$. T_e[N II] behaves oppositely. With λ4686 present, there is little discernable trend with excitation around a median value of 10,300 K; as the excitation drops and λ4686 disappears, this temperature appears first to increase, and then to decrease to values well below 8000 K: for log T_* (central star temperature) <4.7, T_e[N II] = 14,670 log T_* − 57,330. The dispersion in T_e for a specific excitation correlates negatively with O/H as expected.

Combination of the [O III] and [N II] data sets shows that the mean ratio of T_e[N II]/T_e[O III] = $\bar{r}$ varies smoothly and strongly also as a function of overall nebular excitation. As excitation increases from $T_* \approx$ 25,000 K to ~50,000 K, $\bar{r}$ increases from ~0.7 to ~1.1. It then decreases through the onset of He^{+2}, dropping to 0.7 again for the highest levels of ionization, that is, the nebular temperature gradient as inferred from O^{+2} and N^{+} is usually negative with respect to distance from the central star but reverses to positive for nebulae in the midrange of excitation for $T_* \approx$ 50,000 K.

Comparison of [O III] temperatures among major reference sources shows clear systematic differences. The observations by French and by Torres-Peimbert and Peimbert yield the highest values, roughly 1000 K higher than those obtained from Aller and Czyzak and from Barker. No such trends are seen for T_e[N II], possibly because the scatter in the data is considerably larger.

Read the paper by Donald Menzel

PHYSICAL PROCESSES IN GASEOUS NEBULAE

I. ABSORPTION AND EMISSION OF RADIATION

DONALD H. MENZEL

ABSTRACT

In this paper, the first of a series dealing with the physical state of gaseous nebulae, various fundamental formulae are derived. The total emission and absorption of radiation by atomic hydrogen are evaluated, together with the number of transitions to and from any quantum level, discrete or continuous. The equations are thrown into simple homogeneous form. The general equations that determine the statistical equilibrium of the assembly and the partition of atoms into various atomic states are developed. Solution of these equations is deferred until a later paper.

The Interstellar Medium --- HW20220331

due in two weeks

1. Consider a speck of spherical dust grain of a radius a and at a distance of d from a star with a surface temperature of T_* and a stellar radius of R_*. (a) Find the equilibrium temperature T_d of the grain. (b) Plot T_d as a function d. (c) Now replace the dust with the Earth, still at d from the same star, and estimate $T_\oplus$.

2. As in the last question, compute now the temperatures of the 8 planets in the solar system and our Moon versus their distances. Make a plot to show this and mark in the plot the actual average temperature of each object. Comment on possible discrepancies.

3. (extra credit) Find the ‘habitable zone’ of Vega.