Recall continuum radiation

Absorption --- bound-free (ionization); free-free

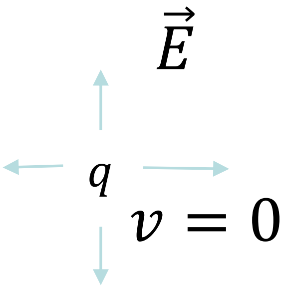
Emission --- (**thermal**) blackbody; bremsstrahlung

(**non-thermal**) synchrotron; Cherenkov

If $v = 0 \rightarrow$ starionary $\vec{E}$ field

If $v = \text{const} \rightarrow$ current $\vec{J} \rightarrow \vec{B}$

If $\frac{d\vec{v}}{dt} \neq 0 \rightarrow$ varying $\vec{E}$, varying $\vec{B} \implies$ EM waves



If q accelerated by B

✓ $\beta \equiv v/c \ll 1 \rightarrow$ cyclotron rad, $h\nu_{\text{giration}}$; $\omega_B = \frac{qB}{\gamma mc}$

✓ $\beta \approx 1 \rightarrow$ synchrotron rad, $\boldsymbol{P} = 4\,\sigma_T \mathrm{c}\,\beta^2\gamma^2\,U_B$

$\gamma = 1/\sqrt{1 - v^2/c^2}$; $\sigma_T = 8\pi \mathrm{r}_0^2/3$; $U_B = \mathrm{B}^2/8\pi$

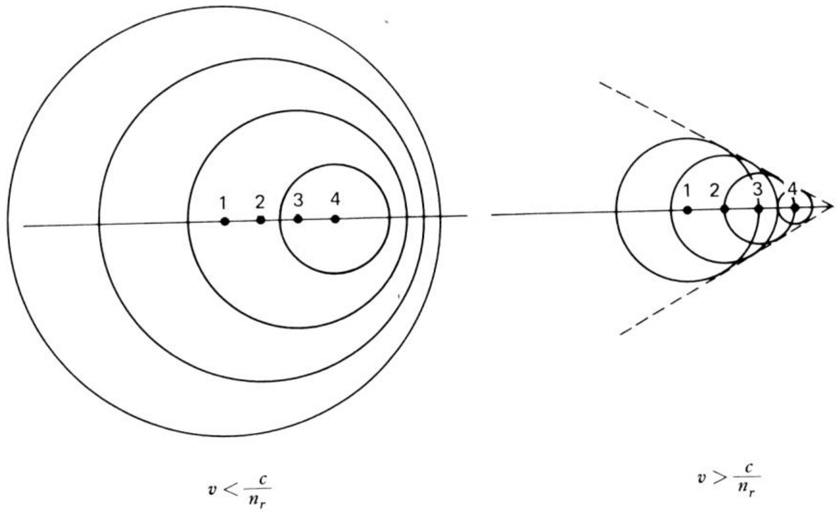
Particle-Photon Scattering

- Photon → (momentum to) a charged particle
  Thomson scattering
- High-energy photon → (momentum to) a charged particle
  Compton scattering

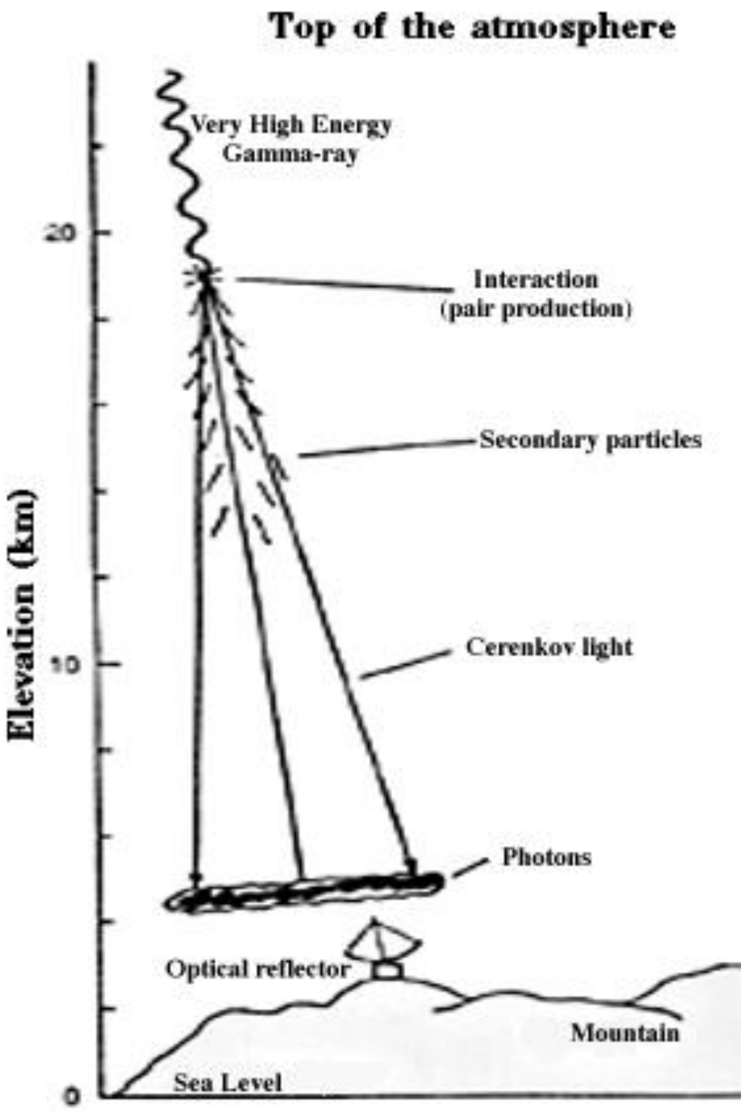
A high-energy charged particle → (momentum to) a photon
  Inverse Compton scattering

Čerenkov Radiation

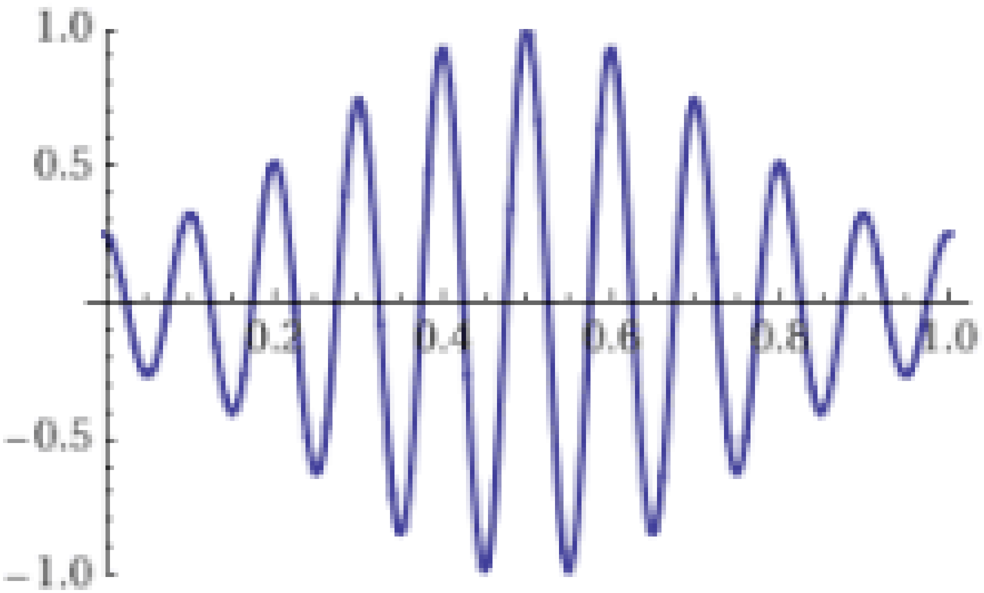
- A charge particle (e.g., an electron) moving uniformly in a vacuum does not radiate.
- Also true if moving uniformly in a dielectric medium, if the velocity is less than the phase velocity of light in the medium.
- When $c/n_r < v < c$ $\longrightarrow$ Cherenkov radiation (e.g., due to energetic CR in the earth's atmosphere)



Consider a sine wave $A\cos(kx - \omega t)$ has the phase velocity $v_p = \frac{\omega}{k}$, whereas the group velocity is $v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k}$



http://www.astro.wisc.edu/~larson/Webpage/Gamma.html



The phase velocity, $v_p = \omega/k$,
e.g., in a wave A $\sin(kx - \omega t)$

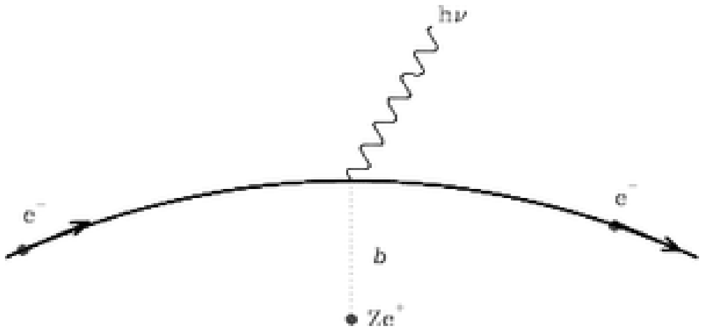
The group velocity, $v_p = \partial\omega/\partial k$
The refractive index $n = c/v_p$
$\omega(k)$ is the dispersion relation

This shows a wave with the group velocity and phase velocity going in different directions. The group velocity is positive, while the phase velocity is negative.

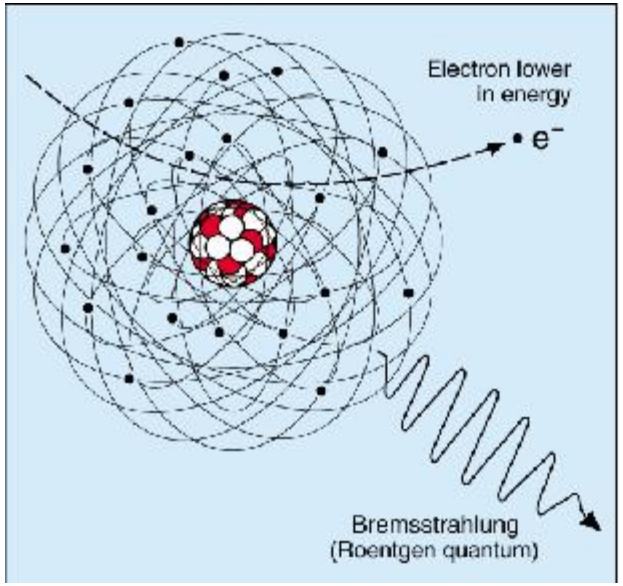
https://en.wikipedia.org/wiki/Phase_velocity

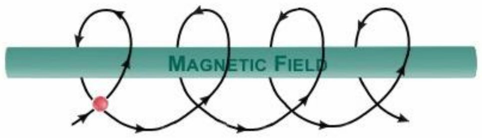
Thermal Emission

Hot, low-density plasma (e.g., HII regions, PNe, SNRs)
→ thermal radio emission

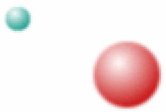


http://www.astro.utu.fi/~cflynn/





c.f.
Synchrotron
Radiation



Emission from Single-speed Electrons

Assuming electrons move rapidly, so nearly in a straight line
→ small-angle scattering

Energy radiated per unit time per unit volume per frequency range is $\frac{dE}{d\omega\, dV\, dt} = \frac{16e^6}{3c^3m^2}\frac{1}{v}\, n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$

After evaluating $b_{\max}$ and $b_{\min}$, one gets

$$\frac{dE}{d\omega\, dV\, dt} = \frac{16\pi e^6}{3\sqrt{3}c^3m^2}\frac{1}{v}\, n_e n_i Z^2\, g_{ff}$$

where g_{ff} is the Gaunt (correction) factor.

Rybicki & Lightman

Thermal Bremsstrahlung Emission

Now consider an assemble of particles; i.e., the average over a thermal distribution of (Maxwellian) speeds.

Emission spectrum

$$j_\nu^{ff} = 6.8 \times 10^{-38} \frac{Z^2 n_e n_i}{\sqrt{T}} \; e^{-h\nu/kT} \, \overline{g}_{ff} \quad [\mathrm{erg\ s^{-1}\ cm^{-3}\ Hz^{-1}}]$$

where $\overline{g}_{ff}$ is the velocity-averaged Gaunt factor ~1.

Total Emission Power (Volume Emissivity)

$$j^{ff} = 1.4 \times 10^{-27} \frac{Z^2 n_e n_i}{\sqrt{T}} \, \overline{g}_B \quad [\mathrm{erg\ s^{-1}\ cm^{-3}}]$$

where $\overline{g}_B$ is the frequency-averaged Gaunt factor ~1.1 to 1.5.

Emission line observations suggest $T_e \approx 10^4$ K, so in radio wavelengths, $h\nu \ll kT$, and $B_\nu \approx 2k\,T\nu^2/c^2$

For free-free emission or bremsstrahlung (breaking radiation)

$$j_\nu \approx \frac{n_e n_i}{T_e^{1/2}} \implies \boxed{\kappa_\nu = \frac{j_\nu}{B_\nu} \approx \frac{n_e n_i}{T_e^{3/2}\ \nu^2}}$$

Considering also the Gaunt factor, $g_{ff} \propto \ln(T/\nu)$

$$\boxed{\tau = \int \kappa_\nu\, ds = \frac{8.235 \times 10^{-2}}{T_e^{1.35}\ \nu_{\mathrm{GHz}}^{2.1}}\ \mathrm{EM}}$$

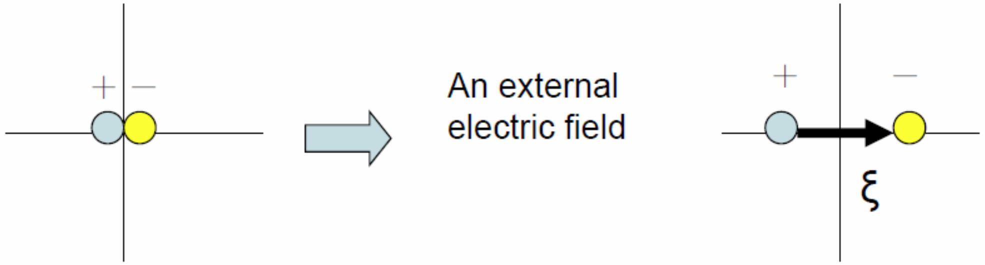
where $\mathrm{EM} = \int n_e n_i\, ds$ [cm^{-6} pc] is the **emission measure**.

$\tau \searrow$ at higher ν (or long λ)

Plasma is more transparent at higher frequencies.

Bright HII regions $n_e \approx n_p \approx 10^3 \ \mathrm{cm}^{-3}$ over a linear scale of $\Delta s \approx 1 \ \mathrm{pc} \ \longrightarrow \ \mathrm{EM} \approx 10^6 \ \mathrm{cm}^{-6} \ \mathrm{pc}$

Plasma Frequency



The dipole moment $\vec{P} = -ne\vec{\xi}$,
points from (−) to (+); n is electron density

Plasma dielectric constant σ;

$\vec{J} = \sigma\vec{E}$

Net charge = 0, so $\vec{D}$(electric displacement) = 0

$\vec{D} = \vec{E} + 4\pi\,\vec{P}$, so $\vec{E} = 4\pi\,ne\vec{\xi}$ (In MKS, $\vec{D} = \epsilon_0\vec{E} + \vec{P}$)

In 1-D, $E_x = 4\pi\,ne\,\xi$, and the force is $F_x = -eE_x$

The equation of motion, $m\dfrac{d^2\xi}{dt^2} = -eE_x = 4\pi\,ne^2\,\xi$

To solve this, use the operator $\dfrac{\hat{d}}{dt} = j\omega$, so $-m\omega^2 = -4\pi\,ne^2$

$-m\omega^2 = -4\pi\, ne^2$

$$\omega_p = \sqrt{\frac{4\pi n e^2}{m}} = 5.6 \times 10^4 \sqrt{n} \text{ rad s}^{-1}$$

(In MKS, $\omega_p = \sqrt{\frac{n_e\, e^2}{\epsilon_0\, m_e}}$)

or

$$\boxed{\nu_p = 8.97 \times 10^3 \sqrt{n} \text{ Hz}}$$

This is the **plasma frequency**

Index of refraction $\boxed{n_r^2 = 1 - \frac{\omega_p^2}{\omega^2},}$ for which the wave number

$$k = 2\pi/\lambda = 2\pi/(v/\nu) = 2\pi T/v = \omega/(c/n_r) \qquad k = \frac{\omega}{c}(1 - \frac{\omega_p^2}{\omega^2})^{1/2}$$

In general EM waves $E \propto \; e^{j(kx-wt)}$ propagate in a plasma with

$$\boxed{k^2 - c^2 = \omega^2 - \omega_p^2}$$ **(dispersion relation)**

If $\omega > \omega_p$, k is real $\longrightarrow$ No attenuation; "*Electrons cannot follow*"

If $\omega < \omega_p$, k is imaginary $\longrightarrow$ "*Electrons resonate.*"

→ absorption → no propagation

In a plasma, the group velocity is

$$v_g = \frac{d\omega}{dk} = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}$$

This is the speed information can be transmitted.

$$n_r^2 = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2}$$

Because of the refractive effects, different frequencies → different wave speed → **interstellar scintillation**

cf terrestrial "seeing" and scintillation (twinkling stars)

- **Metal** --- $\nu_p \sim 10^{15}$ Hz, i.e., in UV region; so metal absorbs EM waves until UV, i.e., transparent to UV radiation
- **Earth's ionosphere** --- n_e depends on height; at H=300 km, $n_e \sim 10^5 \sim 10^6\ \mathrm{cm}^{-3}$, $\nu_p \sim 10^7$ Hz (λ~30 m)

  cf. shortwave radio signals are reflected by ionosphere.
  All high frequency band (HF) 3—30 MHz (100 to 10 m)
- **ISM** --- $n_e \sim$ a few, $\nu_p \sim 10^4$ Hz (very low), radio observations usually $\nu > 10^8\mathrm{Hz} \gg \nu_p \Longrightarrow$ transmission ok

Now back to radio thermal emission…

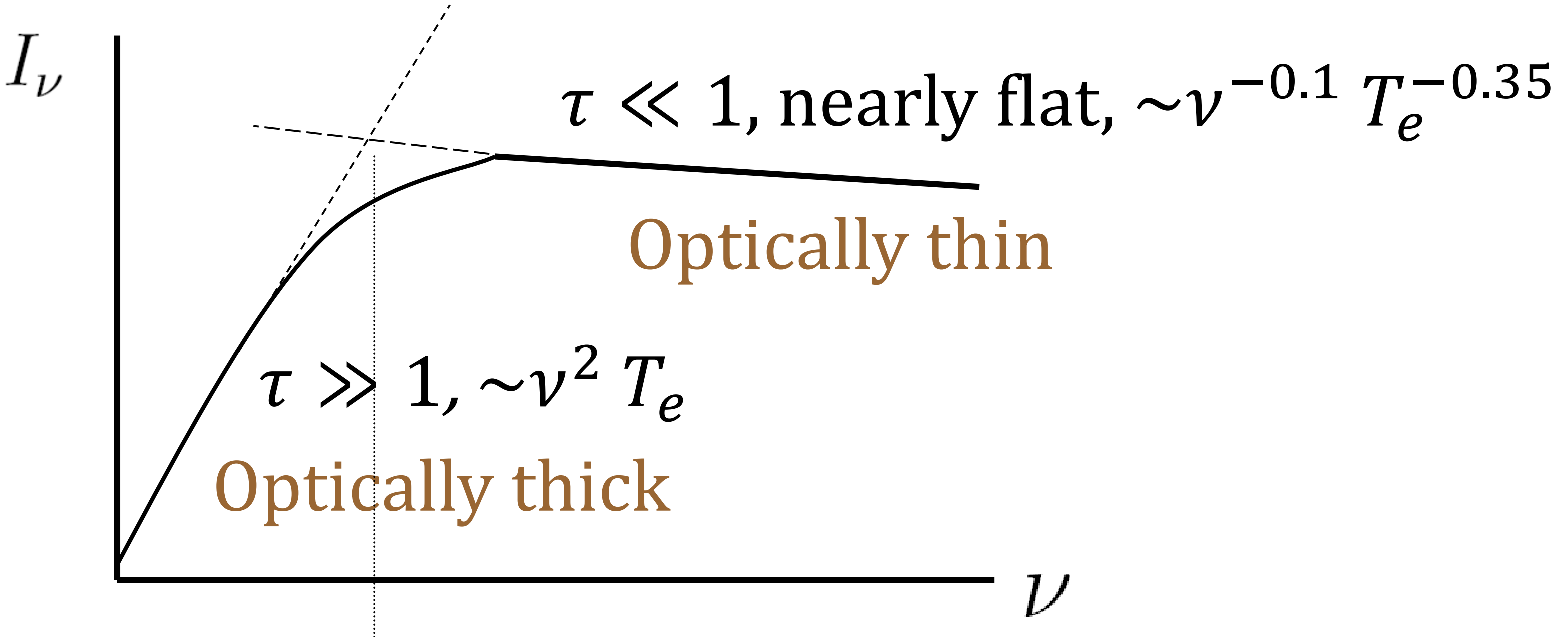
Recall $\tau = \int \kappa_\nu \, ds \propto \dfrac{\mathrm{EM}}{T_e^{1.5}\nu^2}$

✓ At high frequencies, $\tau \ll 1$,

$$I_\nu = B_\nu \tau \propto \frac{2kT\nu^2}{c^2}\frac{EM}{T^{3/2}\nu^2} \propto \frac{EM}{T^{1/2}} \quad \cancel{\longleftrightarrow} \quad \nu$$

✓ At low frequencies, $\tau \gg 1$,

$$I_\nu = B_\nu = \frac{2kT\nu^2}{c^2} \quad \text{Blackbody} \quad \cancel{\longleftrightarrow} \quad n_e$$



$$\tau = 1, \text{at } \nu_{\text{critical}} = 0.3\left(\frac{n_e^2 L}{T^{1.35}}\right) \text{ GHz}$$

Observations at $\tau \gg 1$, get T
Observations at $\tau \ll 1$, get n_e

Advantages of radio observations: less extinction; measures T directly at low freq.

In general, I_ν or $S_\nu \propto \nu^\alpha$, where α is the spectral index.

For H II regions, if optically thin $\alpha \approx 0$

if optically thick $\alpha \approx +2$

For synchrotron radiation, $S_\nu \propto \nu^{-0.8}$ or $T_\mathrm{B} \propto \nu^{-2.8}$

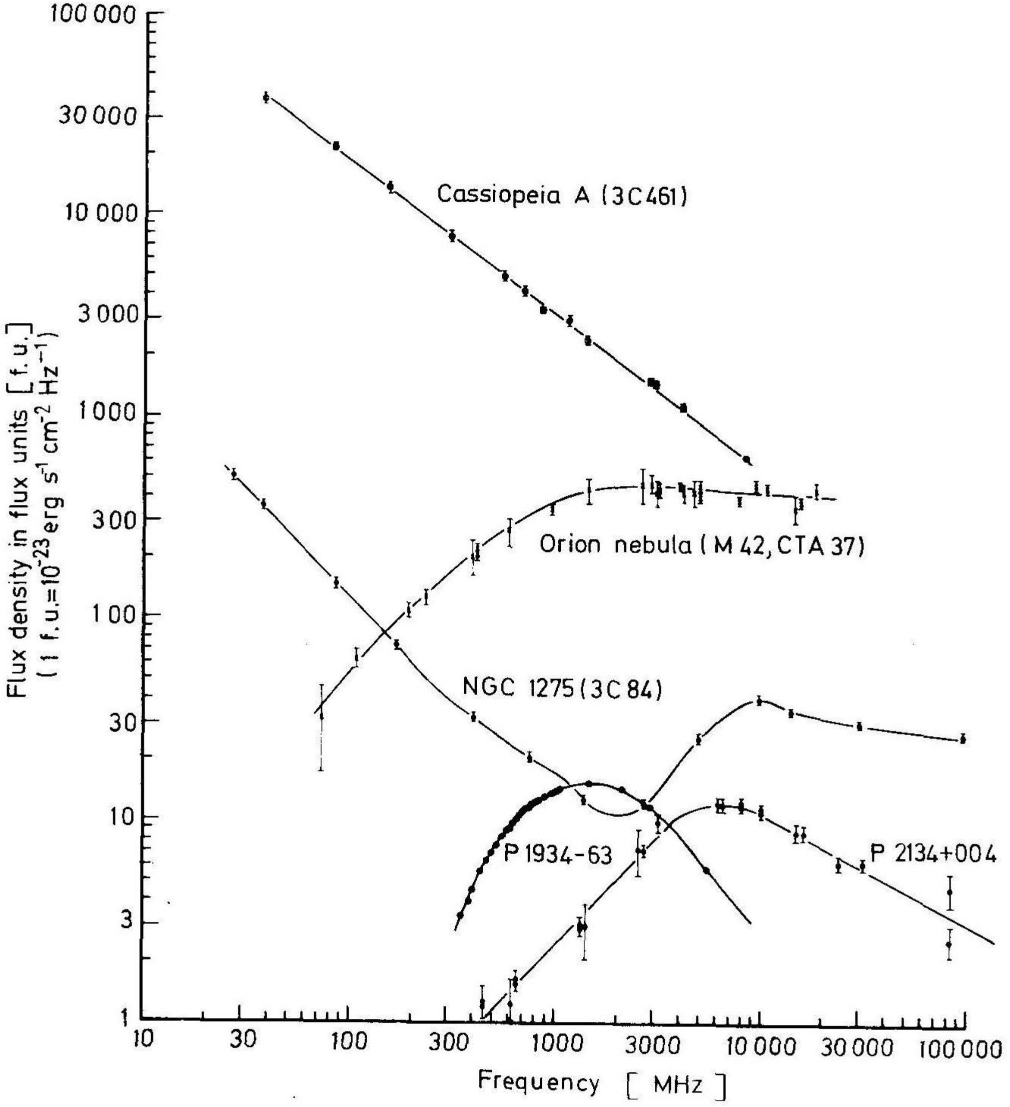
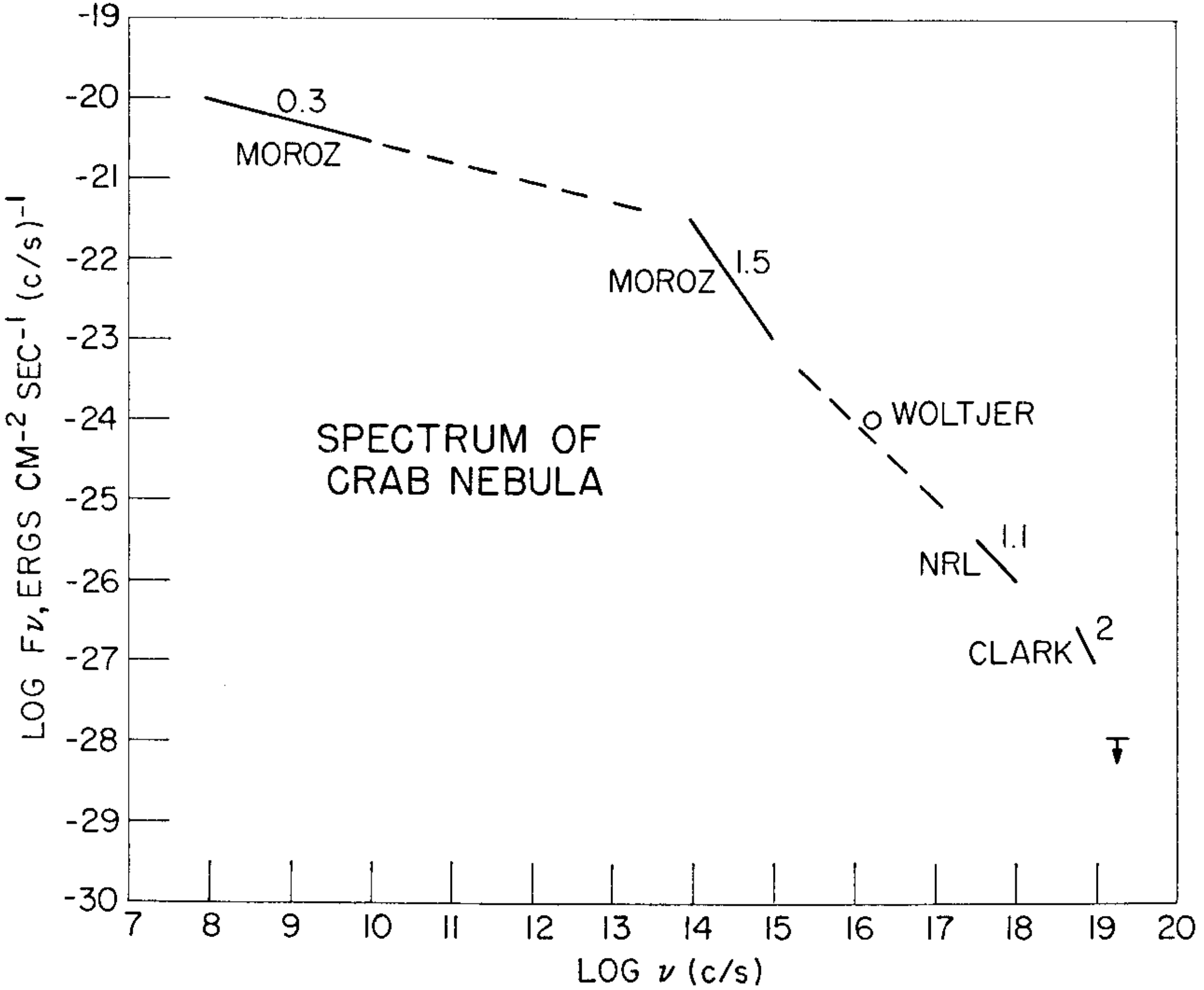


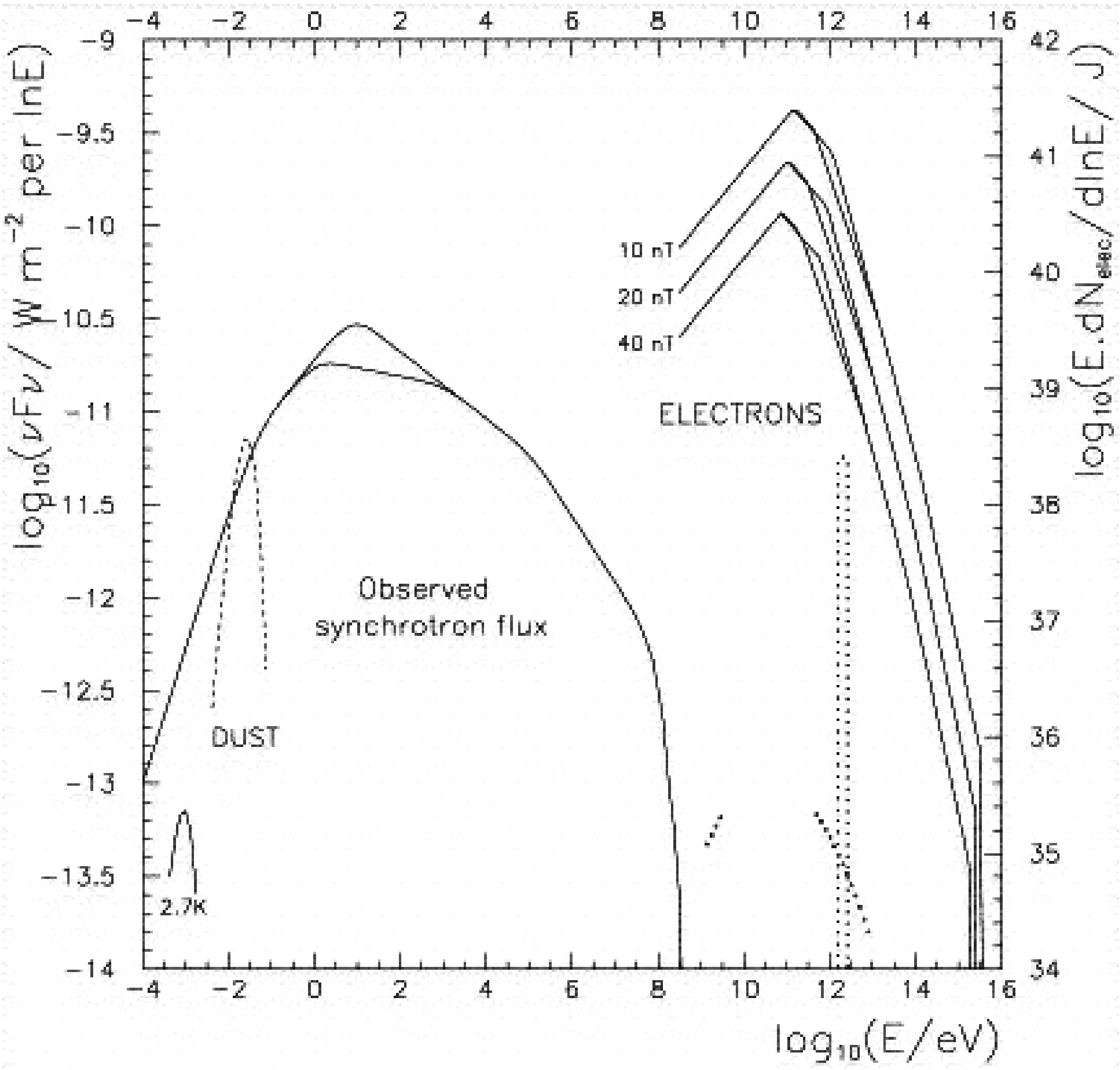
Fig. 4. Radiofrequency spectra of sources exhibiting the power law spectrum of synchrotron radiation (Casseopeia A), the flat spectrum of thermal bremsstrahlung radiation with low frequency self absorption (Orion Nebula), unusual high frequency radiation (NGC 1275), and low frequency absorption processes (P 1934 − 63 and P 2134 + 004). The data for P 2134 + 004 are from E. K. Conklin, and the other data are from Kellermann (1966), Hjellming, and Churchwell (1969), Terzian and Parrish (1970), Kellermann, Pauliny-Toth, and Williams (1969), and Kellermann *et al.* (1971)

Lang



Spectrum of Crab Nebula from radio to X-ray wavelengths

Middlehurst & Aller

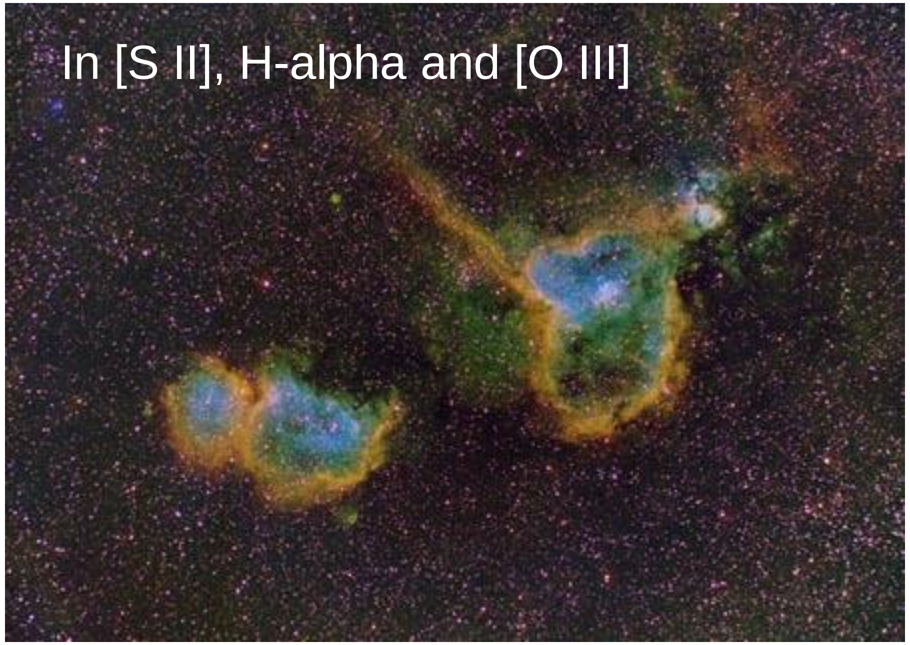


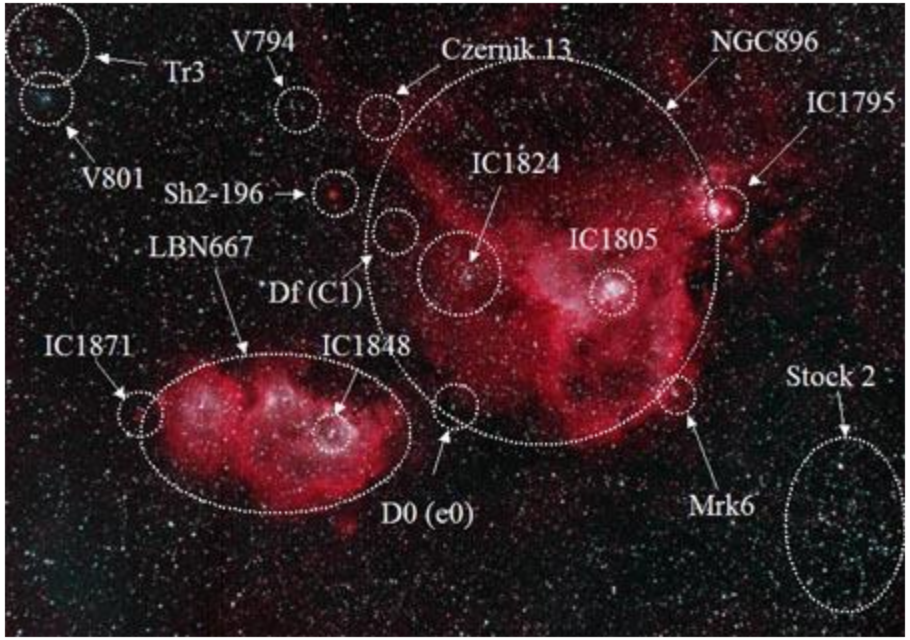
Spectrum of the Crab over a very wide range of energies. The emission is dominated by synchrotron radiation, and at very high energies ($10^{10} - 10^{20}$ eV) there may be an inverse-Compton component.

Hillas (1998), ApJ, 503, 744

http://www.astro.utu.fi/~cflynn/astroII/l4.html

W3= IC 1795





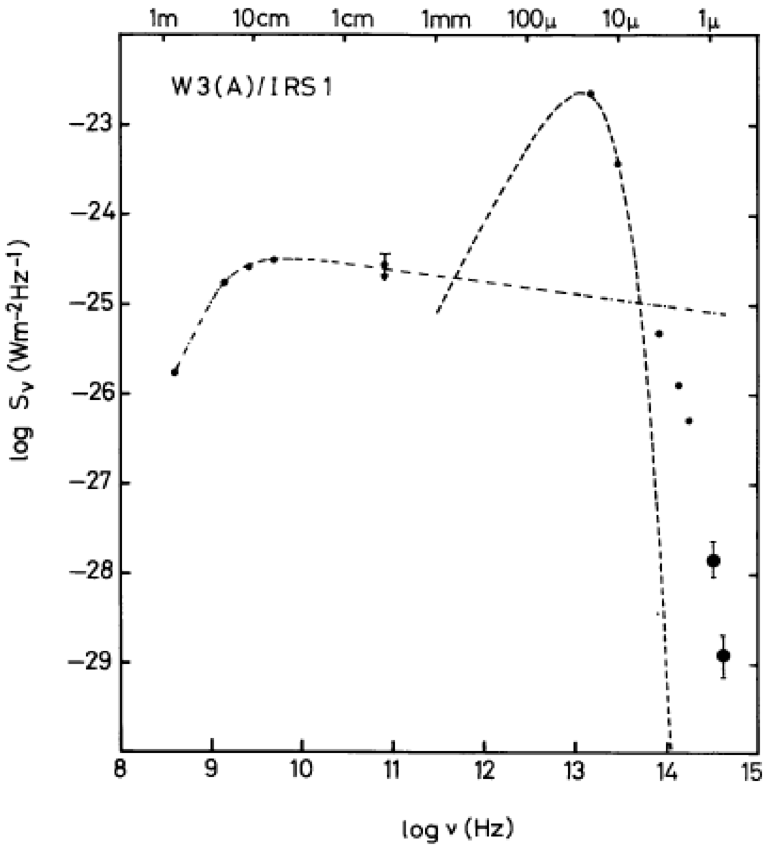
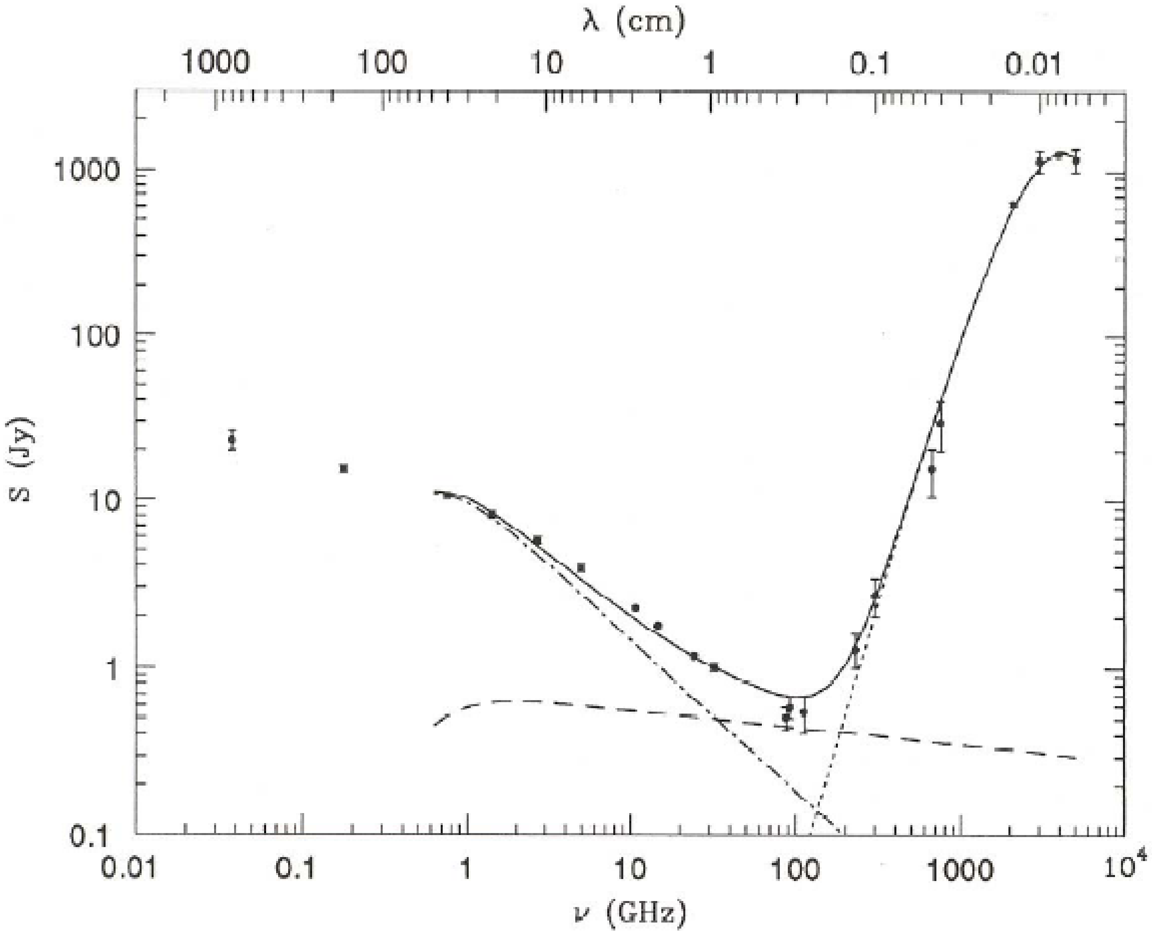


Fig. 2. Energy distribution of IRS 1 according to Wynn-Williams *et al.* (•) together with flux densities in *I* and *R* (●). Dashed lines: Predicted free-free emission from ionized hydrogen and thermal infra-red radiation of dust, represented by Planck-curve with temperature $T = 200$ K

Beetz et al. (1974) A&A, 34, 335



λ (cm)
1000
100
10
1
0.1
0.01
1000
100
10
1
0.1
S (Jy)
0.01
0.1
1
10
100
1000
10^4
ν (GHz)