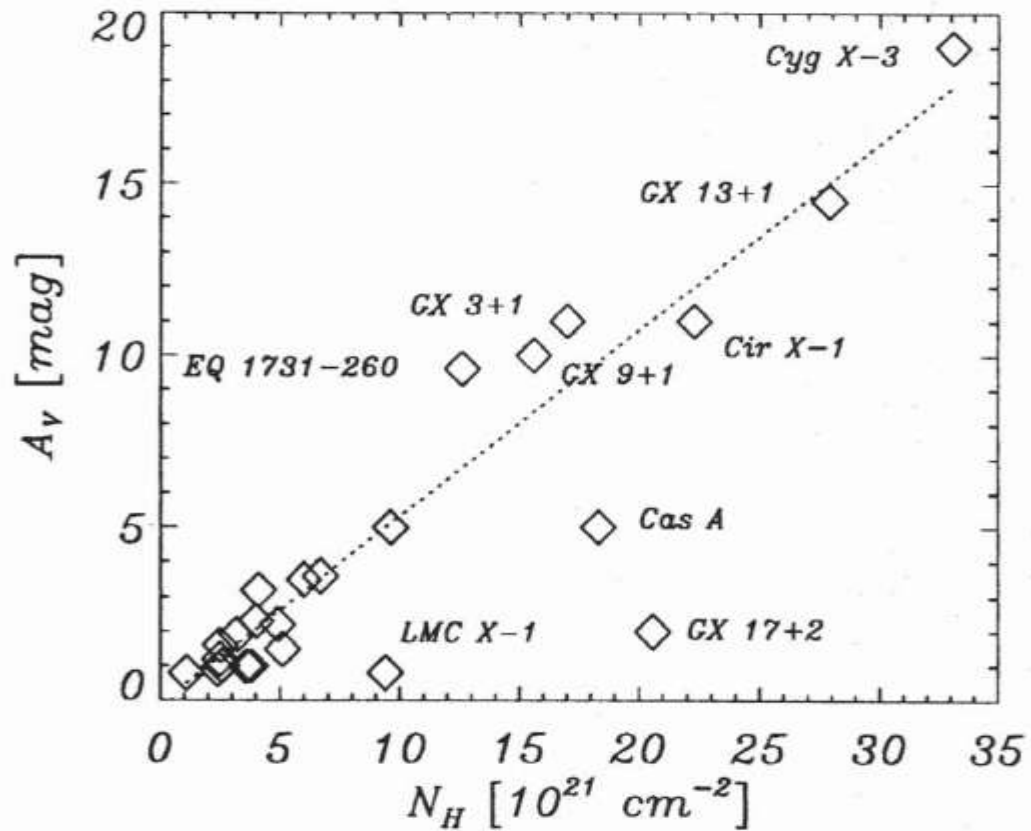


Star Formation

Stars are formed in dense molecular cloud cores, whereas planets are formed, contemporaneously, in young circumstellar disks.

→ Compression of gas from a cloud size $\sim 10^{18}$ cm down to a stellar size $\sim 10^{11}$ cm, i.e., density increases by a factor of $\sim 10^{21}$.



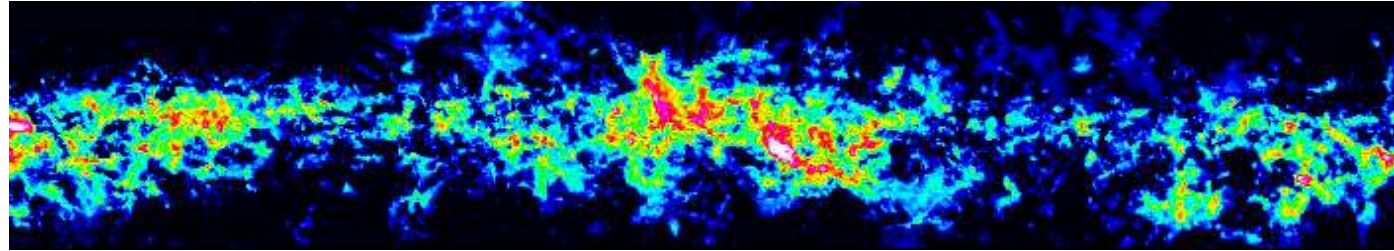
A gas-to-dust ratio ~ 100
(by mass) seems universal.

Fig. 3. Visual extinction vs. equivalent hydrogen column density. The fit (dotted line) does not contain GX 17+2 and LMC X-1. It yields $N_H = 1.79 \pm 0.03 A_V[\text{mag}] \times 10^{21}[\text{cm}^{-2}]$

$$\frac{N_H}{A_V} \approx 1.8 \times 10^{21} \text{ atoms cm}^{-2} \text{ mag}^{-1}$$

Predehl & Schmitt (1995)

Filamentary Molecular Clouds

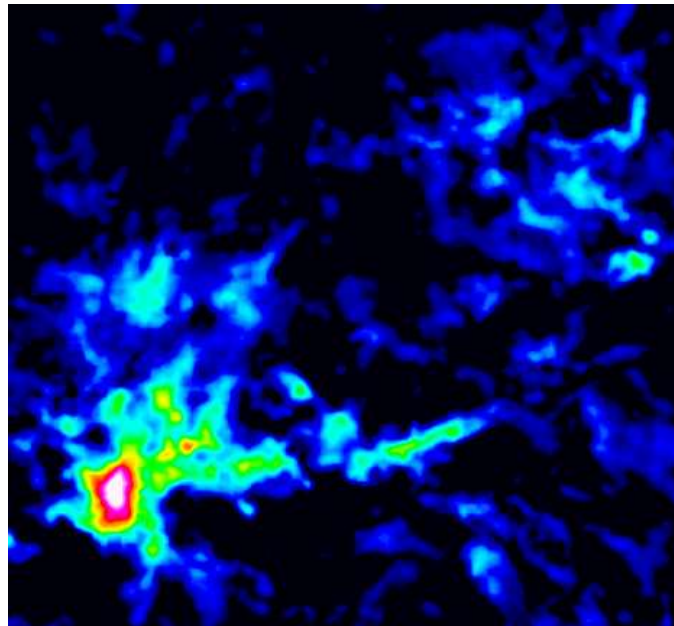


Molecular clumps/ clouds/condensations

$$n \sim 10^3 \text{ cm}^{-3}, D \sim 5 \text{ pc},$$
$$M \sim 10^3 \mathcal{M}_{\odot}$$

Dense molecular cores

$$n \geq 10^4 \text{ cm}^{-3}, D \sim 0.1 \text{ pc},$$
$$M \sim 1\text{-}2 \mathcal{M}_{\odot}$$



Giant Molecular Clouds

$$D = 20 \sim 100 \text{ pc}$$

$$\mathcal{M} = 10^5 \sim 10^6 \mathcal{M}_{\odot}$$

$$\rho \approx 10 \sim 300 \text{ cm}^{-3}$$

$$T \approx 10 \sim 30 \text{ K}$$

$$\Delta v \approx 5 \sim 15 \text{ km}^{-1}$$

Stars are formed in groups → seen as star clusters if gravitationally bound. Groups of young stars are found at the densest parts of the molecular clouds.

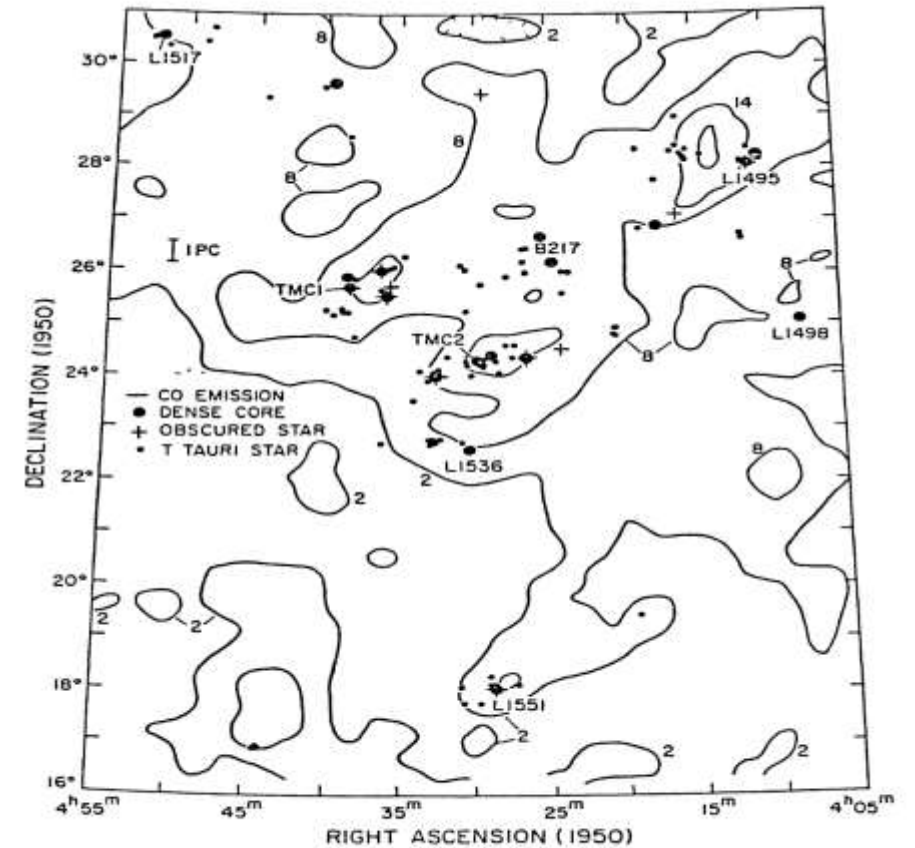
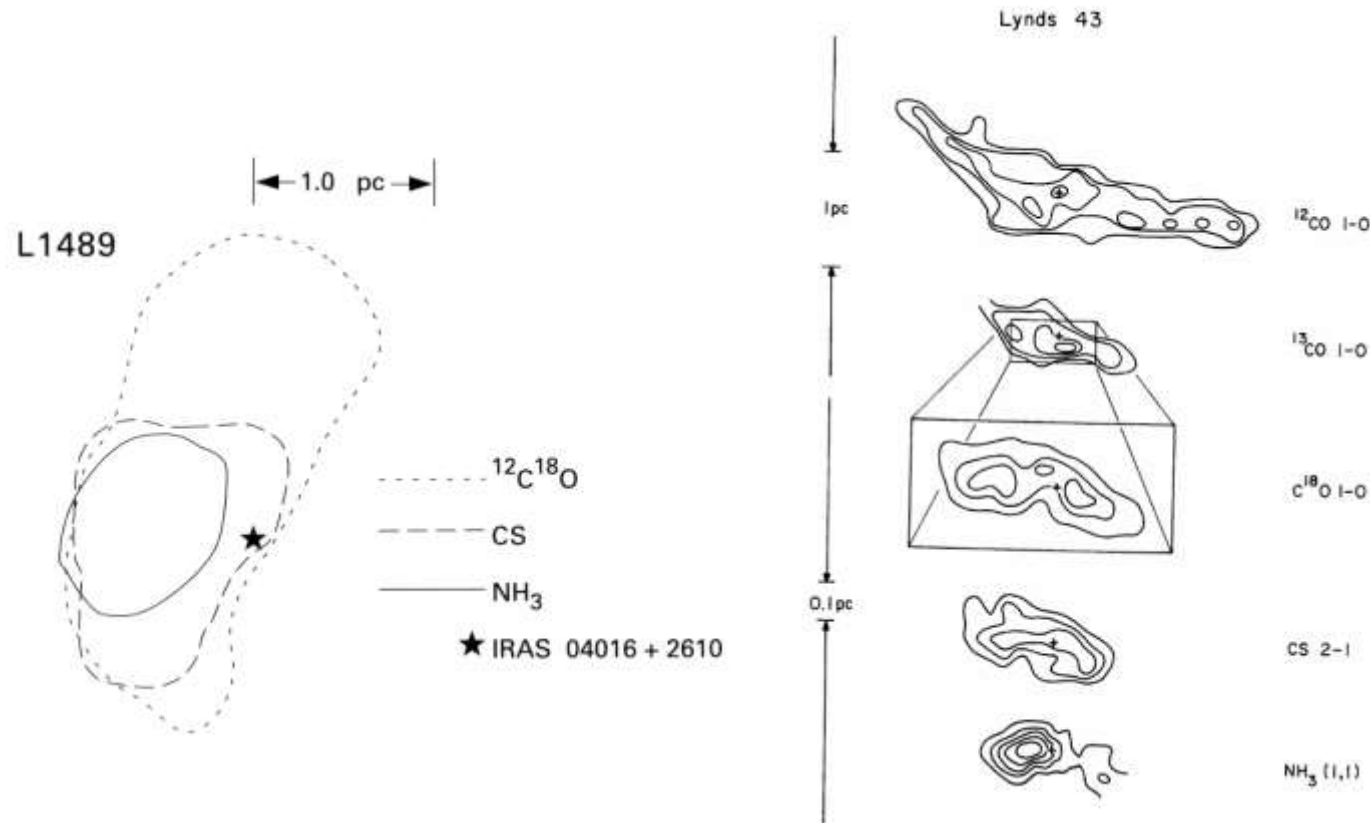


Figure 2 CO contour map of the Taurus molecular cloud with positions of dense NH_3 cores, embedded infrared sources, and visible T Tauri stars (from Myers 1986).

Molecular clouds observed by different tracers ...

Taurus molecular cloud

Nearby Examples

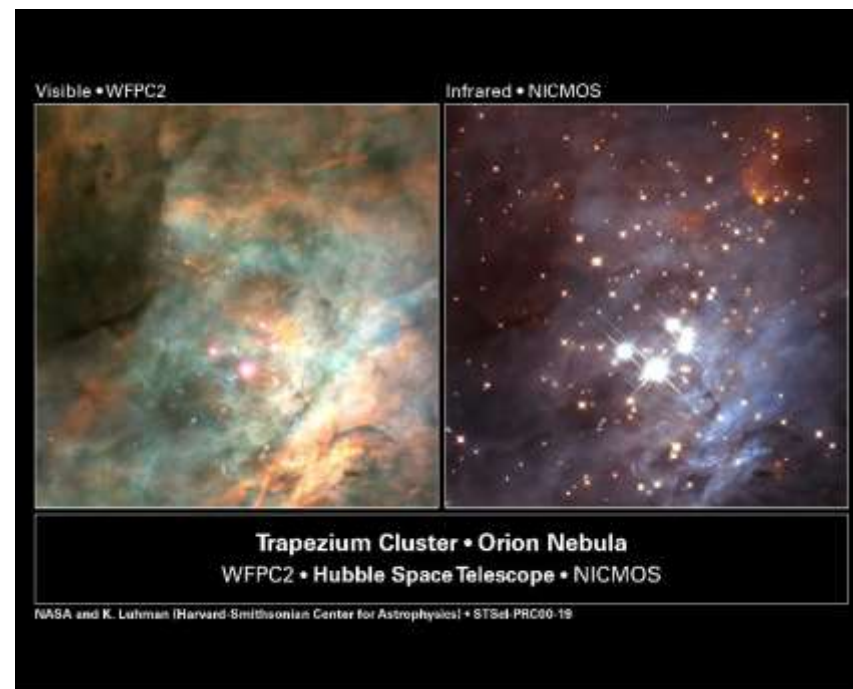
Massive Star-Forming Regions

- *Per OB2* (350 pc)
- *Orion OB Association* (350--400 pc) ... rich

Low-Mass Star-Forming Regions

- *Taurus Molecular Cloud (TMC-1)* (140 pc)
- *Rho Ophiuchi cloud* (130 pc)
- *Lupus* (140 pc)
- *Chamaeleon* (160 pc)
- *Corona Australis* (130 pc)

4/5 in the southern sky ... why?



Stability: The Virial Theorem

In a spherically symmetric cloud of temperature T , for each particle, the equation of motion is $\mathbf{F}_i = m_i \ddot{\mathbf{r}}_i = \dot{\mathbf{p}}_i$, the momentum change with time.

Sum up all particles and take time derivative

$$\begin{aligned} \frac{d}{dt} \sum_i m_i \dot{\mathbf{r}}_i \cdot \mathbf{r}_i &= \sum_i \dot{\mathbf{p}}_i \cdot \mathbf{r}_i + \sum_i \mathbf{p}_i \cdot \dot{\mathbf{r}}_i \\ &= \sum_i \mathbf{F}_i \cdot \mathbf{r}_i + \sum_i m_i \dot{\mathbf{r}}_i \cdot \dot{\mathbf{r}}_i \\ &= E_p + 2E_k \end{aligned}$$

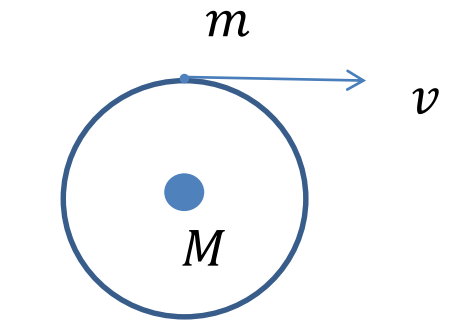
$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i = \text{virial of Clausius}$$

For moment of inertia, $I = \sum_i m_i r_i^2$,

$$\frac{d^2 I}{dt^2} = \frac{d}{dt} \left[\sum_i m_i 2 r_i \dot{r}_i \right]$$

Hence

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2E_k + E_p$$



$$\frac{GmM}{r^2} = m \frac{v^2}{r}$$

To be stable, LHS = 0

$$2E_k + E_p = 0$$

$$2 \left(\frac{1}{2} \right) m v^2 = GmM/r$$

LHS = 0 → stable

LHS < 0 → collapsing

LHS > 0 → expanding

E_K a variety of kinetic energies

- ✓ Kinetic energy of molecules
- ✓ Bulk motion of clouds
- ✓ Rotation
- ✓ ...

E_P a variety of potential energies

- ✓ Gravitation
- ✓ Magnetic field
- ✓ Electrical field
- ✓ ...

Note:

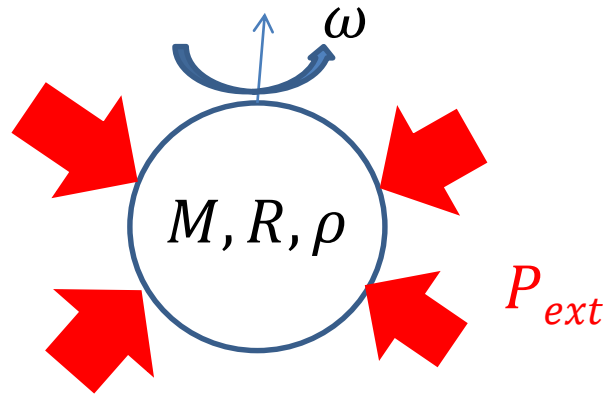
Virial theorem governs the motion status,

whereas the total energy

$$\begin{aligned} E_{\text{total}} &= E_K + E_P \\ &= E_K + \Omega \text{ (mostly)} \end{aligned}$$

governs whether the system is dynamically bound.

A coin flying either upward or downward is bound.



Cloud of mass M , radius R , rotating at ω

$$E_{rot} = \frac{1}{2} I \omega^2 \quad I = \frac{2}{5} M R^2 \quad \Omega = -\frac{3}{5} \frac{GM^2}{R}$$

Generalized virial theorem

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2 \langle E_K \rangle + \int \vec{r} \cdot \vec{F} dm + 3 \int P dV - \oint P \vec{r} \cdot d\vec{s}$$

If $\omega = 0$, and $P_{ext} = 0$

$$2 \cdot \frac{3}{2} \frac{M}{\mu m_H} kT - \frac{3}{5} \frac{GM^2}{R} = 0$$

$$R_J = \frac{1}{5} \frac{GM \mu m_H}{kT}$$

This is the **Jeans length**.

$\mu \approx 2.37$ for solar abundance with H_2

Jeans length = critical spatial wavelength (length scale)

If the perturbation length scale is longer

→ Medium is decoupled from self-gravity → stable

$$M_J = \frac{4}{3}\pi R_J^3 \rho \quad R_J = \left(\frac{15}{4\pi} \frac{kT}{\mu m_H G \rho}\right)^{1/2} \sim \sqrt{\frac{T}{\rho}}$$

$$M_J = \left(\frac{\pi kT}{4\mu m_H G}\right)^{3/2} \sqrt{\frac{1}{\rho}} \sim \frac{T^{3/2}}{\rho^{1/2}}$$

This is the **Jeans mass** ...
the **critical** mass for onset
of gravitational collapse

If cloud mass $M > M_{\text{Jeans}}$ → cloud collapse

Note the above does not consider external pressure, or other internal supporting mechanisms.

A non-magnetic, isothermal cloud in equilibrium with external pressure → a **Bonnor-Ebert sphere** (Bonnor 1956, Ebert 1955)

$$2E_K + E_P - 3P_{\text{ext}}V = 0$$

The potential term may include, other than the gravity, also rotation, magnetic field, etc.

At first, the cloud is optically **thin**.

Contraction → density ↑ → collisions more frequent
→ molecules excited and radiated → radiation escapes
→ cooling → less resistance to the contraction
→ cloud collapse (free fall)

To maintain $2E_K + E_P = 0$, the total energy $E_t = E_K + E_P$ must change. The gravitational energy

$$\Omega \sim -\frac{GM^2}{r} \longrightarrow d\Omega \sim \frac{dr}{r^2}$$

For contraction, $dr < 0$, so $d\Omega < 0$, then

$$dE_t = dE_K + d\Omega = \frac{1}{2} d\Omega = Ldt$$

This means to maintain quasistatic contraction, **half** of the gravitation energy from the contraction is radiated away.

Eventually the cloud becomes dense enough (i.e., optically **thick**) and contraction leads to temperature increase.

The cloud's temperature increases while energy is taken away
→ negative heat capacity

Numerically,

$$M_J = 1.0 \left(\frac{T}{10 \text{ K}} \right)^{3/2} \left(\frac{n_{\text{H}_2}}{10^4 \text{ cm}^{-3}} \right)^{-1/2} [\mathcal{M}_\odot]$$

- H I clouds

$T \approx 100 \text{ K}, n_H \approx 100, R_J \approx 25 \text{ pc}; M_J \approx 300 \mathcal{M}_\odot > M_{\text{obs}}$

So H I clouds are not collapsing.

- Dark molecular clouds

$T \approx 15 \text{ K}, n_H \approx 10^5, M_J \approx 20 \mathcal{M}_\odot < M_{\text{obs}} \approx 100 - 1000 \mathcal{M}_\odot$

So H₂ clouds (dense cores and Bok globules) should be collapsing. But observations show that most are not.

→ There is additional support other than the thermal pressure, e.g., rotation, magnetic field, turbulence, etc.

If $\mathcal{M}_{\text{cloud}} > \mathcal{M}_{\text{crit}} \rightarrow$ supercritical \rightarrow Cloud collapses dynamically
 \rightarrow Massive star formation

If $\mathcal{M}_{\text{cloud}} < \mathcal{M}_{\text{crit}} \rightarrow$ subcritical \rightarrow Cloud collapses quasistatically
 \rightarrow Low-mass star formation

Clouds tend to condense with $\mathcal{M} \sim 10^4 M_{\odot}$, but the observed stellar mass ranges $0.05 \leq \mathcal{M}/M_{\odot} \leq 100$

Why is there a lower mass limit and an upper mass limit for stars?

Cloud collapse \rightarrow (local) density increase \rightarrow (local) M_J decrease
 \rightarrow easier to satisfy $M > M_J$, i.e., cloud becomes more unstable

\rightarrow fragmentation

Formation of a cluster of stars $\sim \sim$



$$\text{Recall } M_J \approx 1.2 \times 10^5 \left(\frac{T}{100 \text{ K}}\right)^{3/2} \left(\frac{\rho_0}{10^{-24} \text{ g cm}^{-3}}\right)^{-1/2} \frac{1}{\mu^{3/2}} [M_\odot]$$
$$\propto T^{3/2} / \rho^{1/2}$$

A small/decreasing M_J favors cloud collapse.

If during collapse, local $M_J \downarrow \rightarrow$ subregions become unstable and continue to collapse to ever smaller (**fragmentation**).

Since during collapse ρ always \uparrow , the behavior of M_J depends on T .

If gravitational energy is radiated away, i.e., $\tau_{\text{cooling}} \ll \tau_{\text{ff}}$ and collapse is isothermal, $T = \text{const}$, so $M_J \propto \rho^{-1/2} \rightarrow$ collapse continues

Equation of motion for a spherical surface at r is

$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2}$$

Dimensional analysis yields

$$\frac{R}{t^2} \sim \frac{GM}{R^2} \Rightarrow t_{ff} \sim \frac{1}{\sqrt{G\rho}}$$

More accurately, $t_{ff} = \left(\frac{3\pi}{32 G \rho_0} \right)^{\frac{1}{2}} = \frac{3.4 \times 10^7}{\sqrt{n_0}} \text{ [yr]} = 35 / \sqrt{\rho_{\text{cgs}}} \text{ [min]}$

It takes the Sun ~ 30 minutes to collapse (the **free-fall time scale**).

Ex: How long does a typical dense molecular core take to collapse?

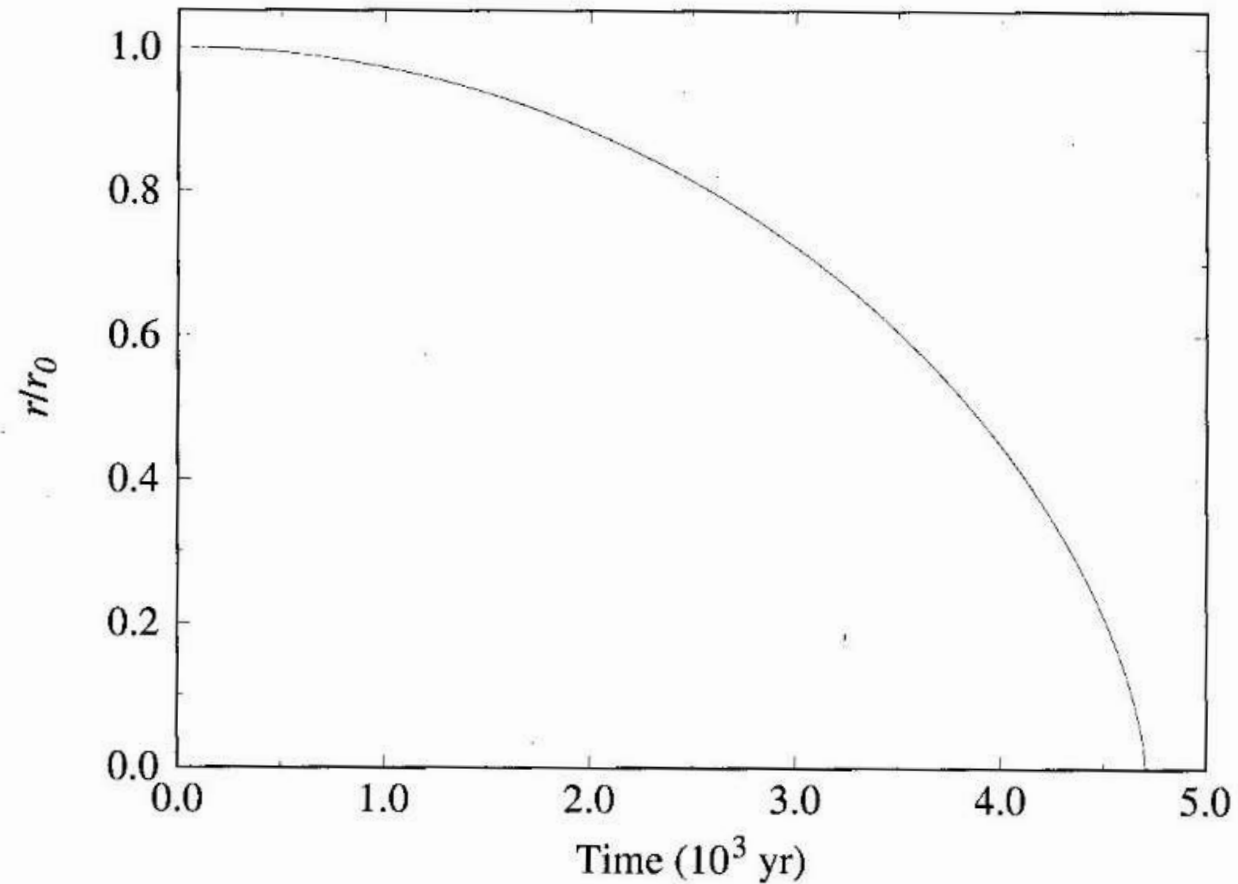


Figure 12.5 The ratio of the radius relative to its initial value as a function of time for the homologous collapse of a molecular cloud. The collapse is assumed to be isothermal, beginning with a density of $\rho_0 = 2 \times 10^{-16} \text{ g cm}^{-3}$.

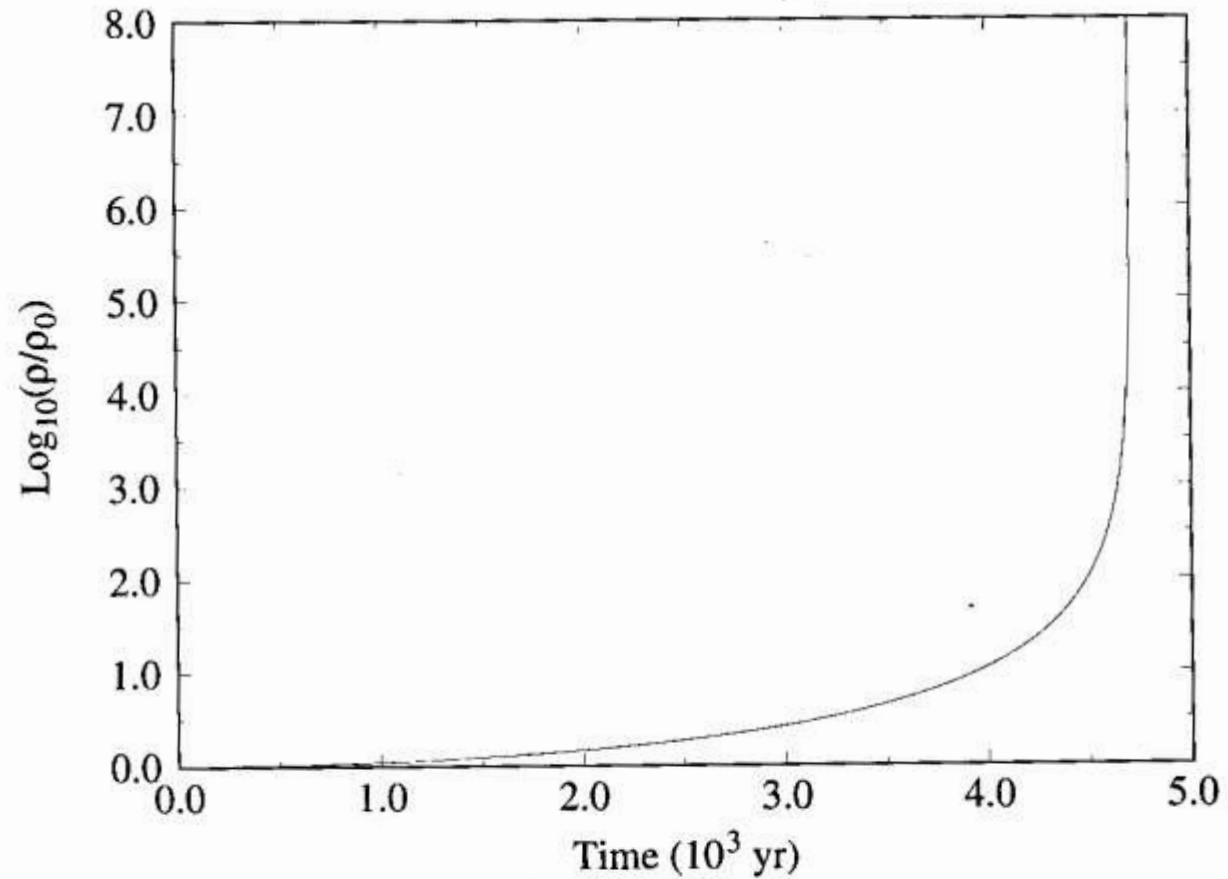


Figure 12.6 The ratio of the cloud's density relative to its initial value as a function of time for the isothermal, homologous collapse of a molecular cloud with an initial density of $\rho_0 = 2 \times 10^{-16} \text{ g cm}^{-3}$.

Carroll & Ostlie

Note that $t_{\text{ff}} \propto \frac{1}{\sqrt{G\rho_0}}$ has no dependence on r_0 .

If ρ_0 is uniform, all m collapse to the center at the same time

→ **homologous collapse**

In reality, ρ_0 is somewhat centrally condensed, as observed,

e.g., $\rho_0 \propto r^{-1}$ to r^{-2} , inner region (small r), $t_{\text{ff}} \downarrow\downarrow$

→ **inside-out collapse**

A protostellar core is formed, followed by material “raining down” → **accretion**

Gravitational energy → kinetic energy → heat

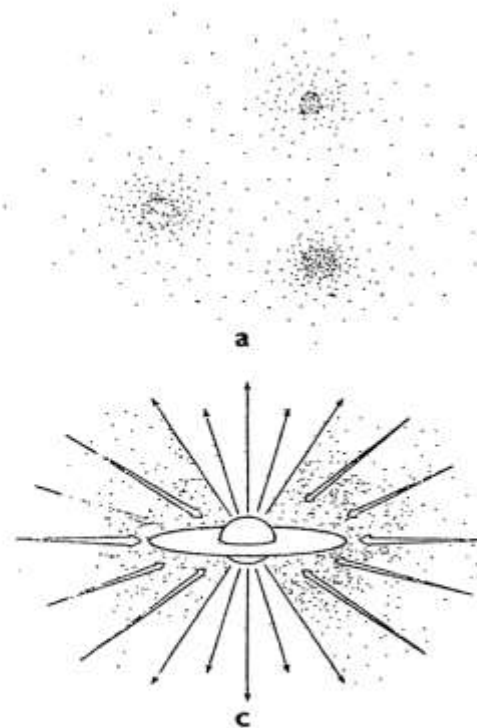
$$L_{\text{acc}} \sim GM_* \dot{M} / R_*$$

STAR FORMATION IN MOLECULAR CLOUDS: OBSERVATION AND THEORY

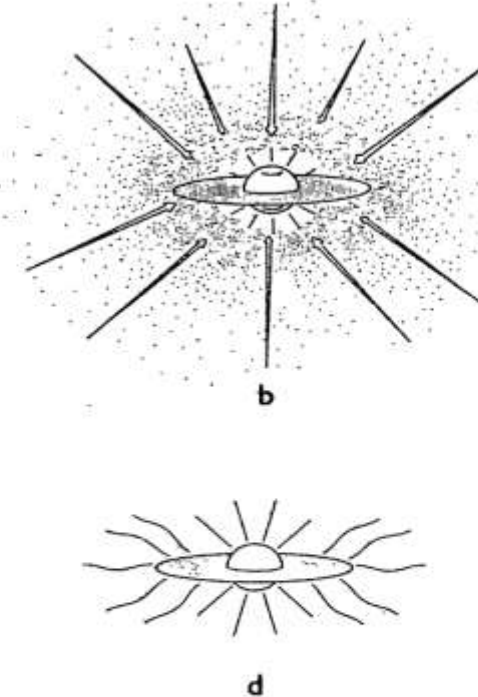
Frank H. Shu, Fred C. Adams, and Susana Lizano

Astronomy Department, University of California, Berkeley, California 94720

Cores form within molecular clouds.



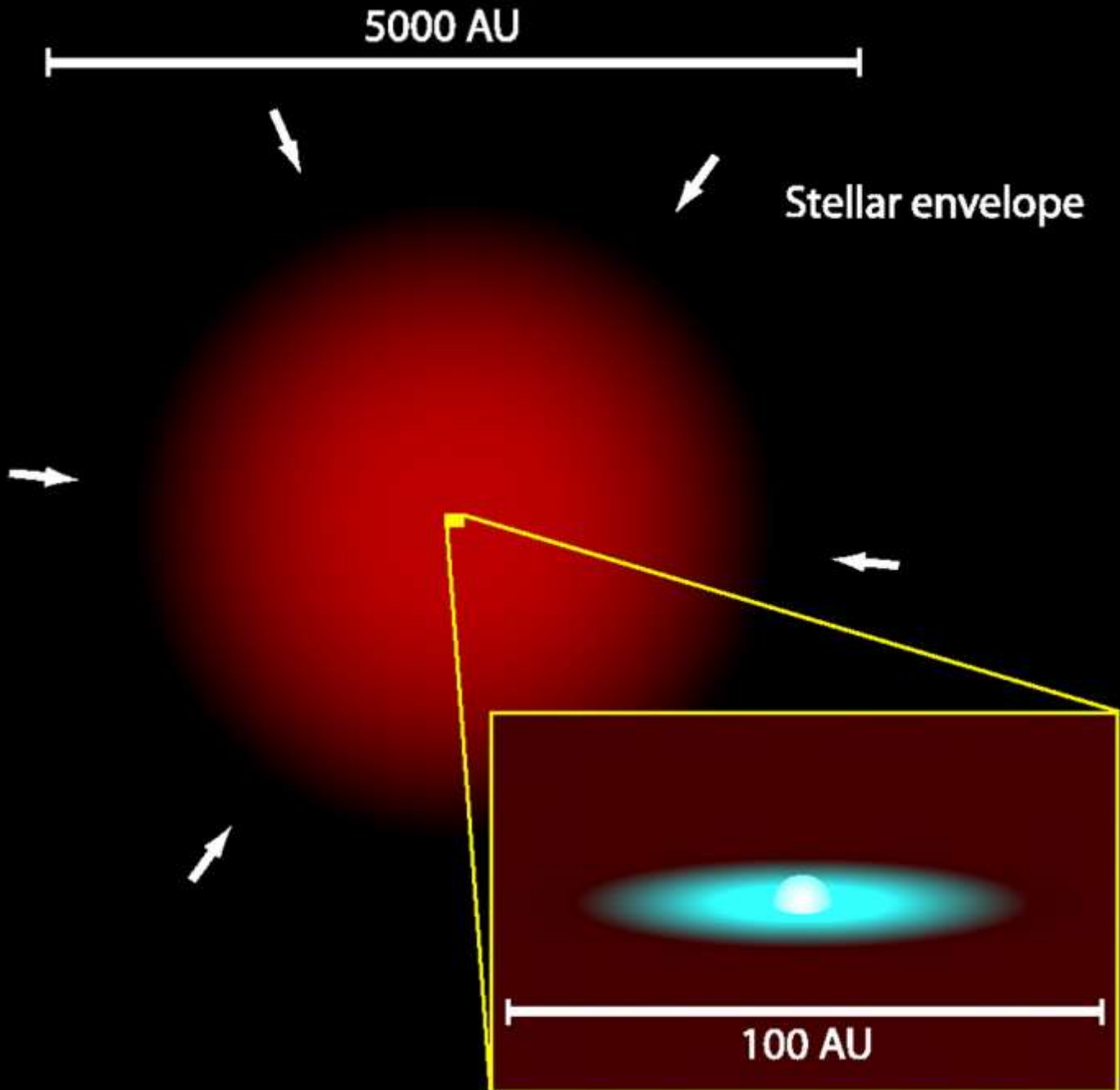
A stellar wind with a bipolar flow forms.



A core collapse inside-out and form a protostar with a toroid.

A star is form with a circumstellar disk.

Figure 7 The four stages of star formation. (a) Cores form within molecular clouds as magnetic and turbulent support is lost through ambipolar diffusion. (b) A protostar with a surrounding nebular disk forms at the center of a cloud core collapsing from inside-out. (c) A stellar wind breaks out along the rotational axis of the system, creating a bipolar flow. (d) The infall terminates, revealing a newly formed star with a circumstellar disk.



Central condensed protostar,
 $r \sim$ a few R_{\odot}

Circumstellar disk,
 $r \sim 100$ au

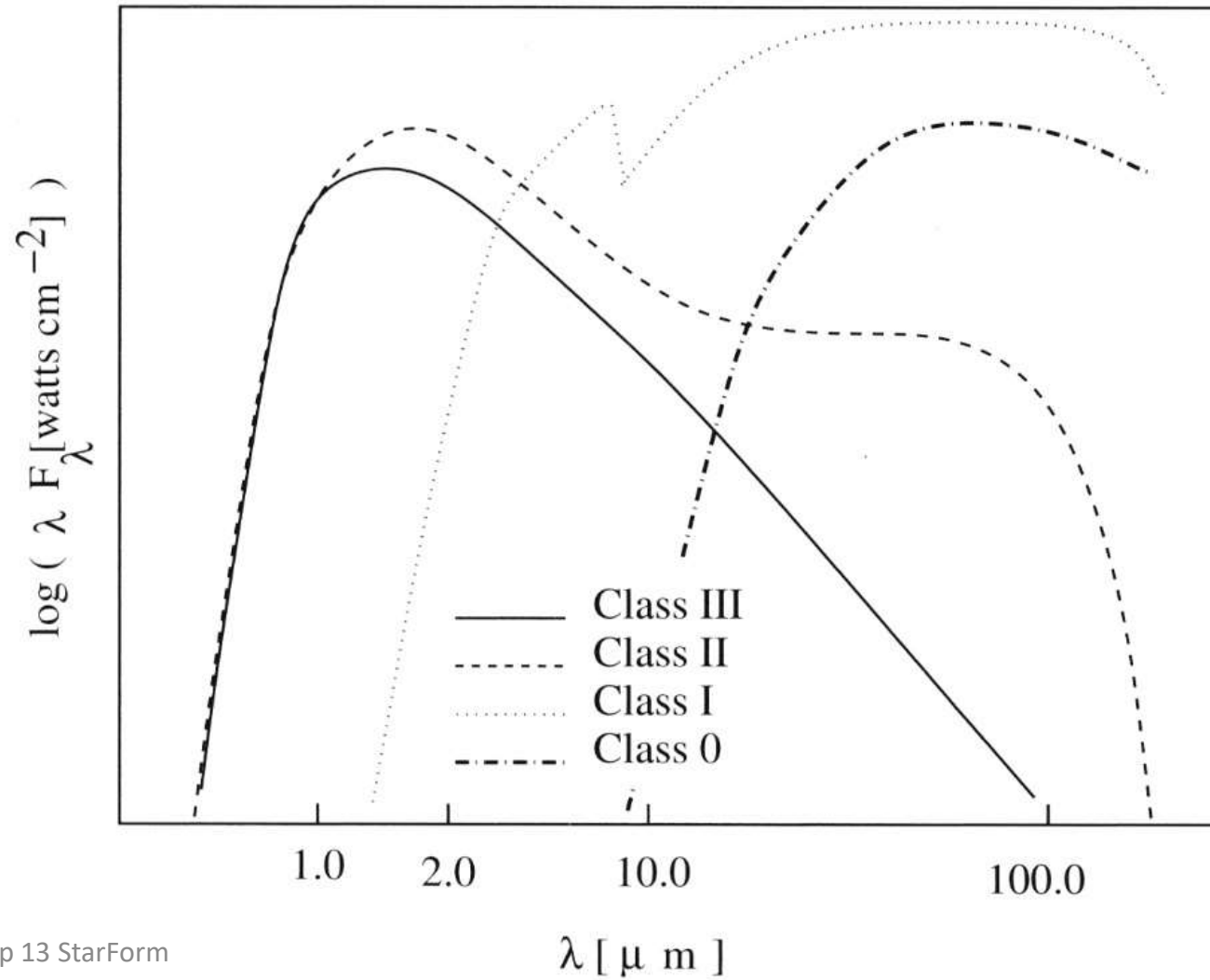
Surrounding envelope,
 $r \sim 5000$ au

Matter accretes from the
envelope via the disk onto
the protostar

Ward-Thomson (2002)

Spectral energy distribution

F_λ vs λ
or $\log \lambda F_\lambda$ vs $\log \lambda$



Spectral index useful to classify a young stellar object
(YSO)

$$\alpha = \frac{d \log (\lambda F_{\lambda})}{d \log (\lambda)}$$

Where λ =wavelength, between 2.2 and 20 μm ; F_{λ} =flux density

Class 0 sources --- undetectable at $\lambda < 20 \mu\text{m}$

Class I sources --- $\alpha > 0.3$

Flat spectrum sources --- $0.3 > \alpha > -0.3$

Class II sources --- $0.3 > \alpha > -1.6$

Class III sources --- $\alpha < -1.6$

→ Evolutionary sequence in decreasing amounts of circumstellar material (disk clearing)

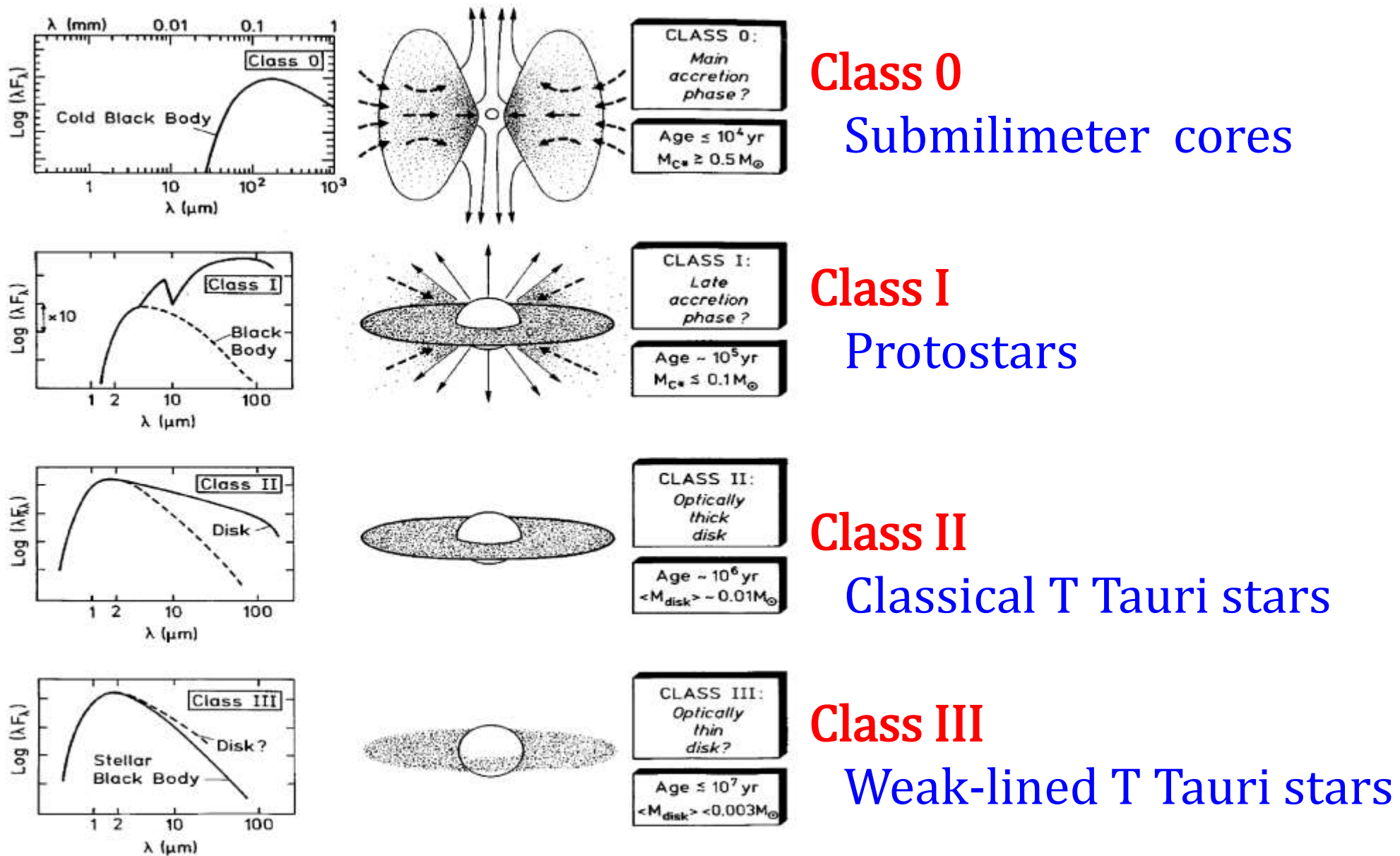


Figure 11 Evolutionary sequence of the spectral energy distributions for low-mass YSOs as proposed by André (1994). The four classes 0, I, II, and III correspond to successive stages of evolution.

Basic Questions in Star Formation

- The rate and efficiency of SF as a function of time and position in the Galaxy, and in external galaxies? How are these quantities measured?
- Cloud fragmentation to form clusters?
- Triggered SF?
- Different processes for high-mass and low-mass?
- Mass spectrum? Typical $0.1 - 1 M_{\odot}$, why?
- Formation of multiple systems?
- What is a protostar observationally?
- Evolution of disks?
- Origin of bipolar outflows?
- Environments for planet formation?
- Role of rotation and magnetic field?

Bodenheimer