PHD QUALIFY EXAMINATION — STELLAR ASTROPHYSICS

2nd October, 1997

(1) (25 points)

A chemically homogeneous spherical star has a density profile $\rho = \rho_c(1 - r/R)$, where R is the radius of the star.

(a) Determine the core pressure in terms of the total mass.

(b) Sketch the density and pressure profiles.

(c) Suppose the core is made up of completely ionized hydrogen. The electrons are completely degenerate and nonrelativistic while the ions are classical. Calculate the minimum mass of the star to ignite thermonuclear reaction. Give a rough estimation of this mass. (You may assume that the degenerate pressure of electron is given by $h^2/5m_e(3/8\pi)^{(2/3)}\, n_e^{(5/3)}$.)

(2) (25 points)

Consider the Sun as a self-gravitating gas sphere.

(a) Using virial theorem, derive an expression for the average temperature inside the Sun in terms of its mass and radius. Can you justify the claim that radiation energy density is negligible inside the Sun?

(b) Suppose the luminosity of the Sun is kept at the present rate, how long will the Sun shine if the only available source of energy is (i) radiation energy, (ii) total thermal energy, and (iii) energy from thermonuclear fusion of hydrogen?

(c) Suppose the photons inside the Sun undergo random walks. Suggest a relation between the mean free path and the time taken by (b) (i). Hence estimate the average opacity inside the Sun?

(3) (25 points)

The 'Hayashi track' refers to an evolutionary sequence of a star in its pre-main sequence phase on the Hersprung-Russel diagram.

(a) Draw the Hayashi track for the sun (you should roughly label the axes), and give a physical explanation of the shape of the track.

(b) How does the Hayashi track of a 1 $M_\odot$ star compare with that of a 2 $M_\odot$ star? Why?

(c) How about a 10 $M_\odot$ star?

(4) (25 points)

Explain the 'Jeans mass', and derive its expression for a collection of gas. Carefully state whatever assumption you need to make. For a typical diffuse interstellar cloud on the galactic disk, the density $\rho \sim 2 \times 10^{-24}$ g cm^{-3}, and the temperature $T \sim 100$ K, its Jeans mass is about 10^5–10^6 $M_\odot$, which far exceeds the mass for individual stars ($0.05 \lesssim M/M_\odot \lesssim 60$). Discuss the implication.