PHD QUALIFY EXAMINATION — GALACTIC AND EXTRAGALACTIC ASTROPHYSICS

25th September, 1998

(1) (25 points)

(a) (5 points) What are the luminosity function $\Phi(M)$, the initial luminosity function $\Psi(M)$ and the initial mass function $\xi(m)$ (M is absolute magnitude and m is mass)? How are they related?

(b) (10 points) The angular velocity of the Galaxy at a star ϖ from the centre of the Galaxy is Ω, and the angular velocity of the Galaxy at the Sun (ϖ_o) is Ω_o. Suppose R is the distance of the star from the Sun, and l is the galactic longitude of the star. Find the general rotation formula, i.e., express $v_{\rm R}$ and $v_{\rm T}$ (the observed radial and tangential velocities of the star relative to the Sun) in terms of the quantities above.

(c) (10 points) If R is much smaller than ϖ and ϖ_o, derive the Oort's formulae: $v_{\rm R} = AR\sin(2l)$, $v_{\rm T} = R[A\cos(2l) + B]$. Describe how to measure the Oort's constants A and B.

(2) (25 points)

(a) (5 points) Describe how to estimate the mass of (i) a spiral galaxy, and (ii) an elliptical galaxy.

(b) (5 points) Describe different phases of the interstellar medium.

(c) (5 points) Describe the physics of the 21-cm line.

(d) (5 points) What is the meaning and assumption of Local Thermodynamic Equilibrium? Can you give an example which violates LTE?

(e) (5 points) What is the nominal value of large scale magnetic field in our Galaxy? Can you suggest a method to measure it?

(3) (25 points)

(a) (15 points) Consider a two-state atom with energies E_1, E_2 ($E_1 < E_2$) and degeneracies g_1, g_2. Suppose upward transition can be achieved by collision with electrons or absorption of a photon of energy $\Delta E = E_2 - E_1$, and downward transition can be achieved by collision with electrons, stimulated emission or spontaneous emission. Let n_1, n_2, n_e, n_γ be the number densities of atoms in state 1, state 2, ambient electrons, ambient photons (of energy ΔE), respectively. Denote the Einstein's coefficients for spontaneous emission, stimulated emission, stimulated absorption, collisional upward and downward transistions by A_{21}, B_{21}, B_{12}, γ_{12} and γ_{21}, respectively. Find an expression for n_2/n_1, and find the conditions under which the Boltzmann distribution $n_2/n_1 = g_2/g_1 \exp(-\Delta E/kT)$ holds.

(b) (10 points) For 21-cm line $A_{21} = 2.85 \times 10^{-15}\ \mathrm{s}^{-1}$, $g_1 = 1$, $g_2 = 3$. Consider a typical HI region with temperature 100 K. Is Boltzmann distribution applicable? What is the total number of 21-cm photons emitted per unit time per unit volume?

(4) (25 points)

(a) (10 points) The effects of electrons in ISM on the propagation of radio waves (with frewuency ω) can be described by the dispersion relation $\epsilon\omega^2 = c^2k^2$, where the plasma dielectric constant ϵ of the ISM is given by (in SI units) $\epsilon = 1 - \omega_\mathrm{p}^2/\omega^2$, where $\omega_\mathrm{p}^2 = n_\mathrm{e}e^2/\epsilon_\mathrm{o} m_\mathrm{e}$. What are the phase and group velocity of the radio wave? Describe the physics of $\omega > \omega_\mathrm{g}$ and $\omega < \omega_\mathrm{g}$.

(b) (10 points) Consider a pulsar at a distance l from us. Assuming $\omega \gg \omega_\mathrm{p}$, show that the time taken by a radio wave from the pulsar to us is

$$t \approx \frac{l}{c} + \frac{e^2}{2\epsilon_\mathrm{o} m_\mathrm{e} c\omega^2} DM \,,$$

where $DM = \int_0^l n_\mathrm{e} \mathrm{d}l$ is called the dispersion measure. Describe how to estimate l and the ISM electron density n_e by radio waves from the pulsar.

(c) (5 points) Describe how the radio wave be affected if there is a large scale magnetic field between the pulsar and us.