Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION — STELLAR ASTROPHYSICS

9:00–13:00, 31st May, 2013

(1) (35 points)
Gravitational binding energy, dynamical timescale.

(a) (5 points) The mass, luminosity, and effective temperature of α Lyr (Vega) are $M = 2.6M_\odot$, $L = 51L_\odot$, and $T_{eff} = 9300$ K, respectively. Estimate the radius of α Lyr. Compare your result with the solar radius.

(b) (10 points) The gravitational binding energy of a star is the amount of energy required to disassemble the star completely. Show the gravitational binding energy of a star with the mass M and radius R. Assume uniform density in the star.

(c) (5 points) Calculate the gravitational binding energy of α Lyr.

(d) (5 points) Can the release of gravitational potential energy be the source of the radiation from α Lyr? Explain your answer.

(e) (5 points) The timescale τ for the physical quantity ϕ is defined as $\tau = \phi/\dot{\phi}$ where $\dot{\phi} = d\phi/dt$. Consider the size of α Lyr, and estimate the dynamical timescale of α Lyr.

(f) (5 points) What are the implications from the results of previous question?

(2) (5 points)
Site of star formation.
A region where we find a group of blue luminous stars is thought to be a site of active and ongoing star formation. Explain why. What else is expected to exist near the site of this kind?

(3) (30 points)
Stellar mass loss and radiation pressure.
A star loses its mass throughout its life, from pre-main sequence outflows, jets, and winds, to main sequence and giant winds.

(a) (5 points) Describe a couple lines of direct observational evidence of the expansion of the outer layers of stars beyond the stellar photosphere into the interstellar medium.

(b) (10 points) A red giant is particularly susceptible to mass loss because of its expanded size and excessive radiation pressure. A photon of energy E carries a momentum of $p = E/c$ and thus can exert a radiation pressure. Show that for blackbody radiation of temperature T, the radiation pressure is $P_{\rm rad} = aT^4/3 = u/3$, where u is the energy density. That is, the blackbody radiation pressure is one-third of the energy density.

(c) (10 points) We would like to estimate the radiation pressure relative to the gas pressure at the center of the Sun. For this, let us first obtain a rough estimate of the gas pressure. The hydrostatic equation requires that $dP/dr = -G(M_r\rho)/r^2$, where M_r is the mass interior to the sphere of radius r. Assuming $\rho = \bar{\rho}$, that is, the Sun has an average density of 1.41 g cm^{-3}, find the central pressure, then using the ideal gas law, show that the central temperature of the Sun is about $T_c \sim 1.44 \times 10^7$ K. Compute the radiation pressure at the center, and the ratio of this pressure to that of the gas.

(d) (5 points) The maximum stellar luminosity, the Eddington limit, is related to the radiation pressure. Qualitatively explain — but do not derive — what this limit is.

(4) (10 points)
White dwarfs.
Show that the volume of a white dwarf is inversely proportional to its mass. Explain the physical reasoning why a more massive white dwarf would have a smaller volume. There exists a maximum mass for a white dwarf, i.e., the Chandrasekhar limit. What happens if a white dwarf has a mass exceeding this limit?

(5) (10 points)
Accretion versus thermonuclear fusion.
An accretion process releases the kinetic energy of a mass, gained from the gravitational energy of the accreted body, into heat and light upon impact. Consider a mass starts at rest infinitely from a neutron star with mass $M = 1.4M_{\odot}$ and radius $R = 10$ km, compute the energy released when this mass is accreted onto the neutron star. Compare this to the rest energy of the mass.

(6) (10 points)
Spectrum of HD 191089.
Figure 1 shows the visible and infrared spectrum of HD191089. Why does this object have two peaks in its spectrum? What are the differences between stars with two peaks in their spectra and stars with single-peaked spectrum? What is the temperature of the material contributing to the secondary peak?

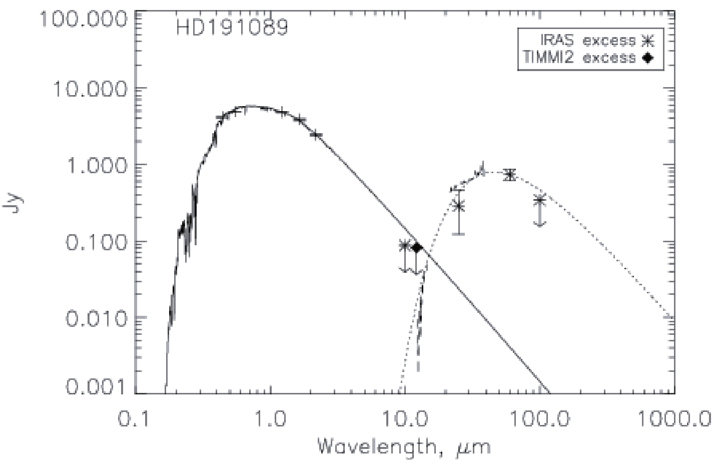

Fig. 1. Figure for question (6). Visible and infrared spectrum of HD191089.

Constants

Speed of light	$c = 3.00 \times 10^{8}$ m s^{-1}
Gravitational constant	$G = 6.67 \times 10^{-11}$ m^{3} kg^{-1} s^{-2}
Planck constant	$h = 6.63 \times 10^{-34}$ J s
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J K^{-1}
Electron volt	$1\,\mathrm{eV} = 1.6 \times 10^{-19}$ J
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ J m^{-2} s^{-1} K^{-4}
Radiation constant	$a = 7.56 \times 10^{-16}$ J m^{-3} K^{-4}
Avogadro constant	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Atomic mass unit	$m_H = 1.66 \times 10^{-27}$ kg
electron mass	$m_e = 9.11 \times 10^{-31}$ kg
proton mass	$m_p = 1.6726 \times 10^{-27}$ kg
neutron mass	$m_n = 1.6749 \times 10^{-27}$ kg
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}$ kg
hydrogen atom mass	1.674×10^{-27} kg
helium-3 atom mass	5.009×10^{-27} kg
helium-4 atom mass	6.648×10^{-27} kg
ideal gas constant	$\mathcal{R} = 8.31 \times 10^{3}$ J kg^{-1} K^{-1}
Solar mass	$M_\odot = 1.99 \times 10^{30}$ kg
Solar radius	$R_\odot = 6.96 \times 10^{8}$ m
Solar luminosity	$L_\odot = 3.85 \times 10^{26}$J s^{-1}
Mean density of the Earth	$\rho_\oplus = 5.51 \times 10^{3}$ kg m^{-3}
Earth mass	$M_\oplus = 5.98 \times 10^{24}$ kg
Earth radius	$R_\oplus = 6.38 \times 10^{6}$ m
Astronomical unit	$1\,AU = 1.50 \times 10^{11}$ m
π	$\pi = 3.14$
cal and J	1 cal = 4.2 J