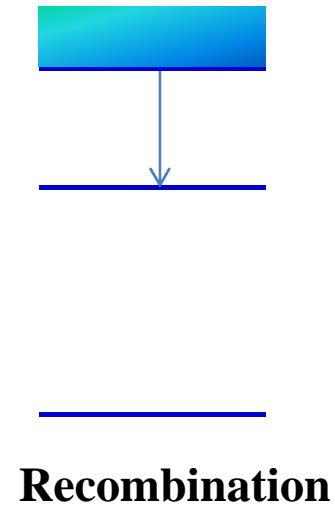
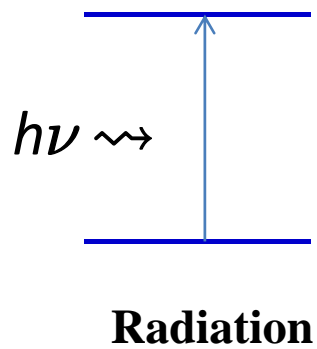
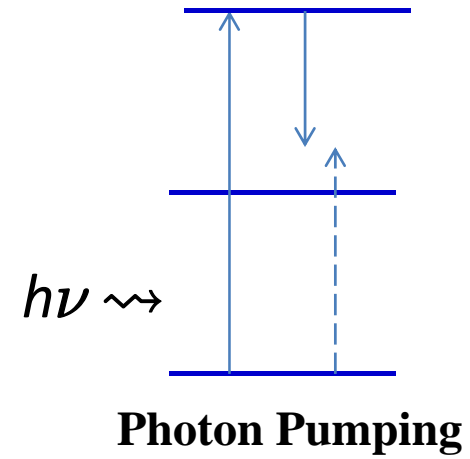
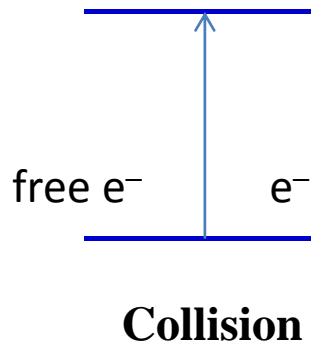


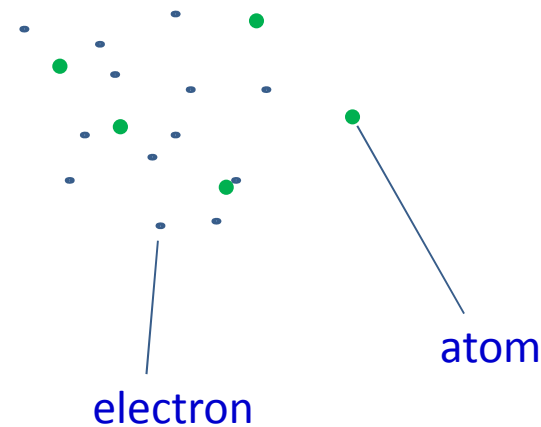
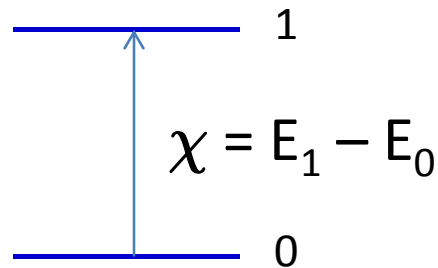
Excitations



Principle of detailed balance

Consider a 2-level system, excitation occurs if the incoming free electrons have kinetic energy

$$\frac{1}{2}mv^2 > \chi$$



Define the **excitation rate coefficient** γ_{01} so that

$$\# \text{ of excitation } s^{-1} \text{ cm}^{-3} (= n_e n_0 v \sigma) \equiv n_e n_0 \gamma_{01}$$

where both n_e and n_0 have units of $[\text{cm}^{-3}]$.

$$\gamma_{01} \equiv \langle \sigma v \rangle = \int_{\chi=\frac{1}{2}mv^2}^{\infty} v \sigma_{01}(v) f(\vec{v}) d^3\vec{v}$$

Here σ_{01} is the excitation cross section, and $f(\vec{v})$ is the Maxwellian distribution function

$$f(v; T) dv = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$$

So

$$\gamma_{01} = \frac{4}{\sqrt{\pi}} \left(\frac{1}{2kT} \right)^{1/2} \int_{\chi=\frac{1}{2}mv^2}^{\infty} v^3 \sigma_{01}(v) e^{-\frac{mv^2}{2kT}} dv$$

This is the upward transition.

Downward transition:

- spontaneous emission, rate = $n_1 A_{10}$
 - deexcitation by collisions, rate = $n_1 n_e \gamma_{10}$,
where $\gamma_{10} = \int_0^\infty v \sigma_{10}(v) f(v) dv = \gamma_{10}(T)$
-

Detailed balance

In steady state, [upwards] = [downwards]

$$n_0 n_e \gamma_{01}(T) = n_1 [A_{10} + n_e \gamma_{10}(T)]$$

$$\frac{n_1}{n_0} = \frac{n_e \gamma_{01}}{A_{10} + n_e \gamma_{10}} = \frac{\gamma_{01}}{\gamma_{10}} \frac{1}{1 + \frac{A_{10}}{n_e \gamma_{10}}} \quad (\text{A})$$

(i) At **high** densities, i.e., $n_e \rightarrow \infty$, (i.e., collisional excitation and deexcitation dominate \rightarrow in TE)

$$\frac{n_1}{n_0} = \frac{\gamma_{01}}{\gamma_{10}}$$

But since $\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\chi/kT}$,

$$\frac{\gamma_{01}}{\gamma_{10}} = \frac{g_1}{g_0} e^{-\chi/kT} \text{ for } n_e \gg 1$$

(ii) At **low** densities, i.e., $n_e \rightarrow 0$

$$\frac{n_1}{n_0} \rightarrow \frac{\gamma_{01}}{\gamma_{10}} \frac{n_e \gamma_{10}}{A_{10}} = \frac{n_e \gamma_{01}}{A_{10}}$$

← upward by collision
← downward by radiation only

This means every collisional excitation is followed by the emission of a photon.

For a radiative transition, the rate of emission of line photons [$\text{s}^{-1} \text{ atom}^{-1}$] ... recall eq. (A)

$$\frac{n_1}{n_0} A_{10} = A_{10} \frac{\gamma_{01}(T)}{\gamma_{10}(T)} \frac{1}{1 + \frac{A_{10}}{n_e \gamma_{10}(T)}}$$

(i) At **high** densities

$$\frac{n_1 A_{10}}{n_0} = A_{10} \frac{\gamma_{01}}{\gamma_{10}} = A_{10} \frac{g_1}{g_0} e^{-\chi/kT} \quad \longleftrightarrow n_e$$

TE

(ii) At **low** densities

$$\frac{n_1 A_{10}}{n_0} = A_{10} \frac{\gamma_{01}}{\gamma_{10}} \frac{n_e \gamma_{10}}{A_{10}} = n_e \gamma_{01} \quad \longleftrightarrow T$$

This is what we had earlier ; i.e., every collisional excitation
 → emission of a line photon.