The Origin of the Initial Mass Function

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We review recent advances in our understanding of the origin of the initial mass function (IMF). We emphasize the use of numerical simulations to investigate how each physical process involved in star formation affects the resulting IMF. We stress that it is insufficient to just reproduce the IMF, but that any successful model needs to account for the many observed properties of star forming regions including clustering, mass segregation and binarity. Fragmentation involving the interplay of gravity, turbulence, and thermal effects is probably responsible for setting the characteristic stellar mass. Low-mass stars and brown dwarfs can form through the fragmentation of dense filaments and disks, possibly followed by early ejection from these dense environments which truncates their growth in mass. Higher-mass stars and the Salpeter-like slope of the IMF are most likely formed through continued accretion in a clustered environment. The effects of feedback and magnetic fields on the origin of the IMF are still largely unclear. Lastly, we discuss a number of outstanding problems that need to be addressed in order to develop a complete theory for the origin of the IMF.

1. INTRODUCTION

One of the main goals for a theory of star formation is to understand the origin of the stellar initial mass function (IMF). There has been considerable observational work establishing the general form of the IMF (e.g., Scalo, 1986, 1998; Kroupa, 2001, 2002; Reid et al., 2002; Chabrier, 2003), but as yet we do not have a clear understanding of the physics that determines the distribution of stellar masses. The aim of this chapter is to review the physical processes that are most likely involved and to discuss observational tests that can be used to distinguish between them.

Understanding the origin of the IMF is crucial as it includes the basic physics that determines our observable universe, the generation of the chemical elements, the kinematic feedback into the ISM and overall the formation and evolution of galaxies. Once we understand the origin of the IMF, we can also contemplate how and when the IMF is likely to vary in certain environments such as the early universe and the Galactic centre.

There have been many theoretical ideas advanced to explain the IMF (cf. Miller and Scalo, 1979; Silk and Taka-  

hashi, 1979; Fleck, 1982, Zinnecker, 1982, 1984; Elmegreen and Mathieu, 1983; Yoshii and Saio, 1985; Silk, 1995,  


Meyer et al., 2000; Larson, 2003, 2005; Zinnecker et  

al., 1993; Zinnecker, 2005; Corbelli et al., 2005 and re-  

ferences therein). Most theories are ‘successful’ in that they are able to derive a Salpeter-slope IMF (Salpeter, 1955) but generally they have lacked significant predictive powers. The main problem is that it is far too easy to develop a theory, typically involving many variables, that has as a goal to explain a population distribution dependent on only one variable, the stellar mass. There have been a large number of analytical theories made to explain the IMF and therefore the probability of any one of them being correct is relatively small. It is thus imperative not only for a model to ‘explain’ the IMF, but also to develop secondary indicators that can be used to assess its likelihood of contributing to a full theory.

Recent increases of computational power have implied that numerical simulations can now include many of the relevant physical processes and be used to produce a measurable IMF that can be compared with observations. This means that we no longer have to rely on analytical arguments as to what individual processes can do but we can include these processes into numerical simulations and can test what their effect is on star formation and the generation of an IMF. Most importantly, numerical simulations provide a wealth of secondary information other than just an IMF and these can be taken to compare directly with observed properties of young stars and star forming regions. We thus concentrate in this review on the use of numerical simulations to assess the importance of the physical processes and guide us in our aim of developing a theory for the origin of the initial mass function.

The initial mass function is generally categorized by a
The IMF is an important feature of the mass distribution of stars. It is characterized by a power-law form, which can be described mathematically as:

$$dN \propto m^{-\alpha} \, dm,$$

where $dN$ is the number of stars, $m$ is the mass of a star, and $\alpha$ is the slope of the IMF. This form is observed in a wide range of stellar populations, from low-mass stars to high-mass stars. The slope $\alpha$ varies depending on the mass range and the type of stellar population.

One of the most remarkable features of IMF research is the occurrence of a marked flattening of the IMF below one solar mass, which is likely a lower mass cut-off. The upper-mass Salpeter-like slope is most important in understanding the role of star formation in affecting the evolution of galaxies and their interstellar media. The IMF for NGC3603, shown as a histogram in log mass, is a good example of how the IMF is set early in the star formation process. It appears that the form of the IMF has converged to a certain degree and is generally described as either a log-normal distribution with a power-law tail or as a series of power-laws. With the caveat that stellar masses (and ages) are difficult to extract during the pre-main sequence contraction phase, most young stellar regions have mass functions that follow a 'normal' IMF. In this case, the IMF appears to be a (near) universal function of star formation in our Galaxy.
may be an indication of the different physics there (Larson, 2005). Firstly, there is the Arches cluster which appears to have a top-heavy IMF in the resolved population (Stolte et al., 2005). Caveats are that this may be influenced by mass segregation in the cluster, incompleteness, and perhaps unresolved binaries. The second case is the Galactic centre where the massive stars are resolved (Paumard et al., 2006), but there appears little evidence for a low-mass pre-main sequence population based on expected X-ray fluxes and on dynamical mass estimates (Nayashin and Sunyaev, 2005).

3. RELEVANT OBSERVATIONAL CONSTRAINTS

It is apparent that many models have been advanced to explain the origin of the IMF. It is equally apparent that just being capable of reproducing the observed IMF is not a sufficient condition. We need observational tests and secondary indicators that can be used to distinguish between the models, be they current or in the future. In theory, most if not all observed properties of young stars (discs, velocities, clusterings) and star forming regions (mass distributions, kinematics) should be explained by a complete model for the IMF. In practice, it is presently unclear what the implications of many of the observed properties are. Here we outline a selection of potential tests that either can be used presently or are likely to be usable in the next several years.

3.1. Young stellar clusters

It is becoming increasingly apparent that most stars form in groups and clusters with the higher-mass stars forming almost exclusively in dense stellar environments. Thus, models for the IMF need to account for the clustered nature of star formation and that the environment is likely to play an important role in determining the stellar masses. For example, models for the IMF need to be able to reproduce the cluster properties in terms of stellar densities, and spatial distributions of lower and higher-mass stars.

One question is whether there is a physical correlation between the star forming environment and the formation of massive stars. A correlation between the mass of the most massive star and the stellar density of companions is seen to exist around Herbig AeBe stars (Testi et al., 1999) although this is not necessarily incompatible with random sampling from an IMF (Bonnell and Clarke, 1999). Recently, Weidner and Kroupa, (2006) have suggested that observations indicate a strong correlation between the most massive star and the cluster mass, and that a random sampling model can be excluded. Estimates of the number of truly isolated massive stars are of order 4 % or less (de Wit et al., 2005). It is therefore a necessary condition for any model for the IMF to explain how massive star formation occurs preferentially in the cores (see below) of stellar clusters where stars are most crowded.

3.2. Mass segregation

Observations show that young stellar clusters generally have a significant degree of mass segregation with the most massive stars located in the dense core of the cluster (Hillenbrand and Hartmann, 1998; Carpenter et al., 2000; Garcia and Mermilliod, 2001). For example, mass segregation is present in the Orion Nebula Cluster (ONC) where stars more massive than five solar masses are significantly more concentrated in the cluster core than are lower-mass stars (Hillenbrand and Hartmann, 1988). This suggests that either the higher-mass stars formed in the centre of the clusters, or that they moved there since their formation. Massive stars are expected to sink to the centre of the cluster due to two-body relaxation but this dynamical relaxation occurs on the relaxation time, inversely proportional to the stellar mass (e.g., Binney and Tremaine, 1987). The young stellar clusters considered are generally less than a relaxation time old such that dynamical mass segregation cannot have fully occurred. N-body simulations have shown that while some dynamical mass segregation does occur relatively quickly, especially of the most massive star, the degree of mass segregation present cannot be fully attributed to dynamical relaxation. Instead, the mass segregation is at least partially primordial (Bonnell and Davies, 1998; Littlefair et al., 2003).

For example, in the ONC at 1 million years, the massive stars need to have formed within three core radii for two-body relaxation to be able to produce the central grouping of massive stars known as the Trapezium (Bonnell and Davies, 1998). Putting the massive stars at radii greater than the half-mass radius of the cluster implies that the ONC would have to be at least 10 dynamical times old (3 to 5 million years) in order to have a 20 % chance of creating a Trapezium-like system in the centre due to dynamical mass segregation. It therefore appears to be an unavoidable consequence of star formation that higher-mass stars typically form in the centre of stellar clusters. A caveat is that these conclusions depend on estimates of the stellar ages. If the systems are significantly older than is generally believed (Palla et al., 2005), then dynamical relaxation is more likely to have contributed to the current mass segregation.

3.3. Binary systems

We know that many stars form in binary systems and that the binary frequency increases with stellar mass. Thus, the formation of binary stars is an essential test for models of the IMF. While the frequency of binaries amongst the lower-mass stars and brown dwarfs is ≈10-30 % (see chapter by Burgasser et al.), this frequency increases to ≥50 % for solar-type stars (Duquennoy and Mayor, 1991) and up to nearly 100 % for massive stars (Mason et al., 1998; Preibisch et al., 2001; Garcia and Mermilliod, 2001).

Of added importance is that many of these systems are very close, with separations less than the expected Jeans or fragmentation lengths within molecular clouds. This implies that they could not have formed at their present separations and masses but must have either evolved to smaller separations, higher masses or both. An evolution in binary separation, combined with a continuum of massive binary
4. NUMERICAL SIMULATIONS

While numerical simulations provide a useful tool to test how the individual physical processes affect the star formation and resulting initial mass function, it should be recalled that each simulation has its particular strengths and weaknesses and that no simulation to date has included all the relevant physical processes. Therefore all conclusions based on numerical simulations should be qualified by the physics they include and their abilities to follow the processes involved.

The majority of the simulations used to study the origin of the IMF have used either grid-based methods or the particle-based Smoothed Particle Hydrodynamics (SPH). Grid based codes use either a fixed Eulerian grid or an Adaptive Grid Refinement (AMR). Adaptive grids are a very important development as they provide much higher resolution in regions of high density or otherwise of interest. This allows grid-based methods to follow collapsing objects over many orders of magnitude increase in density. The resolution elements are individual cells although at least 8 cells are required in order to resolve a 3-d self-gravitating object. Grid-based methods are well suited for including additional physics such as magnetic fields and radiation transport. They are also generally better at capturing shocks as exact solutions across neighbouring grid cells are straightforward to calculate. The greatest weaknesses of grid-based methods is the necessary advection of fluid through the grid cell, especially when considering a self-gravitating fluid. Recently, Edgar et al., (2005) have shown how a resolved self-gravitating binary system can lose angular momentum as it rotates through an AMR grid forcing the system to merge artificially.

In contrast, SPH uses particles to sample the fluid and a smoothing kernel with which to establish the local hydrodynamical quantities. The resolution element is the smoothing length which generally contains \( \approx 50 \) individual particles, but this is also sufficient to resolve a self-gravitating object. Additionally, when following accretion flows, individual particles can be accreted. SPH’s primary asset is that it is Lagrangian and thus is ideally suited to follow the flow of self-gravitating fluids. Gravity is calculated directly from the particles such that it can easily follow a collapsing object. Following fragmentation requires resolving the Jeans mass (Bate and Burkert, 1997) at all points during the collapse. When the Jeans mass is not adequately resolved, fragments with masses below the resolution limit cannot be followed and are forced to disperse into the larger-scale environment. This results in the simulation only determining a lower-limit to the total number of physical fragments which should form. For the same reason, SPH cannot overestimate the number of fragments that form. Tests have repeatedly shown that artificial fragmentation does not occur in SPH (Hubber et al., 2006; Bonnell and Bate, in preparation), as any clumps that contain less than the minimum number of particles cannot collapse, be they gravitationally bound or not. Young stellar objects can be represented by sink-particles that accrete all gas that flows into their sink radius, and are bound to the star (Bate et al., 1995). This permits simulations to follow the dynamics much longer than otherwise possible and follow the accretion of mass onto individual stars. It does exclude the possibility to resolve any discs interior to the sink-radius or their susbsequent fragmentation.

Complicated fluid configurations such as occur in a turbulent medium are straightforward to follow due to the Lagrangian nature of the SPH method. Including radiative transfer and magnetic fields are more complicated due to the disorder inherent in a particle-based code. SPH also smoothes out shock fronts over at least one kernel smoothing length, but generally SPH does an adequate job of establishing the physical conditions across the shock.

SPH’s Lagrangian nature also makes it possible to trace individual fluid elements throughout a simulation in order to establish what exactly is occurring, something that is impossible with grid-based methods. Furthermore, stringent tests can be made on individual particles in order to avoid unphysical results. For example, in the classical Bondi-Hoyle accretion flows, it is necessary to resolve down below the Bondi-Hoyle radius in order to resolve the shock which allows the gas to become bound to the star. This is a grave concern in grid-based codes as otherwise the accretion can be overestimated, but is less of a worry in SPH as particles can be required to be bound before accretion occurs, even when inside the sink-radius of the accretor. This ensures that the accretion is not overestimated but under-resolved flows could result in an underestimation of the accretion rates. Particles that would shock, become bound and accreted are instead free to escape the star.
Fig. 2.— The gravitational fragmentation of molecular cloud is shown from a simulation containing initial structure (Klessen et al., 1998). The gravitational collapse enhances this structure producing filaments which fragment to form individual stars. The time \( t \) is given in units of the free-fall time.

5. PHYSICAL PROCESSES

There are a number of physical processes which are likely to play an important role in the star formation process and thus affect the resulting distribution of stellar masses. These include gravity, accretion, turbulence, magnetic fields, feedback from young stars and other semi-random processes such as dynamical ejectons.

5.1. Gravitational fragmentation

It is clear that gravity has to play an important and potentially dominant role in determining the stellar masses. Gravity is the one force which we know plays the most important role in star formation, forcing molecular clouds with densities of order \( 10^{-20} \) g cm\(^{-3} \) to collapse to form stars with densities of order 1 g cm\(^{-3} \). It is therefore likely that gravity likewise plays a dominant role in shaping the IMF.

Gravitational fragmentation is simply the tendency for gravity to generate clumpy structure from an otherwise smooth medium. It occurs when a subpart of the medium is self-gravitating, that is when gravitational attraction dominate over all support mechanisms. In astrophysics, the one support that cannot be removed and is intrinsically isotropic (such that it supports an object in three dimensions) is the thermal pressure of the gas. Thus thermal support sets a minimum scale on which gravitational fragmentation can occur. The Jeans mass, based on the mass necessary for an object to be bound gravitationally against its thermal support, can be estimated by comparing the respective energies and requiring that \( |E_{\text{grav}}| \geq E_{\text{therm}} \). For the simplest case of a uniform density sphere this yields

\[
M_{\text{Jeans}} \approx 1.1 \left( T_{10} \right)^{3/2} \left( \rho_{19} \right)^{-1/2} M_\odot, \quad (4)
\]

where \( \rho_{19} \) is the gas density in units of \( 10^{-19} \) g cm\(^{-3} \) and \( T_{10} \) is the temperature in units of 10 K. If external pressure is important, then one must use the Bonnor-Ebert mass (Ebert, 1955; Bonnor, 1956) which is somewhat smaller. The corresponding Jeans length, or minimum length scale for gravitational fragmentation is given by

\[
R_{\text{Jeans}} \approx 0.057 \left( T_{10} \right)^{1/2} \left( \rho_{19} \right)^{-1/2} \text{pc}. \quad (5)
\]

This gives an estimate of the minimum initial separation for self-gravitating fragments.

One can see that by varying the temperature and/or the density, it is straightforward to obtain the full range of Jeans masses and thus potentially stellar masses and therefore a variation in either of these variables can produce an IMF. Generally, the temperature is low before star formation and assumed to be nearly isothermal at \( \approx 10K \) such that it is the density which primarily determines the Jeans mass. Other forms of support, such as turbulence and magnetic fields have often been invoked to set the Jeans mass (e.g., McKee and Tan, 2003), but their relevance to gravitational fragmentation is doubtful due to their non-isotropic nature.

Fig. 3.— The IMF that results from isothermal gravitational fragmentation (e.g., Fig. 2) is typically broad and log-normal in shape (Klessen et al., 1998). The stellar masses are measured in terms of the average initial Jeans mass of the cloud.

The primary requirement for gravitational fragmentation is that there exists sufficient initial structure to provide a focus for the gravity. In a smooth uniform sphere, even if subregions are gravitationally unstable they will all collapse and merge together at the centre of the cloud (Layzer, 1963). Some form of seeding is required such that the local
Gravitational fragmentation is unlikely to determine the full mass spectrum. It is difficult to see how gravitational fragmentation could account for the higher-mass stars. These stars are born in the dense cores of stellar clusters where stars are fairly closely packed. Their separations can be used to limit the sizes of any pre-stellar fragments via the Jeans radius, the minimum radius for an object to be gravitationally bound. This, combined with probable gas temperatures, imply a high gas density and thus a low Jeans mass objects such as brown dwarfs (Bate et al., 2002a). Dense circumstellar discs also provide the necessary low Jeans mass in order to form low-mass stars and brown dwarfs. Numerical simulations of gravitational fragmentation can thus explain the characteristic stellar mass and the roughly flat (in log space) distribution of lower-mass stars. These stars are born in the dense cores of stellar clusters. In general, gravitational fragmentation would be expected to instill a reverse mass segregation, the opposite of which is seen in young clusters.

Stellar masses significantly lower than the characteristic stellar mass are also explainable through gravitational fragmentation. In a collapsing region the gas density can increase dramatically and this decreases the Jeans mass. The growth of filamentary structure in the collapse (see Fig. 4), due to the funneling of gas into local potential minima, can then provide the seeds for fragmentation to form very low mass objects such as brown dwarfs (Bate et al., 2002a). Dense circumstellar discs also provide the necessary low Jeans mass in order to form low-mass stars and brown dwarfs. Numerical simulations of gravitational fragmentation can thus explain the characteristic stellar mass and the roughly flat (in log space) distribution of lower-mass stars and brown dwarfs (Bate et al., 2003).

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Fig. 4.— The fragmentation of filamentary structure, and the formation of low-mass stars and brown dwarfs, is shown in a simulation of the formation of a small stellar cluster (Bate et al., 2003).

\[
t_{ff} = \left( \frac{3\pi}{32G\rho} \right)^{1/2}
\]

is shorter than the global free-fall time of the cloud. In principle, any small perturbations can be sufficient as long as the gas remains nearly isothermal (Silk, 1982), but fragmentation is effectively halted once the gas becomes optically thick and the collapse slows down (Tohline, 1982; Boss, 1986). Simple deformations in the form of sheets (Larson, 1985; Burkert and Hartmann, 2004) or filaments (Larson, 1985; Bastien et al., 1991) are unstable to fragmentation as the local free-fall time is much shorter than that for the object as a whole. This allows any density perturbations to grow non-linear during the collapse. More complex configurations (see Figure 2) in the initial density field can equally result in gravitational fragmentation (e.g., Elmegreen and Falgarone, 1996; Elmegreen, 1997, 1999; Klessen et al., 1998; Klessen and Burkert, 2000, 2001). The origin of such density fluctuations can be due to the ‘turbulent’ velocity field seen in molecular clouds (see below) providing the seeds for the gravitational fragmentation (Klessen, 2001; Bate et al., 2003; Bonnell et al., 2003, 2006a).

One of the general outcomes of gravitational fragmentation is that an upper limit to the number of fragments is approximately given by the number of Jeans masses present in the cloud (Larson, 1978, 1985; Bastien et al., 1991; Klessen et al., 1998, Bate et al., 2003). This is easily understood as being the number of individual elements within the cloud that can be gravitationally bound. This results in an average fragment mass that is of order the Jeans mass at the time of fragmentation (e.g., Klessen et al., 1998; Klessen, 2001; Bonnell et al., 2004, 2006a; Clark and Bonnell, 2005; Jappsen et al., 2005). The Jeans criterion can then be thought of as a criterion to determine the characteristic stellar mass and thus provides the foundations for the origin of the IMF. Resulting IMFs are log-normal in shape (Fig. 3, Klessen et al., 1998; Klessen and Burkert, 2001; Klessen, 2001; Bate et al., 2003). The problem is then what determines the Jeans mass at the point of fragmentation. There are two possible solutions. First, that the initial conditions for star formation always have the same physical conditions and thus the same Jeans mass of order a $M_\odot$, which would seem unlikely. The second solution requires some additional thermal physics which sets the Jeans mass at the point where fragmentation occurs (Larson, 2005, Spaans and Silk, 2000).

The coupling of gas to dust may provide the necessary physics to change from a cooling equation of state ($T \propto \rho^{-0.25}$) to one including a slight heating ($T \propto \rho^{0.1}$) with increasing gas densities (Larson, 2005). This provides a method of setting the characteristic stellar mass which is then independent of the initial conditions for star formation. Numerical simulations using a simple cooling/heating prescription to mimic the effects of this transition show that this sets the fragment mass and thus the peak of the IMF (Jappsen et al., 2005; Bonnell et al., 2006a). Indeed, starting from initial conditions with a Jeans mass of $5M_\odot$, which in an isothermal simulation provide a nearly flat (in log mass) IMF up to $\approx 5M_\odot$, the cooling/heating equation of state reduces this characteristic mass to below $1M_\odot$ (Jappsen et al., 2005; Bonnell et al., 2006a), allowing for an upper-mass Salpeter-like slope due to subsequent accretion (see below).

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5.2. Turbulence

It has long been known that supersonic motions are contained within molecular clouds (Larson 1981). These motions are generally considered as being turbulent principally because of the linewidth-size relation $\sigma \propto R^{0.5}$ (Larson, 1981; Heyer and Brunt, 2004) that mimics the expectation for turbulence (Elmegreen and Scalo, 2004) and implies an energy cascade from large to small scales. Alternatively, the clouds could simply contain random bulk motions generated at all scales such as occurs in a clumpy shock (Bonnell et al., 2006b). Nevertheless, for the purposes of this review, we define turbulence as supersonic irregular motions in the clouds that contribute to the support of these clouds (see chapter by Ballesteros-Paredes et al.). It is well known that turbulence or its equivalent can generate density structures in molecular clouds due to supersonic shocks that compress the gas (Elmegreen, 1993; Vazquez-Semadeni, 1994; Padoan, 1995; Stone et al., 1998; Mac Low et al., 1998; Ostriker et al., 1999; Mac Low and Klessen, 2004; Elmegreen and Scalo, 2004). The resultant distribution of density structures, generally referred to as turbulent fragmentation, can either provide the seeds for a gravitational fragmentation (e.g., references above, especially Mac Low and Klessen, 2004), or alternatively could determine the IMF directly at the pre-stellar core phase of star formation (Padoan et al., 1997, 2001; Padoan and Nordlund, 2002).

Turbulent fragmentation provides an attractive mechanism to explain the IMF as it involves only one physical process, which is observed to be ubiquitous in molecular clouds (Elmegreen 1993; Padoan et al., 1997; Padoan and Nordlund, 2002). Multiple compressions result in the formation of sheets and then filaments in the cloud. The density $\rho$ and widths $w$ of these filaments are due to the (MHD) shock conditions such that higher Mach number shocks produce higher density but thinner filaments (Padoan and Nordlund, 2002). Clump masses can then be derived assuming that the shock width gives the three-dimensional size of the clump $(M \propto \rho w^3)$. High velocity shocks produce high density but small clumps, and thus the lowest mass objects. In contrast, lower velocity shocks produce low-density but large shocks which account for the higher-mass clumps. Using the power-spectrum of velocities from numerical simulations of turbulence, and estimates of the density, $\rho$, and width, $w$, of MHD shocks as a function of the flow speed, Padoan and Nordlund, (2002) derive a clump mass distribution for turbulent fragmentation. The turbulent spectrum results in a “universal” IMF slope which closely resembles the Salpeter slope. At lower masses, consideration of the likelihood that these clumps are sufficiently dense to be Jeans unstable produces a turnover and a log normal shape into the brown dwarf regime. This is calculated from the fraction of gas that is over the critical density for a particular Jeans mass, but it does not require that this gas is in a particular core of that mass.

Numerical simulations using grid-based codes have investigated the resulting clump-mass distribution from turbulent fragmentation. While Padoan and Nordlund (2004) have reported results consistent with their earlier analytical models, Ballesteros-Paredes et al. (2006a) conclude that the high-mass end of the mass distribution is not truly Salpeter but becomes steeper at higher masses. Furthermore, the shape depends on the Mach number of the turbulence implying that turbulent fragmentation alone cannot reproduce the stellar IMF (Ballesteros-Paredes et al., 2006a). The difference is attributed to having multiple shocks producing the density structure, which then blurs the relation between the turbulent velocity spectrum and the resultant clump-mass distribution. Thus, the higher mass clumps in the Padoan and Nordlund model have internal motions which will sub-fragment them into smaller clumps (see chapter by Ballesteros-Parades et al.).

The above grid-based simulations are generally not able to follow any gravitational collapse and star formation so the question remains open what stellar IMF would result.
SPH simulations that are capable of following the gravitational collapse and star formation introduce a further complication. These simulations find that most of the clumps are generally unbound and therefore do not collapse to form stars (Klessen et al., 2005; Clark and Bonnell, 2005). It is only the most massive clumps that become gravitationally unstable and form stars. Gravitational collapse requires masses of order the unperturbed Jeans mass of the cloud suggesting that the turbulence has played only a minor role in triggering the star formation process (Clark and Bonnell, 2005). Even then, these cores often contain multiple thermal Jeans masses and thus fragment to form several stars.

In terms of observable predictions, the Padoan and Nordlund turbulent compression model suggests, as does gravitational fragmentation, that the minimum clump separations scales with the mass of the core. Thus, lower-mass clumps can be closely packed whereas higher-mass cores need to be well separated. If these clumps translate directly into stars as required for turbulent compression to generate the IMF, then this appears to predict an initial configuration where the more massive stars are in the least crowded locations. Unless they can dynamically migrate to the cores of stellar clusters fairly quickly, then their formation is difficult to attribute to turbulent fragmentation.

Turbulence has also been invoked as a support for massive cores (McKee and Tan, 2003) and thus as a potential source for massive stars in the centre of clusters. The main idea is that the turbulence acts as a substitute for thermal support and the massive clump evolves as if it was very warm and thus has a much higher Jeans mass. The difficulty with this is that turbulence drives structures into objects and therefore any turbulently supported clump is liable to fragment, forming a small stellar cluster instead of one star. SPH simulations have shown that, in the absence of magnetic fields, a centrally condensed turbulent core fragments readily into multiple objects (Dobbs et al., 2005). The fragmentation is somewhat suppressed if the gas is already optically thick and thus non-isothermal. Heating from accretion onto a stellar surface can also potentially limit any fragmentation (Krumholz, 2006) but is likely to arise only after the fragmentation has occurred. In fact, the difficulty really lies in how such a massive turbulent core could form in the first place. In a turbulent cloud, cores form and dissipate on dynamical timescales suggesting that forming a long-lived core is problematic (Ballesteros-Paredes et al., 1999; Vazquez-Semadeni et al., 2005). As long as the region contains supersonic turbulence, it should fragment on its dynamical timescale long before it can collapse as a single entity. Even MHD turbulence does not suppress the generation of structures which will form the seeds for fragmentation (see chapter by Ballesteros-Paredes et al.).

The most probable role for turbulence is as a means for generating structure in molecular clouds. This structure then provides the finite amplitude seeds for gravitational fragmentation to occur, while the stellar masses are set by the local density and thermal properties of the shocked gas. The formation of lower-mass stars and brown dwarfs directly from turbulent compression is still an open question as it is unclear if turbulent compression can form gravitationally bound cores at such low masses. Turbulent compression is least likely to be responsible for the high-mass slope of the IMF as numerical simulations suggest that the high-end core-mass distribution is not universal and does not follow a Salpeter-like slope (see Fig. 6).

### 5.3. Accretion

![Fig. 7.— A schematic diagram of the physics of accretion in a stellar cluster: The gravitational potential of the individual stars form a larger scale potential that funnels gas down to the cluster core. The stars located there are therefore able to accrete more gas and become higher-mass stars. The gas reservoir can be replenished by infall into the large-scale cluster potential.](image)

Gas accretion is a major process that is likely to play an important role in determining the spectrum of stellar masses. To see this, one needs to consider three facts. First, gravitational collapse is highly non-homologous (Larson, 1969) with only a fraction of a stellar mass reaching stellar densities at the end of a free-fall time. The vast majority of the eventual star needs to be accreted over longer timescales. Secondly, fragmentation is highly inefficient with only a small fraction of the total mass being initially incorporated into the self-gravitating fragments (Larson, 1978; Bate et al., 2003). Thirdly, and most importantly, mm observations of molecular clouds show that even when significant structure is present, this structure only comprises a few percent of the mass available (Motte et al., 1998; Johnstone et al., 2000). The great majority of the cloud mass is in a more distributed form at lower column densities, as detected by extinction mapping (Johnstone et al., 2004). Young stellar clusters are also seen to have 70 to 90 % of their total mass in the form of gas (Lada and Lada, 2003). Thus, a large gas reservoir exists such that if accretion of this gas does occur, it is likely to be the dominant contributor to the final stellar masses and the IMF.

Models using accretion as the basis for the IMF rely essentially on the equation

$$M_\star = \dot{M}_\star t_{\text{acc}}, \quad (7)$$
and by having a physical model to vary either the accretion rate \( \dot{M}_s \) or the accretion timescale \( t_{\text{acc}} \), can easily generate a full distribution of stellar masses. In fact, accretion can be an extremely complex time-dependent phenomenon (e.g., Schmeja and Klessen, 2004) and it may occur in bursts suggesting that we should consider the above equation in terms of a mean accretion rate and timescale. The accretion rates can be varied by being mass dependent (Larson, 1978; Zinnecker, 1982; Bonnell et al., 2001b), dependent on variations of the gas density (Bonnell et al., 1997, 2001a) or on the relative velocity between gas and stars (Bondi and Hoyle, 1944; Bate et al., 2003). Variations in \( t_{\text{acc}} \) (Basu and Jones, 2004; Bate and Bonnell, 2005) can be due to ejections in clusters (Bate et al., 2002a, see §5.6 below) or feedback from forming stars (Shu et al., 2004; Dale et al., 2005, see §5.5 below).

The first models based on accretion (Larson, 1978, 1982; Zinnecker, 1982) discussed how stars compete from the available mass in a reservoir. Stars that accrete slightly more due to their initial mass or proximity to more gas (Larson, 1992) increase their gravitational attraction and therefore their ability to accrete. The depletion of the gas reservoir means that there is less for the remaining stars to accrete. This competitive accretion then provides a reason why there are a few high-mass stars compared to a much larger number of low-mass stars.

In a stellar cluster, the accretion is complicated by the overall potential of the system. Figure 7 shows schematically the effect of the cluster potential on the competitive accretion process. The gravitational potential is the combined potential of all the stars and gas contained in the cluster. This potential then acts to funnel gas down to the centre of the cluster such that any stars located there have significantly higher accretion rates (Bonnell et al., 1997, 2001a). These stars therefore have a greater ability to become high-mass stars due to the higher gas density and due to the fact that this gas is constantly being replenished by infall from the outer part of the cluster. Stars that accrete more are also more liable to sink to the centre of the potential and thereby increase their accretion rates further. It is worthwhile noting here that this process would occur even for a static potential where the stars do not move. The gas is being drawn down to the centre of the potential. It has to settle somewhere, and if it is already a self-gravitating fragment (i.e., a protostar), it will fall into the local potential of one of the stars.

Stars not in the centre of the cluster accrete less as gas is spirited away towards the cluster centre. This ensures that the mean stellar mass remains close to the characteristic mass given by the fragmentation process. Accretion rates onto individual stars depend on the local gas density, the mass of the star and the relative velocity between the gas and the star:

\[
\dot{M}_s \approx \pi \rho v_{\text{rel}} R_{\text{acc}}^2,
\]

where \( R_{\text{acc}} \) is the accretion radius which depends on the mass of the star (see below). The accretion radius is the radius at which gas is irrevocably bound to the star. As a cautionary note, in a stellar cluster the local gas density depends on the cluster potential and the relative gas velocity can be very different from the star’s velocity in the rest frame of the cluster, as both gas and stars are experiencing the same accelerations.

Numerical simulations (Bonnell et al., 2001a) show that in a stellar cluster the accretion radius depends on whether the gas or the stars dominate the potential. In the former case, the relative velocity is low and accretion is limited by the star’s tidal radius. This is given by

\[
R_{\text{tidal}} \approx 0.5 \left( \frac{M_s}{M_{\text{enc}}} \right)^{\frac{1}{3}} R_s,
\]

which measures at what distance gas is more bound to an individual star rather than being tidally sheared away by the overall cluster potential. The tidal radius depends on the star’s position in the cluster, via the enclosed cluster mass \( M_{\text{enc}} \) at the radial location of the star \( R_s \). The alternative is if the stars dominate the potential, then the relative velocity between the gas and the stars can be high. The accretion radius is then the more traditional Bondi-Hoyle radius of the form

\[
R_{\text{BH}} \approx 2GM_s/(v_{\text{rel}}^2 + c_s^2).
\]

It is always the smaller of these two accretion radii which determines when gas is bound to the star and thus should be used to determine the accretion rates. We note again that the relative gas velocity can differ significantly from the star’s velocity in the rest frame of the cluster. Using a simple model for a stellar cluster, it is straightforward to show that these two physical regimes result in two different IMF slopes because of the differing mass dependencies in the accretion rates (Bonnell et al., 2001b). The tidal radius accretion has \( \dot{M}_s \propto M_s^{2/3} \) and, in a \( n \propto r^{-2} \) stellar density distribution results in a relatively shallow \( dN \propto M_s^{-1.5} dM_s \) (cf. Klessen and Burkert, 2000). Shallower stellar density distributions produce steeper IMFs. For accretion in a stellar dominated potential, Bondi-Hoyle accretion in a uniform gas distribution results in an IMF of the form \( dN \propto M_s^{-2} dM_s \) (Zinnecker, 1982). To see this, consider an accretion rate based on equations (8) and (10)

\[
\dot{M}_s \propto M_s^2,
\]

with a solution

\[
M_s = \frac{M_0}{1 - \beta M_0 t}
\]

where \( M_0 \) is the initial stellar mass and \( \beta \) includes the dependence on gas density and velocity (assumed constant in time). From equation (12) we can derive a mass function \( dN = F(M_s) dM_s \) by noting that there is a one-to-one mapping of the initial and final stellar masses (i.e., that the total number of stars is conserved and that there is a monotonic relation between initial and final masses) such that

\[
F(M_s) dM_s = F(M_0) dM_0.
\]
Using equation (12) we can easily derive

\[ F(M_*) = F(M_0) \left( \frac{M_*}{M_0} \right)^{-2}. \] (14)

which, in the case where there is only a small range of initial stellar masses and for \( M_* \gg M_0 \), gives

\[ dN \propto M_*^{-2} dM_*, \] (15)

whereas if the ‘initial’ mass distribution is initially significant and decreasing with increasing masses, then the resulting IMF is steeper. In a stellar cluster with a degree of mass segregation from an earlier gas-dominated phase, this results in a steeper IMF closer to \( dN \propto M_*^{-2.5} dM_* \) (Bonnell et al., 2001b). This steeper IMF is therefore appropriate for the more massive stars that form in the core of a cluster because it is there that the stars first dominate the cluster potential. Although the above is a semi-analytical model and suffers from the pitfalls described in introduction, it is comforting to note that numerical simulations do reproduce the above IMFs and additionally show that the higher-mass stars accrete the majority of their mass in the stellar dominated regime which should, and in this case does, produce the steeper Salpeter-like IMF (Bonnell et al., 2004).

A recent numerical simulation showing the fragmentation of a turbulent molecular cloud and the formation of a stellar cluster is shown in Figure 8. The newly formed stars fall into local potential minima, forming small-N systems which subsequently merge to form one larger stellar cluster. The initial fragmentation produces objects with masses comparable to the mean Jeans mass of the cloud (\( \approx 0.5 M_\odot \)) which implies that they are formed due to gravitational, not turbulent, fragmentation. It is the subsequent competitive accretion which forms the higher-mass stars (Bonnell et al., 2004) and thus the Salpeter-like power law part of the IMF. Overall, the simulation forms a complete stellar population that follows a realistic IMF from 0.1 \( M_\odot \) to 30 \( M_\odot \) (Fig.9 and Bonnell et al., 2003). Accretion forms six stars in excess of 10 \( M_\odot \) with the most massive star nearly 30 \( M_\odot \). Each forming sub-cluster contains a more massive star in its centre and has a population consistent with a Salpeter IMF (Bonnell et al., 2004).

One of the advantages of such a model for the IMF is that it automatically results in a mass segregated cluster. This can be seen from the schematic Figure 7 showing how the stars that are located in the core of the cluster benefit from the extended cluster potential to increase their accretion rates over what they would be in isolation. Thus stars more massive than the mean stellar mass should be relatively mass segregated from birth in the cluster. This is shown in Figure 10 which displays the distribution of low-mass stars with the higher-mass stars located in the centre of individual clusters. There is always a higher-mass star in every (sub)-cluster. Even when individual sub-clusters merge, the massive stars quickly settle into the centre of the combined potential thereby benefitting most from any continuing accretion. One of the strong predictions of competitive accretion is that there is a direct correlation between
the formation of a stellar cluster and the most massive star it contains. Accretion and the growth of the cluster are linked such that the system always has a realistic IMF.

Fig. 10.— The location of the massive stars (dark circles) is shown to be in the centre of individual subclusters of low-mass stars (light circles) due to competitive accretion (cf. Bonnell et al., 2004).

There have recently been some concerns raised that accretion cannot produce the high-mass IMF either due to numerical reasons (Krumholz et al., 2006) or due to the turbulent velocity field (Krumholz et al., 2005a). The numerical concern is that SPH calculations may overestimate the accretion rates if they do not resolve the Bondi-Hoyle radius. However, SPH simulations, being particle based, ensure that unphysical accretion does not occur by demanding that any gas that is accreted is bound to the star. The second concern is that accretion rates should be too low in a turbulent medium to affect the stellar masses. Unfortunately, this study assumes that gravity is negligible on large scales except as a boundary condition for the star forming clump. This cannot be correct in a forming stellar cluster where both gas and stars undergo significant gravitational accelerations from the cluster potential. Furthermore, Krumholz et al. take a virial velocity for the clump to use as the turbulent velocity field (Krumholz et al., 2005a). SPH simulations show that mass accretion occurs from lower velocity gas initially, proceeding to higher velocities when the stellar mass is larger, consistent with both the requirements of the turbulent scaling laws and Bondi-Hoyle accretion (Bonnell and Bate, 2006, in preparation).

5.4. Magnetic Fields

Magnetic fields are commonly invoked as an important mechanism for star formation and thus need to be considered as a potential mechanism for affecting the IMF. Magnetic fields were initially believed to dominate molecular clouds with ambipolar diffusion of these fields driving the star formation process (Mestel and Spitzer, 1956; Shu et al., 1987). Since the realisation that ambipolar diffusion takes too long, and that it would inhibit fragmentation and thus the formation of multiple stars and clusters, and crucially that supersonic motions are common in molecular clouds, the perceived role of magnetic fields has been revised to one of increasing the lifetime of turbulence (Arons and Max, 1975; Lizano and Shu, 1989). More recently, it has been shown that magnetic fields have little effect on the decay rate of turbulence as they do not fully cushion shocks (Mac Low et al., 1997; Stone et al., 1997). Still, magnetic fields are likely to be generally present in molecular clouds and can play an important, if still relatively unknown role.

There have been many studies into the evolution of MHD turbulence and structure formation in molecular clouds (e.g., Ostriker et al., 1999, Vazquez-Semadeni et al., 2000; Heitsch et al., 2001; Tilley and Pudritz, 2005; Li and Nakamura, 2004; see chapter by Balesteros-Paredes et al.). These simulations have found that both MHD and pure HD simulations result in similar clump-mass distributions. One difference is that the slightly weaker shocks in MHD turbulence shift the clump-masses to slightly higher masses.

One potential role for magnetic fields which has not been adequately explored is that they could play an important role in setting the characteristic stellar mass in terms of an effective magnetic Jeans mass. Although in principle this is easy to derive, it is unclear how it would work in practice as magnetic fields are intrinsically non-isotropic and therefore the analogy to an isotropic pressure support is difficult to make. Recent work on this by Shu et al. (2004) has investigated whether magnetic levitation, the support of the outer envelopes of collapsing cores, can set the characteristic mass. Inclusion of such models into numerical simulations is needed to verify if such processes do occur.

5.5. Feedback

Observations of star forming regions readily display the fact that young stars have a significant effect on their environment. This feedback, including jets and outflows from low-mass stars, and winds, ionisation, and radiation pressure from high-mass stars, is therefore a good candidate to halt the accretion process and thereby set the stellar masses (Silk, 1995; Adams and Fatuzzo, 1996). To date, it has been difficult to construct a detailed model for the IMF from feedback as it is a rather complex process. Work is ongoing to include the effects of feedback in numerical models of star formation but have not yet been able to generate stellar mass functions (Li and Nakamura, 2005). In these models, feedback injects significant kinetic energy into the system which appears to quickly decay away again (Li and Nakamura, 2005). Overall, the system continues to evolve (collapse) in a similar way to simulations that neglect both feedback and magnetic fields (e.g., Bonnell et al., 2003). Nevertheless, we can perhaps garner some insight from recent numerical simulations including the effects of ionisation from massive stars (Dale et al., 2005). The inclusion of ionisation from an O star into a simulation of the formation of a stellar cluster shows that the intrinsically isotropic
radiation escapes in preferential directions due to the non-uniform gas distributions (see also Krumholz et al., 2005b). Generally, the radiation decreases the accretion rates but does not halt accretion. In more extreme cases where the gas density is lower, the feedback halts the accretion almost completely for the full cluster. This implies that feedback can stop accretion but probably not differentially and therefore does not result in a non-uniform $f_{\text{acc}}$ which can be combined with a uniform $\dot{M}$ to form a stellar IMF.

Feedback from low-mass stars is less likely to play an important role in setting the IMF. This is simply due to the well collimated outflows being able to deposit their energy at large distances from the star forming environment (Stanke et al., 2000). As accretion can continue in the much more hostile environment of a massive star where the feedback is intrinsically isotropic, it is difficult to see a role for well collimated outflows in setting the IMF.

### 5.6. Stellar interactions

The fact that most stars form in groups and clusters, and on smaller scales in binary and multiple systems, means that they are likely to interact with each other on timescales comparable to that for gravitational collapse and accretion. By interactions, we generally mean gravitational interactions (Reipurth and Clarke, 2001) although in the dense cores of stellar clusters this could involve collisions and mergers (Bonnell et al., 1998, Bally and Zinnecker, 2005). These processes are essentially random with a probability given by the stellar density, velocity dispersion and stellar mass. Close encounters with binary or higher-order systems generally result in an exchange of energy which can eject the lower-mass objects of the encounter (Reipurth and Clarke, 2001). Such an event can quickly remove an accreting star from its gas reservoir, thereby truncating its accretion and setting the stellar mass. This process is what is seen to occur in numerical simulations of clustered star formation (Bate et al., 2002a, 2003) where low-mass objects are preferentially ejected. These objects are often then limited to being brown dwarfs whereas they could have accreted up to stellar masses had they remained in the star forming core (see also Price and Podsialowski, 1995).

Numerical simulations including the dynamics of the newly formed stars have repeatedly shown that such interactions are relatively common (McDonald and Clarke, 1995; Bonnell et al., 1997; Sterzik and Durisen, 1998, 2003; Klessen and Burkert, 2001; Bate et al., 2003, Bate and Bonnell, 2005). especially in small-N or subclusters where the velocity dispersion is relatively low. Thus, such a mechanism should populate the entire regime from the smallest Jeans mass formed from thermal (or turbulent) fragmentation up to the characteristic mass. This results in a relatively flat IMF (in log mass) for low-mass objects (Klessen and Burkert, 2001; Bate et al., 2003, Bate and Bonnell, 2005; Bate, 2005; Delgado-Donate et al., 2004).

Stellar mergers are another quasi-random event that could occur in very dense cores of stellar clusters involving mergers of intermediate or high-mass single (Bonnell et al., 1998; Bonnell and Bate, 2002; Bally and Zinnecker, 2005) or binary (Bonnell and Bate, 2005) stars. In either case, mergers require relatively high stellar densities of order $10^8$ and $10^9$ stars pc$^{-3}$ respectively. These densities, although higher than generally observed are conceivably due to a likely high density phase in the early evolution of stellar clusters (Bonnell and Bate, 2002; Bonnell et al., 2003). In fact, estimates of the resolved central stellar density in 30 Doradus and the Arches cluster are of order a few $\times 10^5$ M$_\odot$ pc$^{-3}$ (Hoffmann et al., 1995, Stolte et al., 2002). Such events could play an important role in setting the IMF for the most massive stars.

### 5.7. Summary of processes

From the above arguments and the expectation of the different physical processes, we can start to assess what determines the stellar masses in the various regimes (Fig. 11). A general caveat should be noted that we still do not have a thorough understanding of what magnetic fields and feedback can do but it is worth noting that in their absence we can construct a working model for the origin of the IMF. First of all, we conclude that the characteristic stellar mass and the broad peak of the IMF is best attributed to gravitational fragmentation and the accompanying thermal physics which sets the mean Jeans mass for fragmentation. The broad peak can be understood as being due to the dispersion in gas densities and temperature at the point where fragmentation occurs. Turbulence is a necessary condition in that it generates the filamentary structure in the molecular clouds which facilitates the fragmentation, but does not itself set the median or characteristic stellar mass.

Lower-mass stars are most likely formed through the gravitational fragmentation of a collapsing region such that the increased gas density allows for lower-mass fragments. These fragments arise in collapsing filaments and circumstellar discs (Bate et al., 2002a). A crucial aspect of this mechanism for the formation of low-mass stars and brown dwarfs is that they not be allowed to increase their mass significantly through accretion. If lower-mass stars are indeed formed in gas dense environments to achieve the low Jeans masses, then subsequent accretion can be expected to be significant. Their continued low-mass status requires that they are ejected from their natal environment, or at least that they are accelerated by stellar interactions such that their accretion rates drops to close to zero. The turbulent compressional formation of low-mass objects (Padoan and Nordlund, 2004) is potentially a viable mechanism although conflicting simulations have raised doubts as to whether low-mass gravitationally bound cores are produced which then collapse to form stars.

Lastly, we conclude that the higher-mass IMF is probably due to continued accretion in a clustered environment. A turbulent compression origin for higher mass stars is problematic as the core-mass distribution from turbulence does not appear to be universal (Ballesteros-Paredes
Furthermore, the large sizes of higher mass prestellar cores generated from turbulence suggest that they should be found in low stellar density environments, not in the dense cores of stellar clusters. Nor should they then be in close, or even relatively wide, binary systems. In contrast, the ability of the cluster potential to increase accretion rates onto the stars in the cluster centre is a simple explanation for more massive stars in the context of low-mass star formation. Continuing accretion is most important for the more massive stars in a forming cluster because it is these that settle, and remain, in the denser central regions. This also produces the observed mass segregation in young stellar clusters. The strong mass dependency of the accretion rates ($\propto M^2$) results in a Salpeter-like high-mass IMF.

Continued accretion and dynamical interactions can also potentially explain the existence of closer binary stars, and the dependency of binary properties on stellar masses (Bate et al., 2002b; Bonnell and Bate, 2005; Sterzik et al., 2003; Durisen et al., 2001). Dynamical interactions harden any existing binary (Sterzik and Durisen, 1998, 2003; Kroupa, 1995) and continued accretion increases both the stellar masses and the binding energy of the system (Bate and Bonnell, 1997; Bonnell and Bate, 2005). This can explain the higher frequency of binary systems amongst massive stars, and the increased likelihood that these systems are close and of near-equal masses.

![Schematic IMF](image)

Fig. 11.— A schematic IMF showing the regions that are expected to be due to the individual processes. The peak of the IMF and the characteristic stellar mass are believed to be due to gravitational fragmentation, while lower mass stars are best understood as being due to fragmentation plus ejection or truncated accretion while higher-mass stars are understood as being due to accretion.

6. OUTSTANDING PROBLEMS

There are outstanding issues which need to be resolved in order to fully understand the origin of the IMF. Some of these involve detailed understanding of the process (e.g., massive star formation, mass limits) whereas others include new observations which may be particularly useful in determining the origin of the IMF.

6.1. Clump-mass spectrum

In order for the stellar IMF to come directly from the clump-mass spectra observed in molecular clouds (e.g., Motte et al., 1998; Johnstone et al., 2000), a one-to-one mapping of core clump to stellar mass is required. The high frequency of multiple systems even amongst the youngest stars (Duchêne et al., 2004) makes a one-to-one mapping unlikely for masses near solar and above. At lower masses, the reduced frequency of binary systems (e.g., Lada, 2006) means that a one-to-one mapping is potentially viable. Another potential difficulty is that some, especially lower mass, clumps are likely to be transient (Johnstone et al., 2000). Simulations commonly report that much of the lower-mass structure formed is gravitationally unbound (Klessen, 2001; Clark and Bonnell, 2005; Tilley and Puurijärvi, 2005). Furthermore, as such mass spectra can be understood to arise due to purely hydrodynamical effects without any self-gravity (e.g., Clark and Bonnell, 2006), the relevance for star formation is unclear. If the clump-mass spectrum does play an integral role in the origin of the IMF, then there should be additional evidence for this in terms of observational properties that can be directly compared. For example, the clustering and spatial mass distribution of clumps should compare directly and favorably to that of the youngest class 0 sources (e.g., Elmegreen and Krakowski, 2001).

6.2. Massive stars

The formation of massive stars, with masses in excess of $10M_\odot$, is problematic due to the high radiation pressure on dust grains and because of the dense stellar environment in which they form. The former can actually halt the infall of gas and thus appears to limit stellar masses. Simulations to date suggest that this sets an upper-mass limit to accretion somewhere in the 10 to 40 $M_\odot$ range (Wolfire and Casella, 1986; Yorke and Sonnhalter, 2002; Edgar and Clarke, 2004). Clearly, there needs to be a mechanism for circumventing this problem as stars as massive as 80-150 $M_\odot$ exist (Massey and Hunter, 1998; Weidner and Kroupa, 2004; Figer, 2005). Suggested solutions include disc accretion and radiation beaming, ultra high accretion rates that overwhelm the radiation pressure (McKee and Tan, 2003), Rayleigh Taylor instabilities in the infalling gas (Krumholz et al., 2005) and stellar collisions (Bonnell et al., 1998; Bonnell and Bate, 2002, 2005). The most complete simulations of disk accretion (Yorke and Sonnhalter, 2002) suggest that radiation beaming due to the star’s rapid rotation, combined with disc accretion can reach stellar masses of order 30 to 40 $M_\odot$, although with low efficiencies. What is most important for any mechanism for massive star formation is that it be put into the context of forming a full IMF (e.g., Bonnell et al., 2004). The most likely scenario for massive star formation involves a combination of many of the above processes, competitive accretion in order to set the distribution of stellar masses, disk accretion, radiation beaming and potentially Rayleigh-Taylor instabilities.
or even stellar mergers to overcome the radiation pressure. Any of these could result in a change in the slope of high-mass stars reflecting the change in physics.

6.3. Mass limits

Observationally, it is unclear what limits there are on stellar masses. At low masses, the IMF appears to continue as far down as is observable. Upper-mass limits are on firmer ground observationally with strong evidence of a lack of stars higher than \( \approx 150 \, M_\odot \) even in regions where statistically they are expected (Figer, 2005; Oey and Clarke, 2005; Weidner and Kroupa, 2004). Physically, the only limitation on the formation of low-mass objects is likely to be the opacity limit whereby an object cannot cool faster than it contracts, setting a lower limit for a gravitationally bound object (Low and Lynden-Bell, 1976; Rees, 1976; Boyd and Whitworth, 2005). This sets a minimum Jeans mass of order \( 3 - 10 \) Jupiter masses. At the higher-end physical limits could be set by radiation pressure on dust or electrons (the Eddington limit), or by physical collisions.

6.4. Clustering and the IMF

Does the existence of a bound stellar cluster affect the high-mass end of the IMF? If accretion in a clustered environment is responsible for the high-mass IMF, then there should be a direct link between cluster properties and the presence of high-mass stars. Competitive accretion models require the presence of a stellar cluster in order for the distributed gas to be sufficient to form high-mass stars. Thus, a large-N cluster produces a more massive star than does the same number of stars divided into many small-N systems (e.g., Weidner and Kroupa, 2006). The combined number of stars in the small-N systems should show a significant lack of higher-mass stars. Evidence for such an environmental dependence on the IMF has recently been argued based on observations of the Vela D cloud (Massi et al., 2006). The six clusters together appear to have a significant lack of higher-mass stars in relation to the expected number from a Salpeter-like IMF and the total number of stars present. A larger statistical sample of small-N systems is required to firmly establish this possibility.

7. SUMMARY

We can now construct a working model for the origin of the IMF based on the physical processes known to occur in star formation and their effects determined through numerical simulations (Fig. 11). This working model attributes the peak of the IMF and the characteristic stellar mass to gravitational fragmentation and the thermal physics at the point of fragmentation. Lower-mass stars and brown dwarfs are ascribed to fragmentation in dense regions and then ejection to truncate the accretion rates while higher-mass stars are due to the continued competitive accretion in the dense cores of forming stellar clusters. It is worth noting that all three physical processes are primarily due to gravity and thus in combination provide the simplest mechanism to produce the IMF.

There is much work yet to be done in terms of including additional physics (magnetic fields, feedback) into the numerical simulations that produce testable IMFs. It is also important to develop additional observational predictions from the theoretical models and to use observed properties of star forming regions to determine necessary and sufficient conditions for a full theory for the origin of the IMF. For example, competitive accretion predicts that high-mass star formation is linked to the formation of a bound stellar cluster. This can be tested by observations: the existence of significant numbers of high-mass stars in non-clustered regions or small-N clusters would argue strongly against the accretion model.

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