Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION — STELLAR ASTROPHYSICS

29th May, 2006

Please answer 5 out of the following 6 problems.

(1) (20 points)
Describe the Hayashi track with its assumptions. Draw the track for 1 $M_{\odot}$ star in the Hertzsprung-Russell diagram. Describe qualitatively the evolutionary sequence of a 1 $M_{\odot}$ star from the zero-aged main sequence to the very end of its life as a star, including the phases of the main sequence, red giant branch, horizontal branch, asymptotic giant branch, ejection of a planetary nebula, and a white dwarf. On the same H-R diagram, label each axis, mark the evolutionary phases, and explain in a separate text the important processes or structure changes during each stage, such as the kind of thermonuclear reactions, the location of those reactions, and the energy transport mechanisms, etc.

(2) (20 points)

(a) (10 points) Derive the (i) dynamical timescale $\tau_{\rm dynamical}$, (ii) thermal timescale $\tau_{\rm thermal}$ and (iii) nuclear timescale $\tau_{\rm nuclear}$ of a self-gravitating gas sphere.

(b) (10 points) Estimate the values of these timescales of the Sun, and compare them to the lifetime of the Sun. Give the interpretation of the result.

(3) (20 points)
The simple evolutionary model of stars can be described in the plane of (log$T$, logρ).

(a) (8 points) Divide the plane into different physical status of the gas. Explain how the borders are determined.

(b) (6 points) Draw the evolutionary track of stars with the mass of 1 $M_{\odot}$ and 10 $M_{\odot}$ using their temperature and density at the center of stars (T_c and ρ_c). Describe how these tracks are derived.

(c) (6 points) Describe the evolutions of 1 and 10 $M_{\odot}$ stars using the diagram you draw. Show the important stages in the evolutionary path, and endings of these stars. Draw more regions, lines, and curves in the diagram if necessary.

(4) (20 points)

(a) (10 points) Derive the mass-radius relation for white dwarfs. Where would white dwarfs of different masses be located in the H-R diagram? Quantitatively explain why there should be a mass limit for white dwarfs. What is this limit called? Write down —do not derive— its numerical expression.

(b) (10 points) For a one-solar mass carbon white dwraf with a surface temperature $T_{\rm eff} \sim 10^4$K, the radius would be $\sim 6 \times 10^3$ km, and the central density would be $\sim 3 \times 10^7$ g cm^{-3}. Find the escape velocity from the surface of the white dwarf. How does this white dwarf's luminosity compared with that of the sun? Estimate the central temperature of the white dwarf.

(5) (20 points)
A Strömgren sphere is the region of ionized hydrogen (an H II region) surrounding a hot, young OB star. For an O star, the radius of the Strömgren sphere can be a few to a few tens of parsecs.

(a) (10 points) Derive a symbolic expression (i.e., instead of a quantitative derivation) how the Strömgren radius is related to the properties of the star and the surrounding gas.

(b) (10 points) In reality, the gas contains not only hydrogen (with an ionization energy of 13.6 eV from the ground level)) but also helium (with an ionization energy of 24.6 eV from He I to He II, and 54.4 eV from He II to He III). Describe how the surrounding gas is organized in different layers in terms of different ionization species.

(6) (20 points) Estimate numerically the natural, Doppler and collisional linewidths for hydrogen in the solar photosphere. Carefully state the principle in each case. For your reference, the lifetime of an excited state of hydrogen is about 10^{-8} s. The effective temperature and gas number density in the solar photosphere are, respectively, $T_{\rm eff} \sim 5780$ K and $n \sim 1.5 \times 10^{17} {\rm cm}^{-3}$.

$a = 4\sigma/c = 7.55 \times 10^{-16}$ J m^{-3} K^{-4}
$c = 3.00 \times 10^{8}$ m s^{-1}
$G = 6.67 \times 10^{-11}$ N m^{2} kg^{-2}
$h = 6.626 \times 10^{-34}$ J s
$k_{\rm B} = 1.38 \times 10^{-23}$ J K^{-1}
$m_e = 9.11 \times 10^{-31}$ kg $= 9.11 \times 10^{-28}$ g
$m_{\rm H} = 1.67 \times 10^{-27}$ kg $= 1.67 \times 10^{-24}$ g
$N_{\rm A} = 6.02 \times 10^{23}$ mol^{-1}
eV $= 1.6 \times 10^{-19}$ J $= 1.6 \times 10^{-12}$ erg
$L_\odot = 3.86 \times 10^{26}$ W $= 3.86 \times 10^{33}$ erg s^{-1}
$M_\odot = 1.99 \times 10^{30}$ kg $= 1.99 \times 10^{33}$ g
$R_\odot = 6.96 \times 10^{8}$ m
$T_{\rm eff\odot} = 5,780$ K