Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION — STELLAR ASTROPHYSICS

9:00–13:00, 20th May, 2015

(1) (10 points) **Observations of stellar properties**
Describe the method to estimate the following stellar parameters (each item carries 2 points):
(a) the spectral type and luminosity class, (b) rotational period, (c) mass, (d) age, and (e) metallicity.

(2) (10 points) **Stellar opacity**
(a) (5 points) Free-free absorption is one of primary sources of stellar opacity. However, it is impossible for a free electron to absorb an incident photon in vacuum space. Prove it.
(b) (5 points) Explain why free-free absorption is still one of the primary sources of stellar opacity.

(3) (10 points) **Radiation transfer**
Suppose the central density of the Sun is 153 g cm^{-3} and the mean opacity at the center of the Sun is $\kappa = 2.17\ \mathrm{cm}^2\ \mathrm{g}^{-1}$.
(a) (5 points) Calculate the mean free path of a photon at the center of the Sun.
(b) (5 points) Calculate the average time (in unit of year) it would take for a photon travel from the center to the surface of the Sun if the mean free path remain constant for the photon's journey to the surface. Take the radius of the Sun as 7×10^{10} cm. (You can ignore the fact that identifiable photons are constantly destroyed and created through absorption, scattering and emission.)

(4) (15 points) **Stellar stability**
(a) (5 points) If the structure of a star is spherical symmetric and the source function is isotropic, show that inside the star the radiation pressure gradient is

$$\frac{\mathrm{d}P_\mathrm{r}}{\mathrm{d}r} = -\frac{\rho\bar{\kappa}}{c}\,F\,,$$

where r is the distance from the center of the star, $\rho(r)$ is the mass density, $\bar{\kappa}(r)$ is the mean opacity, and P_r and F are the radiation pressure and flux, respectively.
(b) (5 points) From (a) show that the star is stable only if the mass to luminosity ratio

$$\frac{M}{L} > \frac{\bar{\kappa}}{4\pi G c}\,.$$

(c) (5 points) The Eddington luminosity of a star can be evaluated by the equation above. If the star is composed by pure hydrogen, the Eddington Luminosity is

$$L_\mathrm{edd} = 1.25 \times 10^{38}\left(\frac{M}{M_\odot}\right)\ \mathrm{erg\ s^{-1}}\,.$$

However, if a star is composed by both hydrogen and helium and the mass fraction of hydrogen is X, what is the Eddington luminosity of this star if all the hydrogen and helium are complete ionized?

(5) (15 points) **Stellar structure**

(a) (5 points) Use the hydrostatic equilibrium equation, mass conservation equation and the polytropic equation of state, $P(\rho) = K\rho^{(n+1)/n}$, where P is the pressure, ρ is the mass density, n is the polytropic index and K is a constant, and the definitions of the following variables $\rho = \rho_c[\phi(\xi)]^n$, where ρ_c is the central mass density, and $r = a\xi$ with

$$a = \sqrt{\frac{(1+n)K\rho_c^{(1-n)/n}}{4\pi G}}$$

to derive the differential equation for $\phi(\xi)$, which is the famous Lane-Emden equation.

(b) (5 points) The Lane-Emden equation has a physical and analytic solution for $n = 1$. Find this solution and don't forget to specify and justify the boundary conditions you use.
[Hint: Replace $\phi = y/\xi$ in the equation.]

(c) (5 points) From the solution of (b), find the density $\rho(r)$, the pressure $P(r)$, the radius and the total mass of this star.

(6) (10 points) **Energy balance**

The energy generation rate for proton-proton and CN reactions are, respectively,

$$\epsilon_{\rm pp} \approx 2.5 \times 10^6 \rho X_{\rm H}^2 \left(\frac{10^6}{T}\right)^{2/3} \exp\left[-33.8\left(\frac{10^6}{T}\right)^{1/3}\right] \text{ erg g}^{-1}\text{ s}^{-1},$$

$$\epsilon_{\rm CN} \approx 8 \times 10^{27} \rho X_{\rm H} X_{\rm CN} \left(\frac{10^6}{T}\right)^{2/3} \exp\left[-153.2\left(\frac{10^6}{T}\right)^{1/3}\right] \text{ erg g}^{-1}\text{ s}^{-1},$$

where ρ is the density of the gas in g cm^{-3}, and $X_{\rm H}$ and $X_{\rm CN}$ are H and C, N mass fractions. Here assume $X_{\rm CN} = 0.005X_{\rm H}$. When the Sun reaches the zero-age main sequence phase, the H burning takes place in the core out to $0.22R$, where the radius $R = 0.87R_\odot$. Adopt a constant temperature 10^7 K and a density 75 g cm^{-3} in the core.

(a) (3 points) Give a physical reasoning why the Sun had a radius only 87% of its present value?

(b) (3 points) Estimate which of the nuclear processes, the p-p or the CN, predominates the energy generation?

(c) (4 points) What are the expected luminosity and the surface temperature of the Sun at zero-age main sequence?

(7) (20 points) **Cloud collapse**

A uniform and isothermal dark cloud of mass M, radius R, and temperature T, with 25% hydrogen molecules (by mass) and the rest of helium, is on the verge of Jeans instability.

(a) (4 points) What is the mean molecular weight of this cloud?

(b) (4 points) What is the density of the cloud in g cm^{-3} and in terms of number density cm^{-3}?

(c) (6 points) Suppose the cloud contracts under its own gravity to become 5 order smaller in radius, and is now rotationally supported, estimate the equatorial speed of the cloud rotation. What is the velocity dispersion in km s^{-1} across the cloud?

(d) (6 points) Assuming the contraction is adiabatic, what is the temperature of the cloud now?

(8) (10 points) **Stellar evolution**

Draw a complete evolutionary track of the Sun in the Hertzsprung-Russell diagram, from the pre-main sequence contraction, through the zero-age main sequence, terminal-age main sequence, red-giant branch, helium flash, horizontal branch, asymptotic giant branch, planetary nebula ejection, to eventually the white dwarf stage. Clearly label each axis of the diagram. In a separate text, explain the energy source of the Sun at each of the above epoch.