Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION 2017 — STELLAR ASTROPHYSICS

1. **[TOTAL: 10%] Magnitudes:**
   The magnitude system is widely used for optical astronomy.

   (a) **[5%]** Some photons from astronomical objects are absorbed or scattered in the atmosphere of the Earth and do not reach to the telescope. We often assume that the amount of atmospheric extinction is proportional to the airmass X. Show that the airmass is roughly equal to $\sec z$ for the range $z < 60°$. Here, z is zenith distance.

   (b) **[5%]** Show that the absolute magnitude M of a star can be expressed as

   $$M - m = 5 - 5 \log d,$$

   where m is the apparent magnitude and d is the distance to the star.

2. **[TOTAL: 15%] Energy generation in stars:**
   Large amount of energy is generated in stars. Here, we consider the source of energy generated at interior of stars.

   (a) **[10%]** In order to form a star, a cloud of interstellar gas has to be gathered. During this process, gravitational potential energy is converted into thermal energy. When a huge interstellar cloud becomes a sphere of a solar radius, how much energy is released? Describe your method of calculations, and show the amount of energy released as a function of stellar radius R and stellar mass M. Can the current rate of solar energy generation be explained by the release of gravitational potential energy? Discuss quantitatively.

   (b) **[5%]** The mass is also a form of energy, and the mass can be converted into the other form of energy. In the interior of the Sun, 4 hydrogen nuclei are converted into a helium nucleus, and the energy is released. Can this kind of nuclear fusion be the source of solar energy? Explain your answer.

3. **[TOTAL: 15%] Structure of a star:**
   A rough picture of the internal structure of a star can be obtained from the assumption of the hydrostatic equilibrium.

   (a) **[5%]** Consider a small volume element in a star. Assume the hydrostatic equilibrium, and derive the equation

   $$\frac{dP}{dr} = -\rho g,$$

   where r is the distance from the center of the star, P is the pressure at r, ρ is the density, g is the gravitational acceleration.

   (b) **[10%]** Using the equation above, give a rough estimate for the pressure and temperature at the center of the Sun. Describe the process of your calculation.

4. **[TOTAL: 10%] Dynamical timescale of stars:**
The characteristic timescale τ for the physical quantity x is defined by

$$\tau = \frac{x}{\dot{x}},$$

where $\dot{x}$ is the rate of change of the quantity x.

   (a) **[5%]** Here, we consider expansion or contraction of a star. Show that the dynamical timescale τ_{dyn} of a star is expressed by

$$\tau_{dyn} \sim \sqrt{\frac{R^3}{2GM}},$$

   where R is the radius of a star, M is the mass of a star, and G is the gravitational constant.

   (b) **[5%]** Give a rough estimate for the dynamical timescale of the Sun, and discuss the implication from your result.

5. **[TOTAL: 10%] Mass of eclipsing binary system:**
Here we derive the mass function of a binary star system, assuming the inclination angle $i = 90°$ (hence no need to worry inclination dependence). Let the two stars in this binary system has mass m_1 and m_2, with distance to center-of-mass of r_1 and r_2, and an orbital period P.

   (a) **[2%]** Show that the mass ratio can also be expressed in terms of their velocity v_1 and v_2.

   (b) **[4%]** Using Kepler's 3$^{\rm rd}$ Law, show that the sum of their mass has the following expression.

$$m_1 + m_2 = \frac{P}{2\pi G}(v_1 + v_2)^3$$

   (c) **[1%]** Combine results from (a) and (b), show that:

$$f_m = \frac{m_2^3}{(m_1 + m_2)^2} = \frac{P}{2\pi G} v_1^3$$

   Where f_m is called the mass function.

   (d) **[3%]** Assume that the radius of the secondary star in this eclipsing binary system is found to be similar to the Earth, this implies the secondary star is either a white dwarf or an exoplanet. Discuss how can you use the result from (c) to determine the nature of this secondary star. What other information (e.g. from observation etc) do you need?

6. **[TOTAL: 25%] Period-mean density relation for pulsating stars:**

   (a) **[3%]** Show that the equation of hydrostatic equilibrium leads to the following equation:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{4}{3}\pi G r \rho^2,$$

   where P is (gas) pressure, r is the radius, and ρ is mean density of the star. What assumption(s) you have to make in your derivation?

   (b) **[3%]** Show that at give r, the pressure can be expressed as:

$$P(r) = \frac{2}{3}\pi G \rho^2 (R^2 - r^2),$$

   where R is the radius of the star.

(c) **[6%]** Assuming the pulsating period Π is the total time for a sound wave crossing the entire star, where the adiabatic sound speed is given as $v_s = \sqrt{\gamma P/\rho}$. Show that

$$\Pi = \sqrt{\frac{3\pi}{2\gamma G\rho}}.$$

This is the famous period-mean density relation for pulsating stars: $\Pi\rho^{1/2} \sim$ constant. Note the integration $\int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \sin^{-1}\frac{x}{a}$.

(d) **[5%]** Demostrated that the Kepler's 3$^{\rm rd}$ Law can also be used to reproduce the above period-mean density relation.

(e) **[8%]** If we observe the following two pulsating stars in Large Magellanic Cloud:

- A RR Lyrae with observed period of $\Pi = 0.5$ days and $m_V = 19$ mag.
- A Cepheid with observed period of $\Pi = 10$ days and $m_V = 14$ mag.

Using Figure 1 to estimate the mass ratio between these two pulsating stars. List out the assupmtions made. If this RR Lyrae has a mass of $0.5M_\odot$, what is the estimated mass of this Cepheid?

7. **[TOTAL: 15%] Stellar evolution:**
Describe the major evolutionary stages for a Solar mass ($1M_\odot$) star and a high mass star (say, $> 10M_\odot$), starting from main-squence until the end of their life. Sketch on H-R diagram for their evolutionary tracks/stages if necessary.

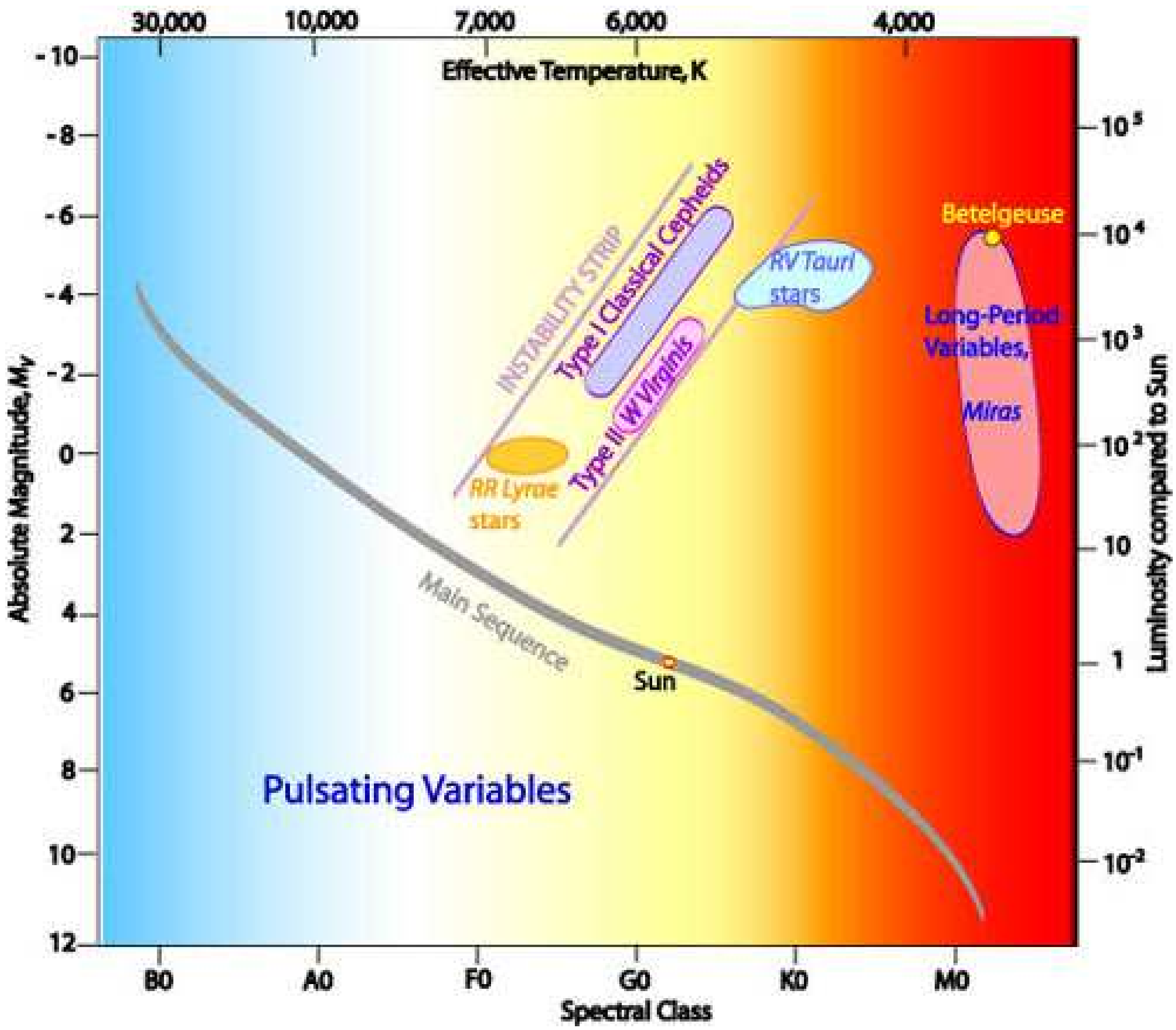

Fig. 1.— H-R diagram for classical pulsating stars.

Constants

Speed of light	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
Radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.6726 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.6749 \times 10^{-27}\,\mathrm{kg}$
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.674 \times 10^{-27}\,\mathrm{kg}$
helium-3 atom mass	$5.009 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.648 \times 10^{-27}\,\mathrm{kg}$
ideal gas constant	$\mathcal{R} = 8.31 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
Earth mass	$M_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus = 6.38 \times 10^{6}\,\mathrm{m}$
Astronomical unit	$1\,\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$
π	$\pi = 3.14$
cal and J	$1\,\mathrm{cal} = 4.2\,\mathrm{J}$