Institute of Astronomy, National Central University

PHD QUALIFYING EXAMINATION 2020 — STELLAR ASTROPHYSICS

1. **[TOTAL: 30%]** Explain each of the following terms succinctly: (1) Chandrasekhar-Schönberg limit; (2) Roche limit; (3) diffraction limit; (4) Eddington limit; (5) Eddington approximation; (6) Kramers opacity; (7) Gould belt; (8) Gamow peak; (9) Strömgren sphere; (10) Bonnor-Ebert sphere.

2. **[TOTAL: 20%]**

   (a) For an isothermal molecular cloud of temperature T, mass M, and an initial density ρ, starting from virial theorem, show that the Jeans mass $M_{\rm Jeans} \propto T^{3/2}\rho^{-1/2}$.

   (b) Numerically, $M_{\rm Jeans} = \left(\frac{T}{10K}\right)^{3/2}\left(\frac{n_{H_2}}{10^4 {\rm cm}^{-3}}\right)^{-1/2} M_\odot$, where n_{H_2} is the volume number density of hydrogen molecules. A typical Bok globule inside an HII region, has $n_{H_2} \approx 10^4 {\rm cm}^{-3}$, diameter ≈ 0.1 pc across, and $T \approx 10$ K. Estimate the mass of the globule, and compare this with its Jeans mass, $M_{\rm Jeans}$. Comment on the dynamical status of the globule.

3. **[TOTAL: 20%]**

   (a) **[7%]** Derive the hydrostatic equilibrium equation:

   $$\frac{{\rm d}P}{{\rm d}r} = -\rho\frac{GM_r}{r^2}$$

   where M_r is enclosed mass at radius r.

   (b) **[5%]** Show that at stellar atmosphere, the hydrostatic equilibrium equation can be expressed as:

   $$\frac{{\rm d}P}{{\rm d}\tau} = -\frac{GM}{R^2}\frac{1}{\kappa}$$

   where τ is optical depth, and κ is opacity.

   (c) **[8%]** Using the hydrostatic equilibrium equation given in (a), show or argue that it will lead to the following result:

   $$2E_{\rm kin} = -E_{\rm pot}$$

   where $E_{\rm kin} = \frac{3}{2}NkT$ is the kinetic energy of the monoatomic ideal gas and $E_{\rm pot} \propto \frac{GM^2}{R}$ is the gravitational potential energy

4. **[TOTAL: 20%]** The main-sequence stars follow a specific mass-radius relation.

   (a) **[4%]** Why the main-sequence stars cannot have arbitrary radius for a given mass?

   (b) **[4%]** The mass-radius relation can be expressed as a power law $R \propto M^\alpha$, what is the value, or range, of α? Why you pick this value, or range of value?

(c) **[6%]** Describe how to measure the radius of a star? What kind of observational data are needed? What assumptions need to be made?

(d) **[6%]** Describe how to measure the mass of a star? What kind of observational data are needed? What assumptions need to be made?

5. **[TOTAL: 10%]** Energy was generated deep inside the star via nuclear reaction.

(a) **[5%]** List and briefly explain two evidence to support the idea that nuclear reaction did take place in interior of our Sun, or other main-sequence stars.

(b) **[5%]** Estimate the nuclear reaction time scale for our Sun. What assumptions you have to made such that the value is closer to the age of the Sun (or Earth)?

Constants

Speed of light	$c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
Radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.6726 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.6749 \times 10^{-27}\,\mathrm{kg}$
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.674 \times 10^{-27}\,\mathrm{kg}$
helium-3 atom mass	$5.009 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.648 \times 10^{-27}\,\mathrm{kg}$
ideal gas constant	$\mathcal{R} = 8.31 \times 10^3\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot = 6.96 \times 10^8\,\mathrm{m}$
Solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
Earth mass	$M_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus = 6.38 \times 10^6\,\mathrm{m}$
Astronomical unit	$1\,\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$
π	$\pi = 3.14$
cal and J	$1\,\mathrm{cal} = 4.2\,\mathrm{J}$