Qualifying Exam

Stellar Astronomy

May 2025

1 Stellar Stability

Main-sequence stars remain structurally stable. The Sun for example is expected to have a main-sequence lifetime of about 10 billion (10^{10}) years.

a. Derive the hydrostatic equilibrium equation of a stable star with mass, pressure, and density dependence on radius. Clearly explain each symbol used in the equation. Describe which forces are balancing each other. What happens to this equilibrium at the center of a star?

b. Main-sequence stars are undergoing nuclear fusion in the core, converting a total of 4 protons to a helium nucleus, liberating energy in the process. There are two kinds of such series of reactions, one called proton-proton chain, and the other the CNO cycle. Describe each chain of reactions, e.g., what kind of nuclei participate in the fusion process.

c. What is the Schönberg-Chandrasekhar limit? Estimate the main-sequence lifetime of the Sun applying the knowledge of this limit. The Sun has an age of about 5 billion years. How is this known?

d. What happens to the structure of the Sun when hydrogen is depleted in the core? How does the Sun's location change in the Hertzsprung-Russell diagram from the main sequence? It helps to draw a schematic.

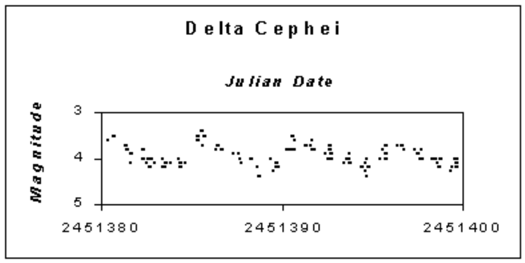
(20%)

2 Stellar Instability

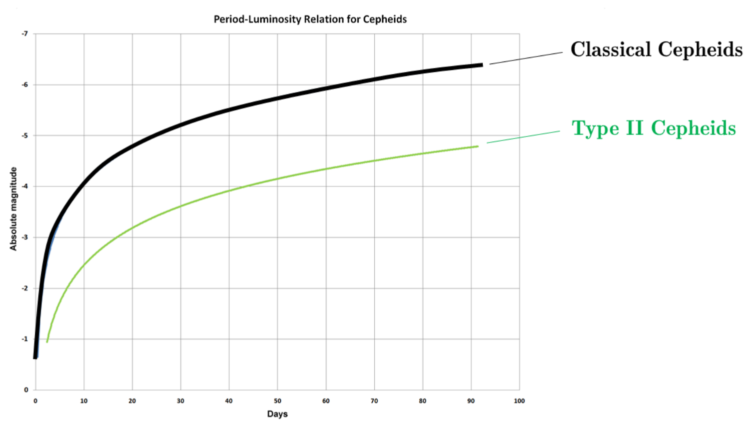
Cepheids are pulsating variables. Located in the instability strip of the Hertzsprung-Russell diagram, they vary in diameter and temperature periodically.

a. Explain the reason why a Cepheid variable pulsates, and why this warrants a period-luminosity relation.

b. Delta Cep is the prototype of classical cepheids, and has an apparent magnitude of $m_V = 3.75$. The following figure shows a segment of its light curve taken from the AAVSO. What is its approximate period? What is the variation amplitude in magnitude?



c. From the following period-luminosity relation, estimate roughly from the distance inferred in (b) the distance to Delta Cep. In 2002, the *HST* measured a parallax distance of 273 pc with a 4% error, whereas in 2015 the *Hipparcos* data suggested 244 ± 10 pc. The star has the latest *Gaia* parallax measurement of 3.5551 ± 0.1475 milliarcsecond. How does the Gaia result compare to the two previous ones?



(30%)

3 Basic mathematics and physics

a. Calculate the following. Give a brief description how to carry out the calculation.

$$\frac{d}{dx}\left(\frac{1}{x^4}\right)$$

(1%)

b. Calculate the following. Give a brief description how to carry out the calculation.

$$\frac{d}{dx}\left(\sin^3 x\right)$$

(1%)

c. Calculate the following. Give a brief description how to carry out the calculation.

$$\frac{d}{dx}\left(x \log x\right)$$

(1%)

d. Calculate the following. Give a brief description how to carry out the calculation.

$$\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$

(1%)

e. Calculate the following. Give a brief description how to carry out the calculation.

$$\int x \cos x dx$$

(1%)

f. Simplify the following.

$$\frac{10^\alpha}{10^\beta}10^\gamma$$

(1%)

g. Astronomers often use the following approximation for $|x| \ll 1$.

$$(1+x)^n \sim 1 + nx$$

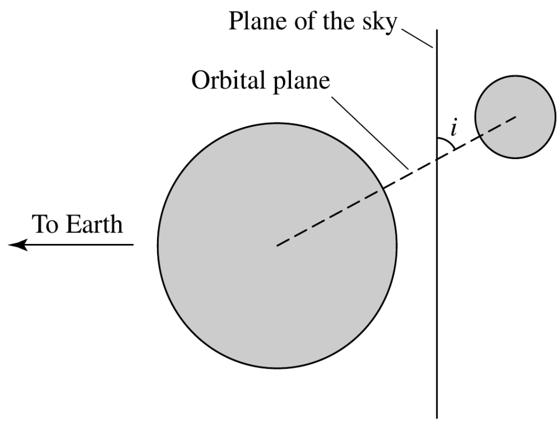
Give a brief description how to derive this approximation. (1%)

h. Calculate the average density of the star Betelgeuse at its maximum size. The mass of Betelgeuse is $\sim 15\ M_\odot$, and the radius of Betelgeuse is $\sim 1000\ R_\odot$ at the maximum. (1%)

4 Eclipsing binary

Assume that two stars are in circular orbits about a mutual centre of mass and are separated by a distance a. Assume also that the angle of inclination i and their stellar radii are r_1 and r_2. Find an expression for the smallest angle of inclination that will just barely produce an eclipse. Describe the method of your calculation.

(3%)



5 Mean free path

a. The density of the photosphere of the Sun is about $\rho = 2.1 \times 10^{-4}$ kg m^{-3}. Assume that the photosphere of the Sun is purely composed of hydrogen atoms. Calculate the number density of hydrogen atoms in the photosphere of the Sun. Describe your answer. (5%)

b. Calculate the mean free path of hydrogen atoms in the photosphere of the Sun. Describe your answer. (5%)

6 The Crab supernova remnant

a. The angular size of the Crab SNR is 4 arcmin $\times$ 2 arcmin and its distance from Earth is approximately 2000 pc. Estimate the linear dimensions of the nebula. Describe your answer. (5 %)

b. The Crab SNR is expanding with the velocity of 1500 km/sec. Using this measured expansion rate of the Crab SNR and ignoring any accelerations since the time of the supernova explosion, estimate the age of the nebula. Describe your answer. (5 %)

7 Mass

An exoplanet 51 Peg b is orbiting around a star 51 Peg. The orbital period of this exoplanet is 4.23077 days and the semimajor axis of the orbit this exoplanet is 0.051 au. Use these information to determine the mass of star 51 Peg. Describe your answer.

(5%)

8 Gravitational potential energy

Suppose there is a star with the core mass of 2.5 $M_\odot$ and core radius of 50-km. How much gravitational potential energy would be released if the core of this star collapses? Give your answer in J. Describe your answer.
(5%)

9 Mass loss

a. At what rate is the Sun's mass decreasing due to nuclear reactions? Express your answer in solar masses per year. Describe your answer. (3%)

b. At a distance of 1 au from the Sun, the solar wind has a typical velocity of 500 km/s with a typical proton density of 7×10^6 m^{-3}. Calculate the mass loss rate of the Sun due to solar wind. Express your answer in solar masses per year. Describe your answer. (3%)

c. Assuming that the nuclear reaction rate and solar wind mass loss rate remain constants, estimate the amount of total mass loss of the Sun over its entire main-sequence lifetime. Express your answer in solar masses. Describe your answer. (3%)

Constants

Speed of light	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron volt	$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
Radiation constant	$a = 7.56 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Atomic mass unit	$m_H = 1.66 \times 10^{-27}\,\mathrm{kg}$
electron mass	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
proton mass	$m_p = 1.6726 \times 10^{-27}\,\mathrm{kg}$
neutron mass	$m_n = 1.6749 \times 10^{-27}\,\mathrm{kg}$
helium-4 nucleus mass	$m_{He4} = 6.643 \times 10^{-27}\,\mathrm{kg}$
hydrogen atom mass	$1.674 \times 10^{-27}\,\mathrm{kg}$
helium-3 atom mass	$5.009 \times 10^{-27}\,\mathrm{kg}$
helium-4 atom mass	$6.648 \times 10^{-27}\,\mathrm{kg}$
Bohr radius	$5.29 \times 10^{-11}\,\mathrm{m}$
ideal gas constant	$\mathcal{R} = 8.31 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Solar mass	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$
Earth mass	$M_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus = 6.38 \times 10^{6}\,\mathrm{m}$
Astronomical unit	$1\,\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$
parsec	$1\,\mathrm{pc} = 3.09 \times 10^{16}\,\mathrm{m}$
π	$\pi = 3.14$
cal and J	$1\,\mathrm{cal} = 4.2\,\mathrm{J}$