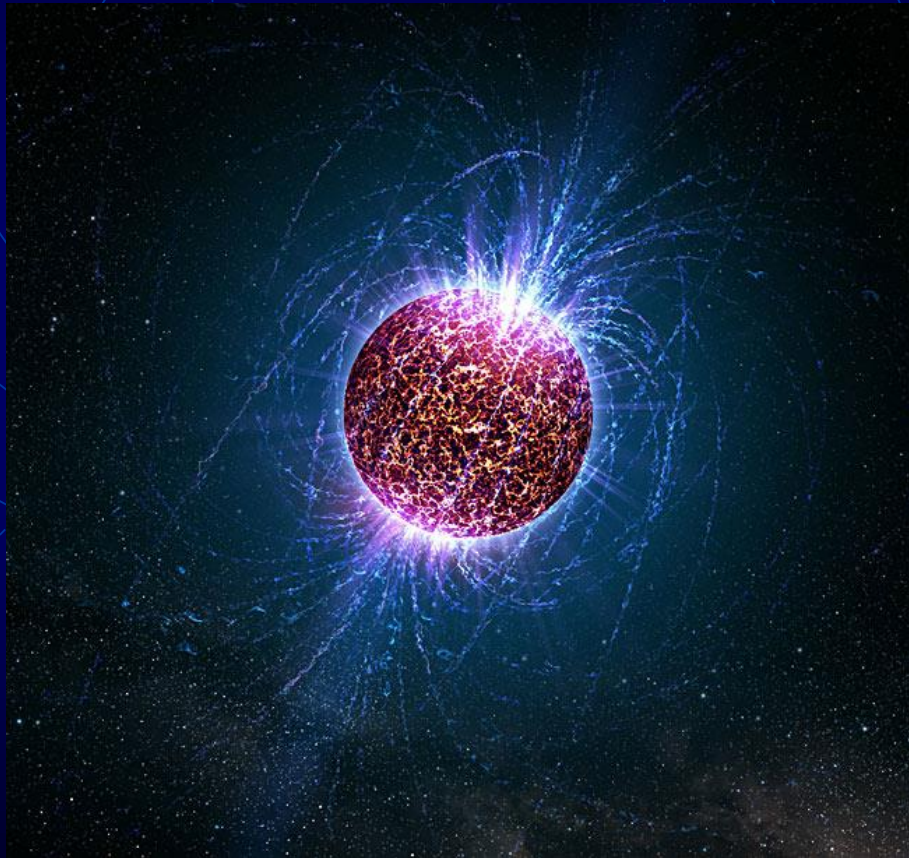


# Compact Objects

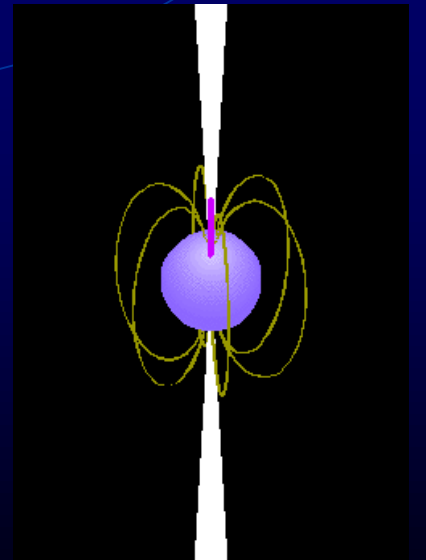
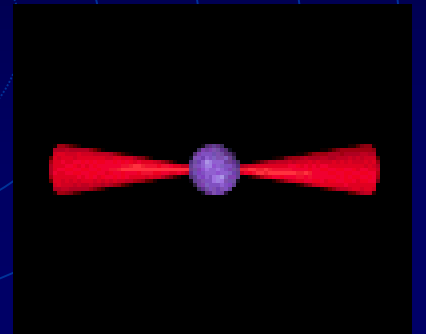
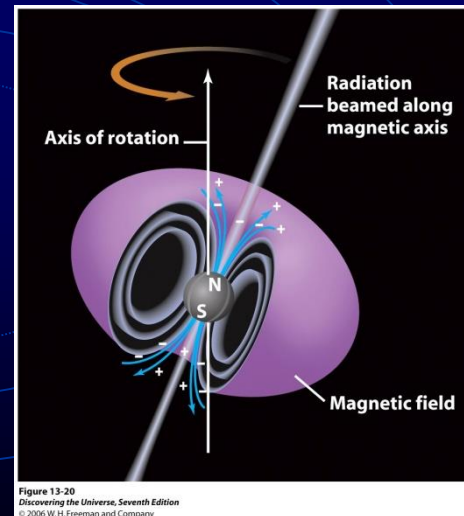
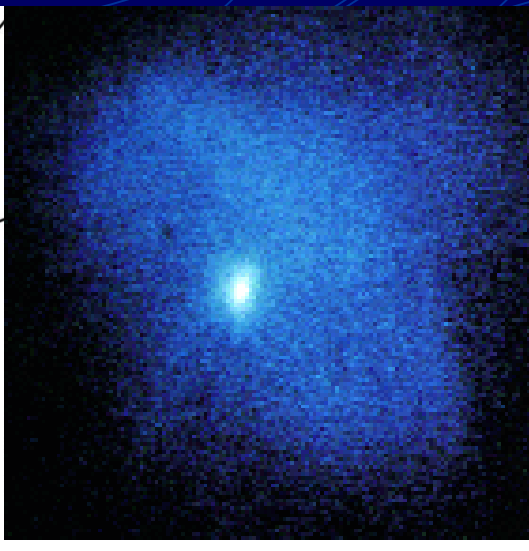
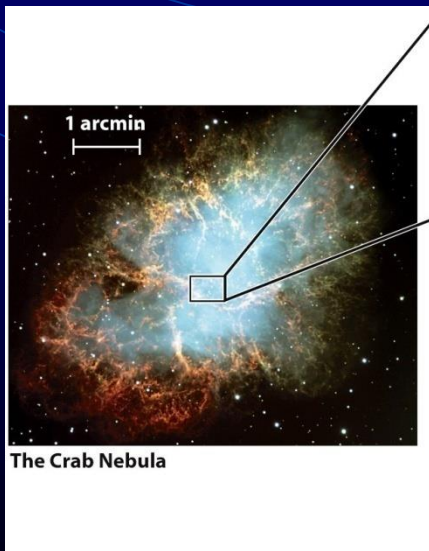
## --- Neutron Stars and Black Holes

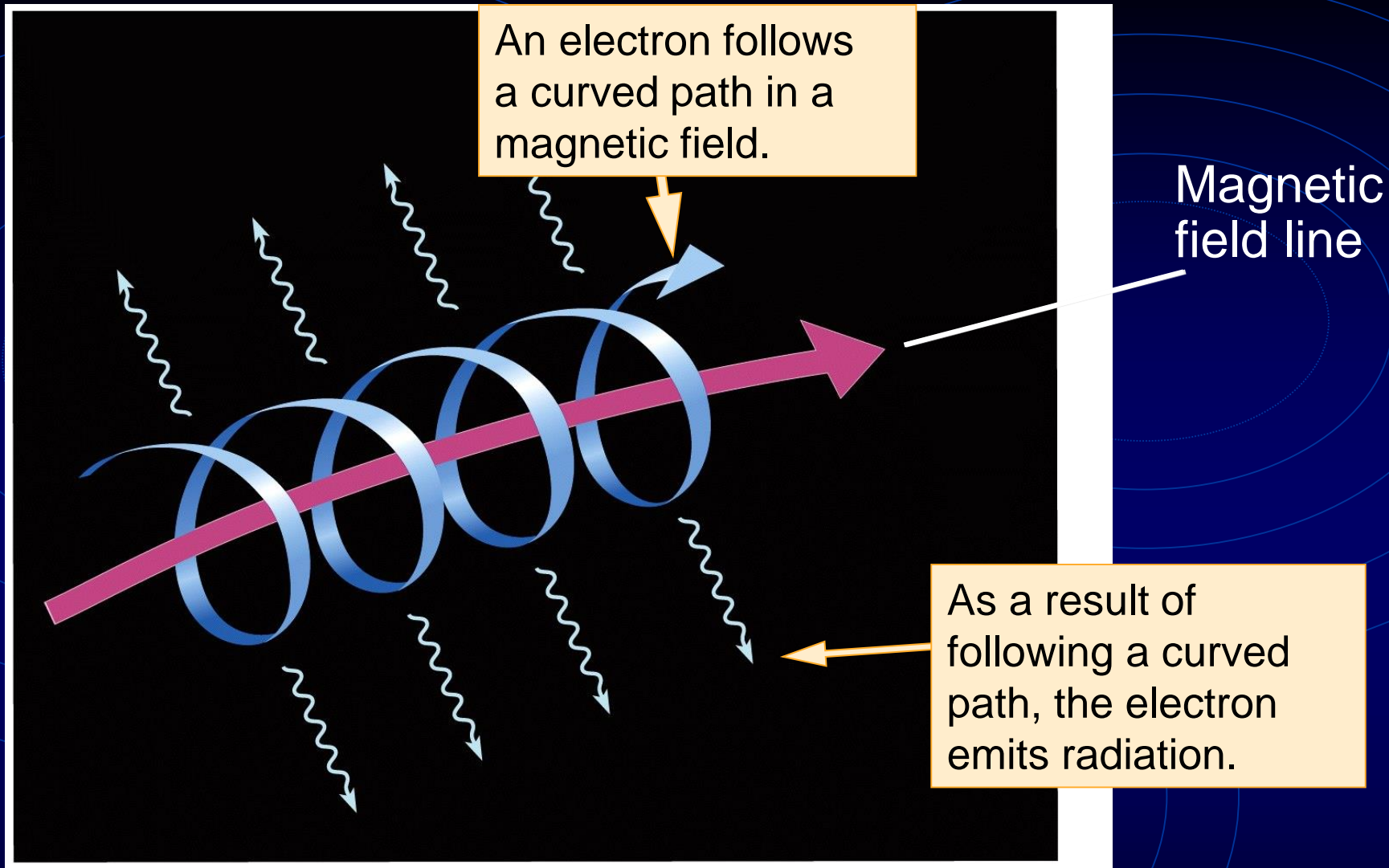


Credit: Casey Reed/Penn State University

[http://www.wvu.edu/depts/skywise/img/blackhole\\_42.jpg](http://www.wvu.edu/depts/skywise/img/blackhole_42.jpg)

- 若天體質量大，以致收縮後連電子簡併壓力都撐不住  
→ **中子星 (neutron star)**
- 中子星比白矮星還小得多。太陽 (D~140 萬 km) 核心成為白矮星，大小約與地球 (1萬多 km) 相當
- 中子星更小，直徑只有10~15 km (cf 中壢市)
- 磁場強大、旋轉快速 → **脈衝星 (pulsar)**  
每秒發出數百到數千次脈衝 (燈塔效應)



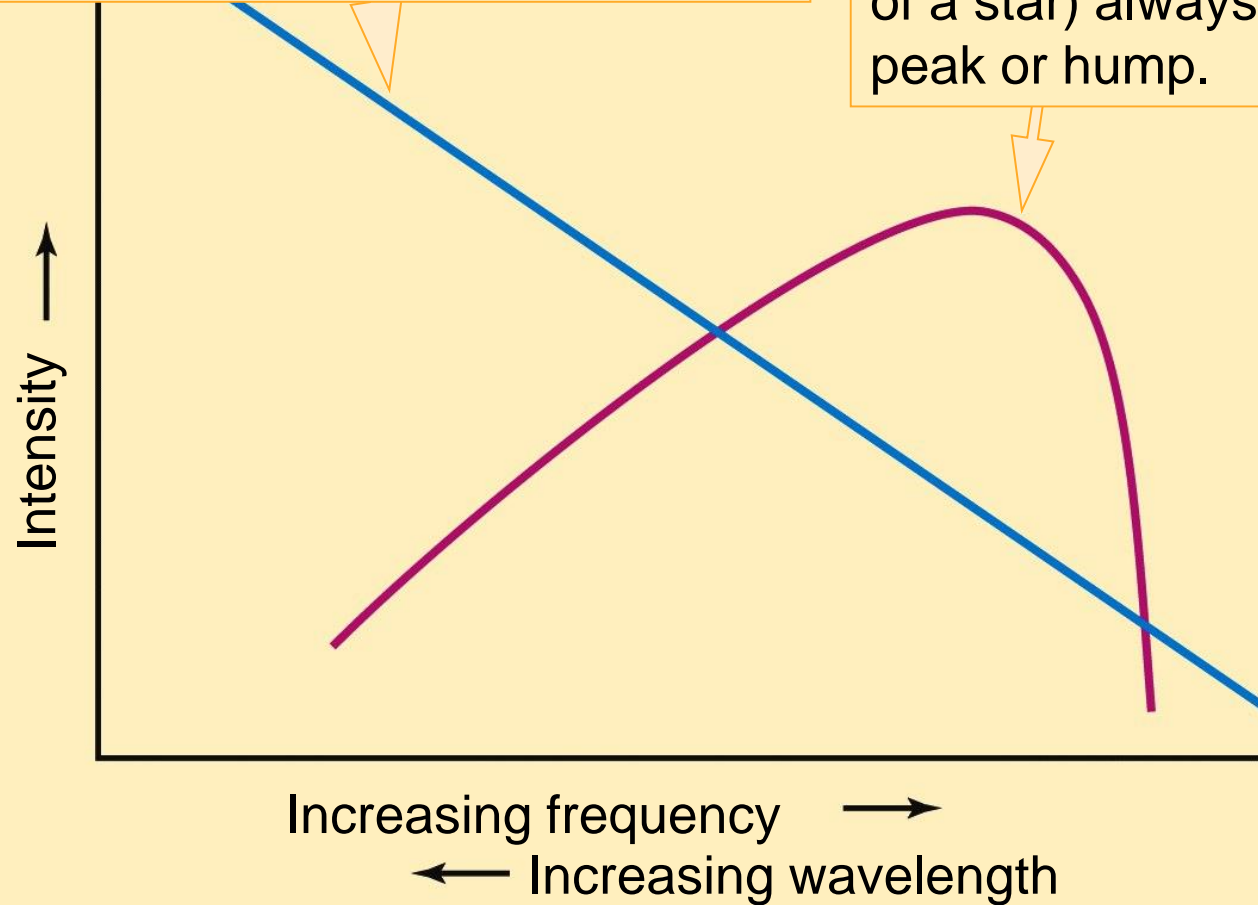


電子被磁場加速 → 放出  
同步輻射 (synchrotron radiation)

$a \perp v$   
polarized

The spectrum of synchrotron radiation shows a steady decline with increasing frequency.

A blackbody spectrum (similar to the spectrum of a star) always has a peak or hump.



Synchrotron radiation differs from blackbody radiation in spectral shapes.

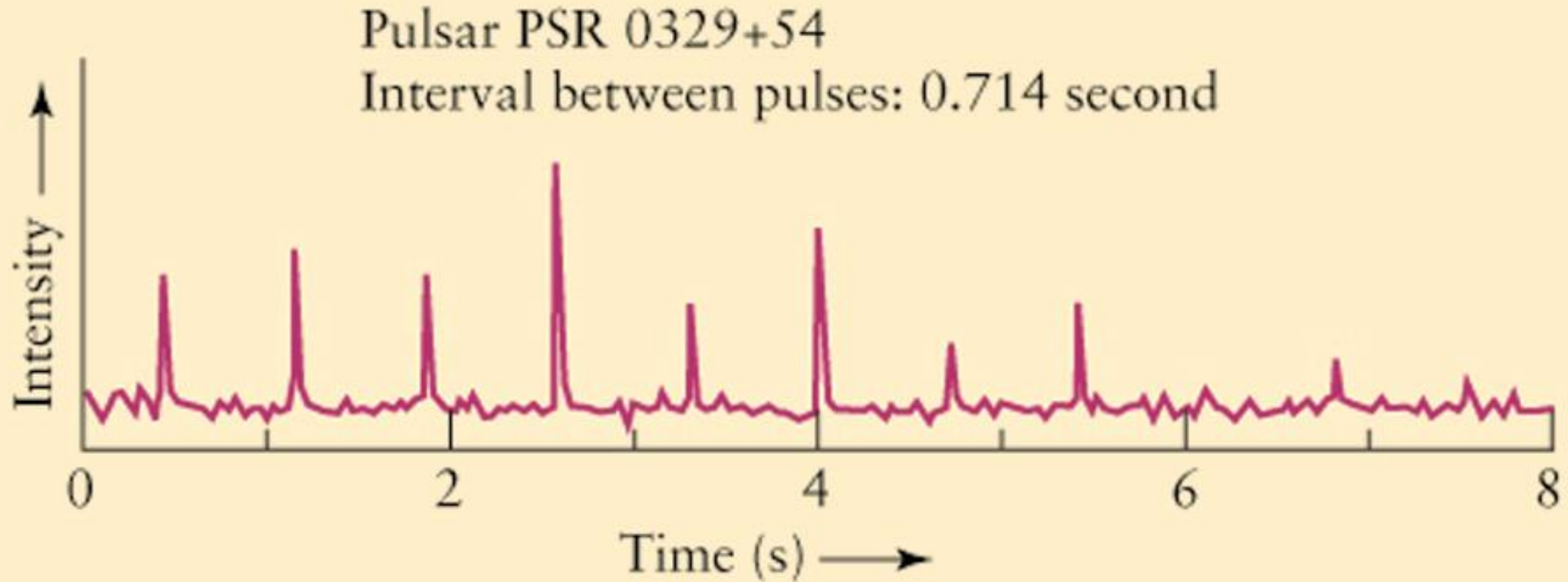
# Angular Momentum Conservation

$$J = m \omega r^2$$

- Consider how fast the Sun (rotation period of  $\sim 30$  days, radius about 700,000 km) would rotate if it were to shrink to  $r \sim 15$  km, if angular momentum is conserved.
- Period =  $2\pi/\omega$
- $P_{\text{after}}/P_{\text{before}} = \omega_{\text{before}}/\omega_{\text{after}} = (r_{\text{after}}/r_{\text{before}})^2$
- $P_{\text{after}} = 30 \text{ days} \times 86400 \text{ s/day} (15/7 \times 10^5)^2$   
 $\sim 0.001 \text{ s, i.e., milliseconds}$

cf The break-up speed  $\rightarrow P > 3.8 \times 10^{-4} \text{ s}$ , so OK

PSR 0329+54  $P \sim 0.714$  s



Pulsars 輻射 後果 → 自轉越來越慢

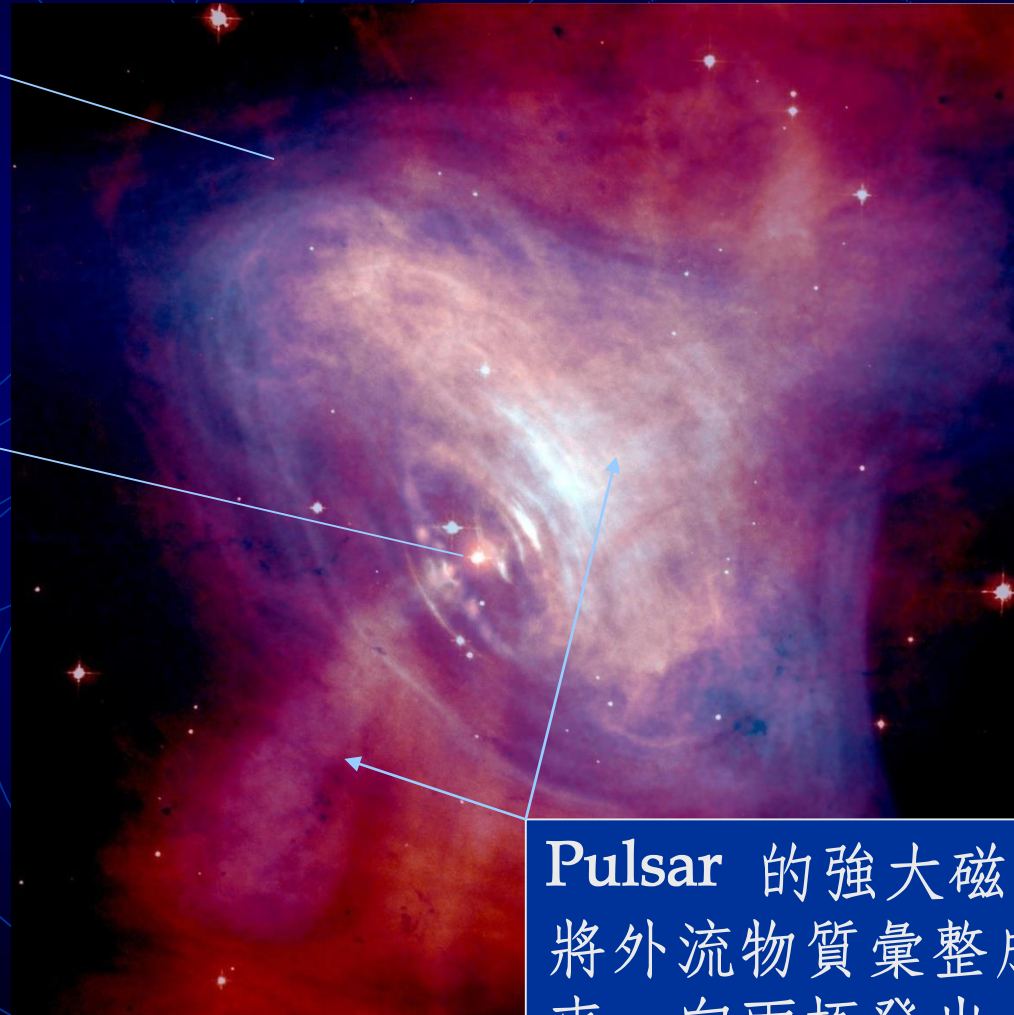


A slow-motion movie of the Crab Pulsar taken at 800 nm using a Lucky Imaging camera from Cambridge U, showing the bright pulse and fainter interpulse ...

脈衝星放出高速物質，產生衝擊波，發射高能 X 射線

The Crab pulsar

The Crab pulsar by HST (in red) and Chandra (in blue)



Pulsar 的強大磁場將外流物質彙整成束，向兩極發出

- A neutron star consists mostly of neutrons  
Assuming  $1.5 M_{\odot}$ ,  $r = 10$  km, the density is  $\rho \sim 4.8 \times 10^{14} \text{ g cm}^{-3}$   
→ nuclear density  $\sim 10^{15} \text{ g cm}^{-3}$
- A neutron star is very hot, supported by the Pauli exclusion force between neutrons.
- Structure of a neutron star is understood currently only by mathematical models.
- 一湯匙的中子星物質，重達50億公噸
- 表面存在非常薄 ( $\sim 1$  m) 的大氣層，為普通的原子、電子
- 越往內部，密度越高 → 自由中子



# Gravitational Red Shift

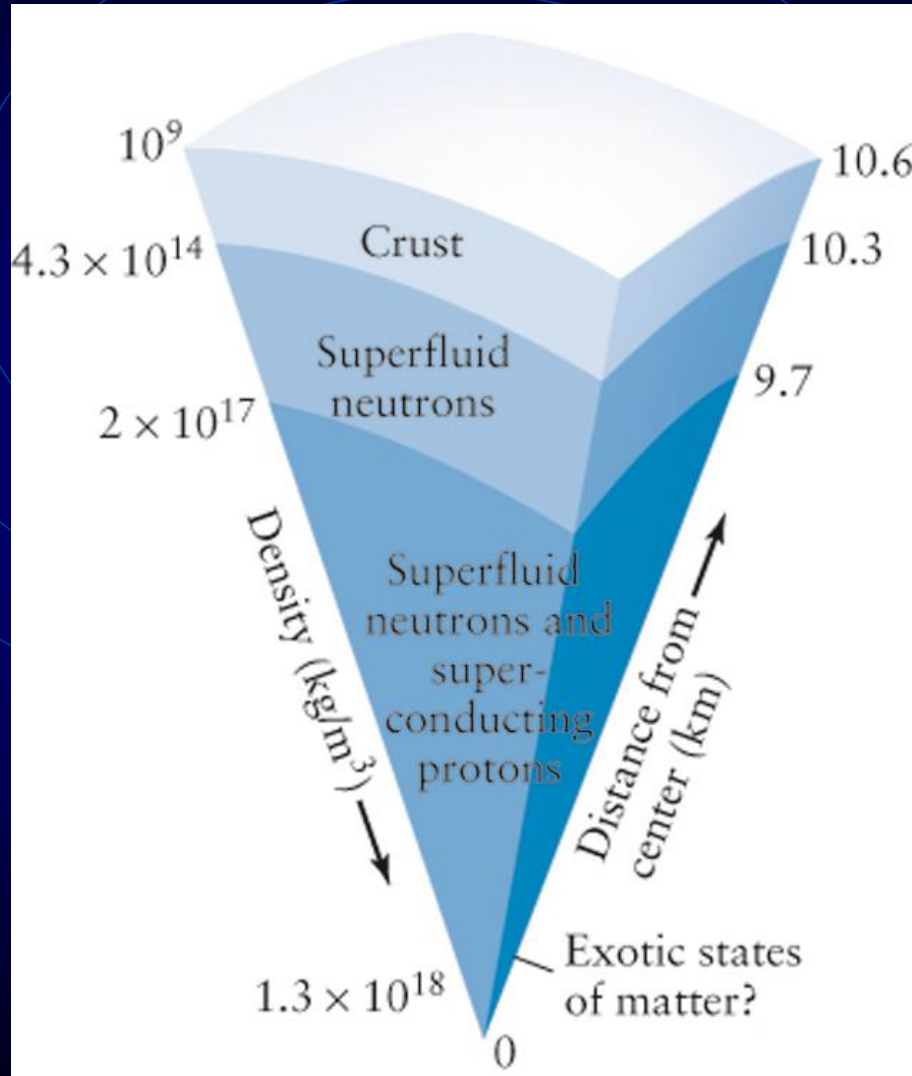
$$\text{Total mass-energy } \mathcal{E} \equiv mc^2 = m_0c^2 + \frac{m_0 GM}{r} = m_0c^2 \left(1 + \frac{GM}{rc^2}\right)$$

... conversion of potential  
to kinetic energy

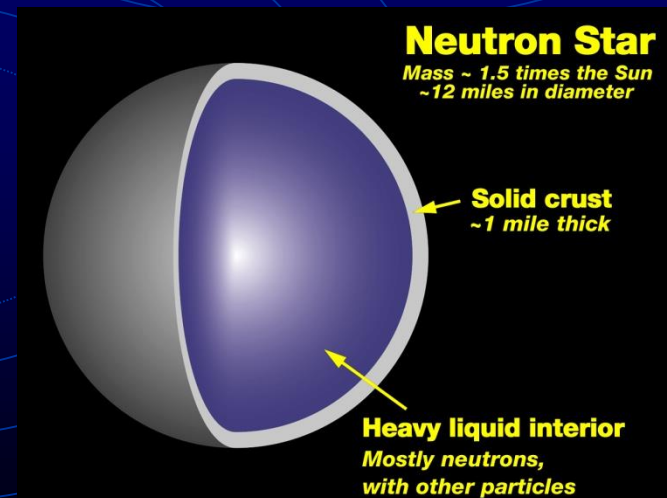
A photon loses energy, so its frequency is red shifted)

$$\begin{aligned} \nu &= \nu_0 \left[1 - \frac{GM}{rc^2}\right] \\ \frac{\Delta\nu}{\nu_0} &= \frac{\nu' - \nu_0}{\nu_0} = -\frac{GM}{rc^2} \end{aligned}$$

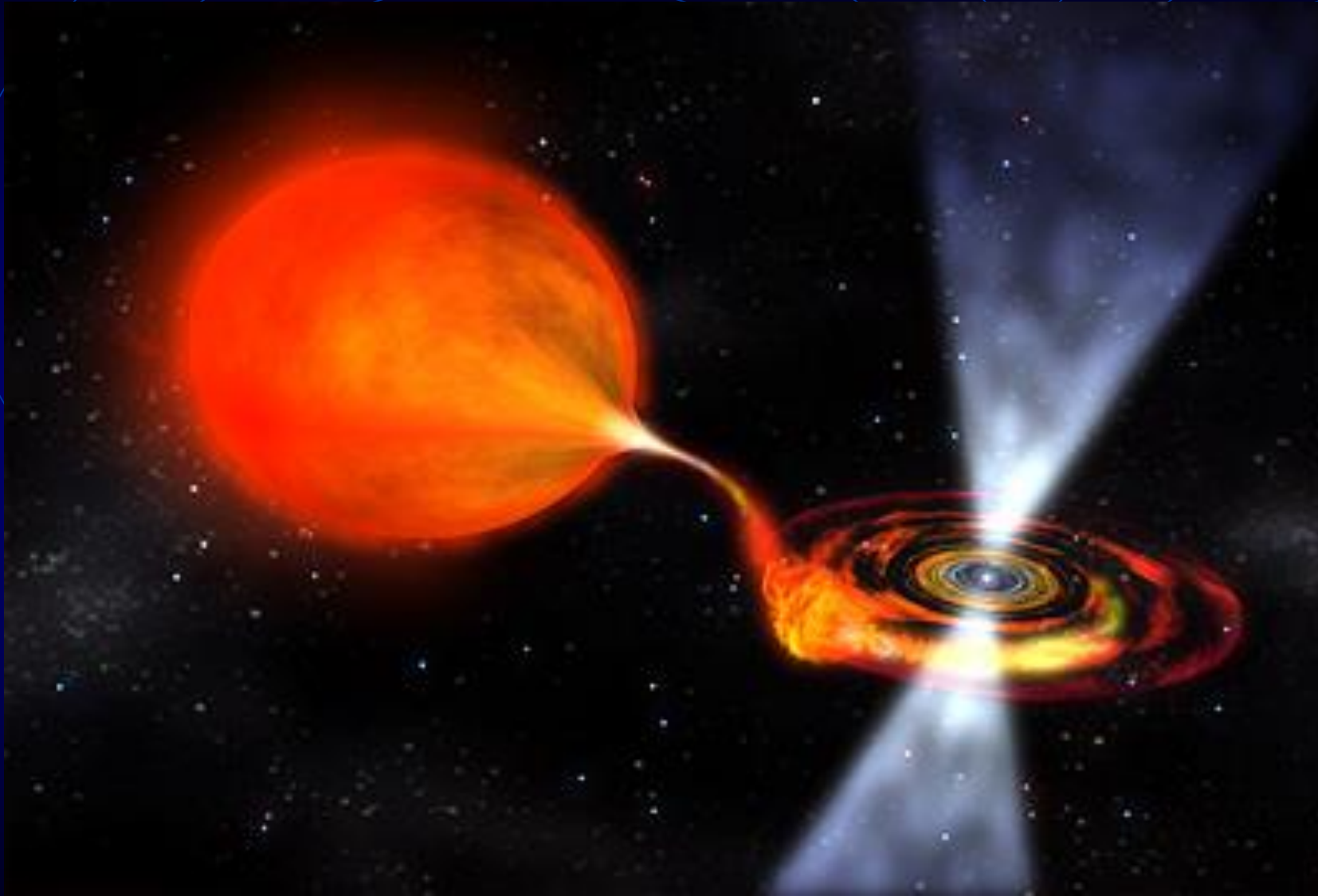
# 中子星內部結構



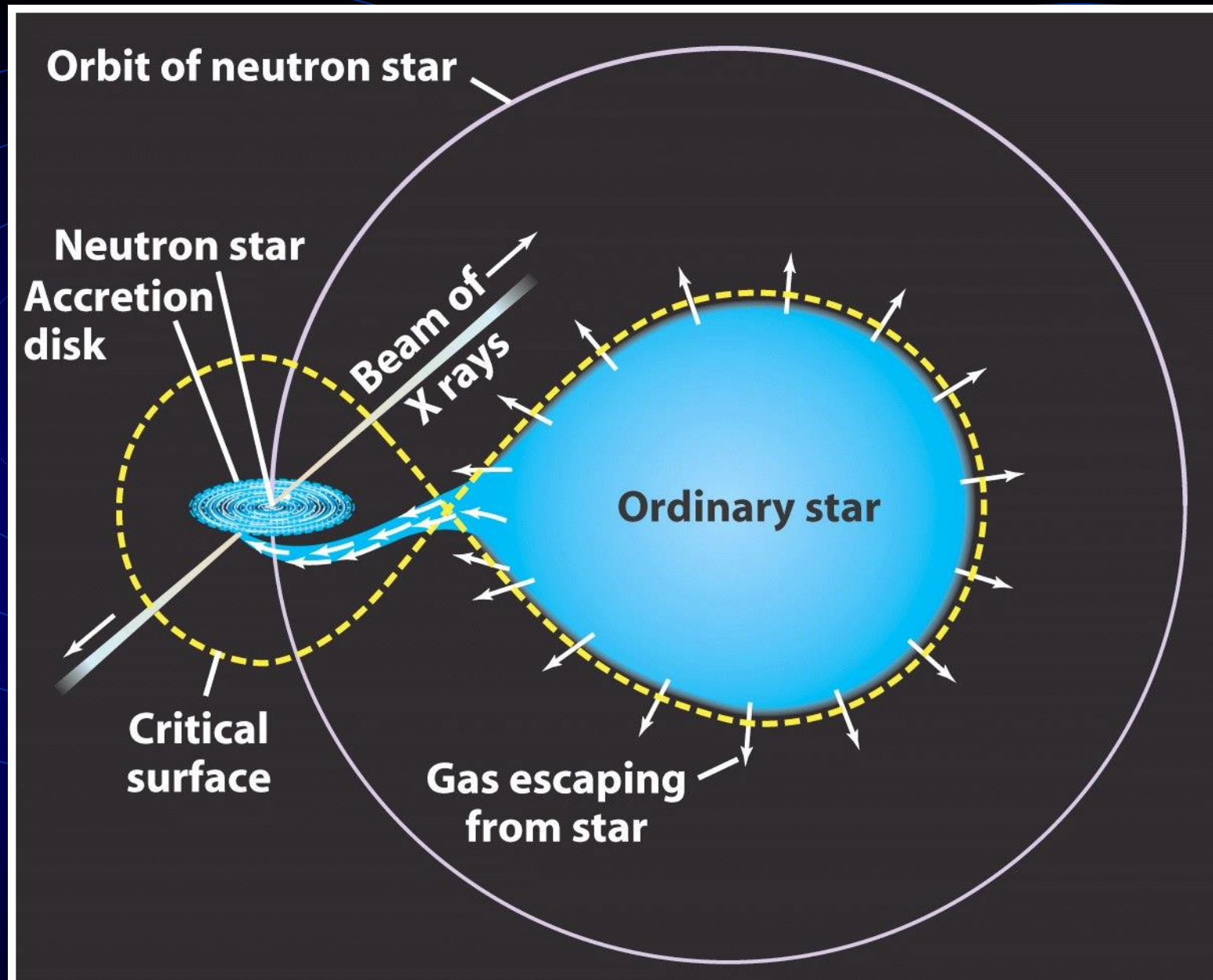
理論計算 → 內部處於  
超流體 (superfluidity)  
與  
超導體 (superconductivity)  
狀態



# 中子星與吸積作用



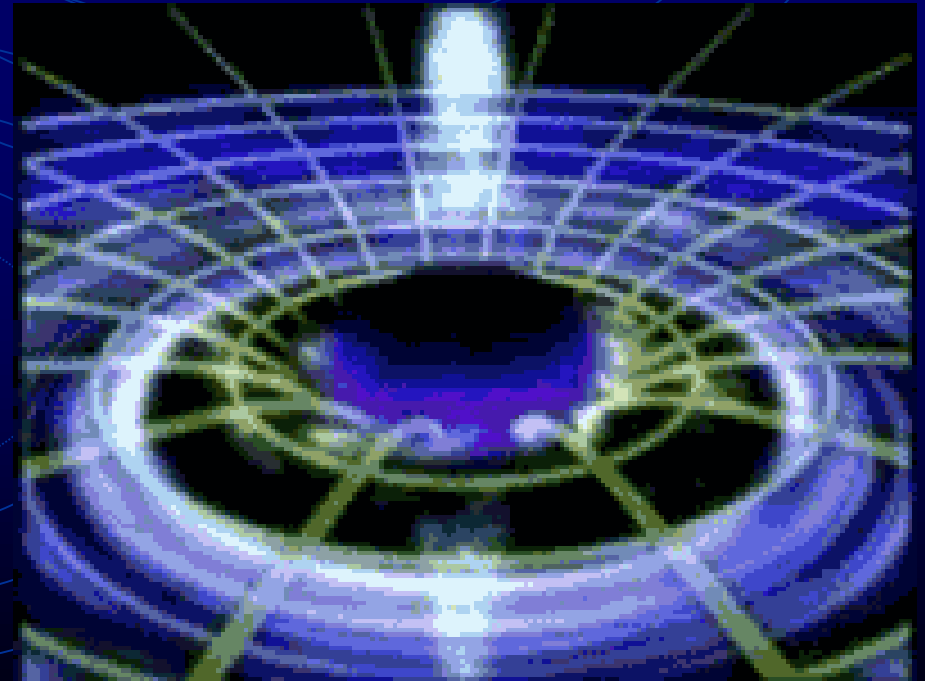
銀河系中據估計  
可能存在上億顆  
中子星，但如果  
並非pulsars 或是  
雙星系統中有吸  
積活動，中子星  
不易被偵測到



A pulsating X-ray Source

# 大質量恆星晚年

- 例如核心質量大於太陽3倍
- 連中子或夸克的支撐力量都抵擋不住巨大的萬有引力  
→ 星體塌縮
- 強大的萬有引力連光線都跑不出來  
→ **黑洞 (black hole)**
- 黑洞是種簡單的質能狀態，  
其「表面」的逃脫速度等於光速

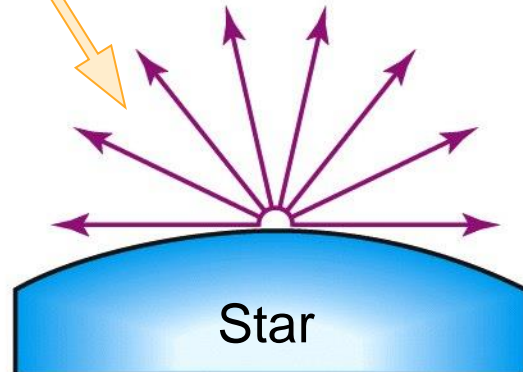


# 脫離速度 (escape velocity)

- 拋個銅板向上...銅板向上飛，達到最高點後停止，接著向下飛
- 如果用力拋個銅板向上...
- 但是如果真的很用力（夠快），越高處離地心越遠，引力越弱，便無法讓銅板停止
- 這個開始拋速度稱做「逃脫速度」
- 地球的 escape velocity 為 40,200 km/h 或 11 km/s；大於這個速度毋須額外力量就可脫離地球

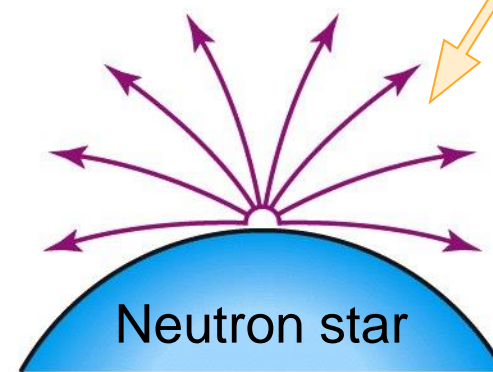


1. A supergiant star has relatively weak gravity, so emitted photons travel in essentially straight lines.



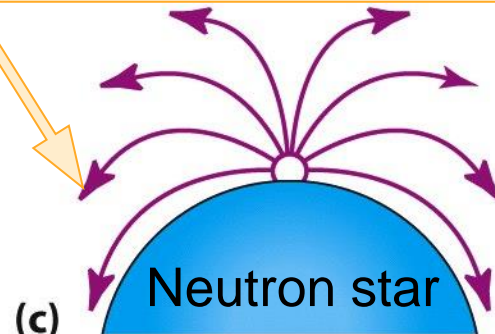
(a)

2. As the star collapses into a neutron star, the surface gravity becomes stronger and photons follow curved paths.



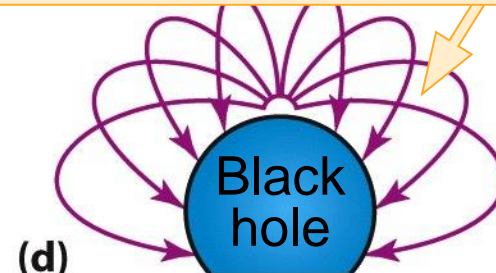
(b)

3. Continued collapse intensifies the surface gravity, and so photons follow paths more sharply curved.



(c)

4. When the star shrinks past a critical size, it becomes a black hole: Photons follow paths that curve back into the black hole so no light escapes.



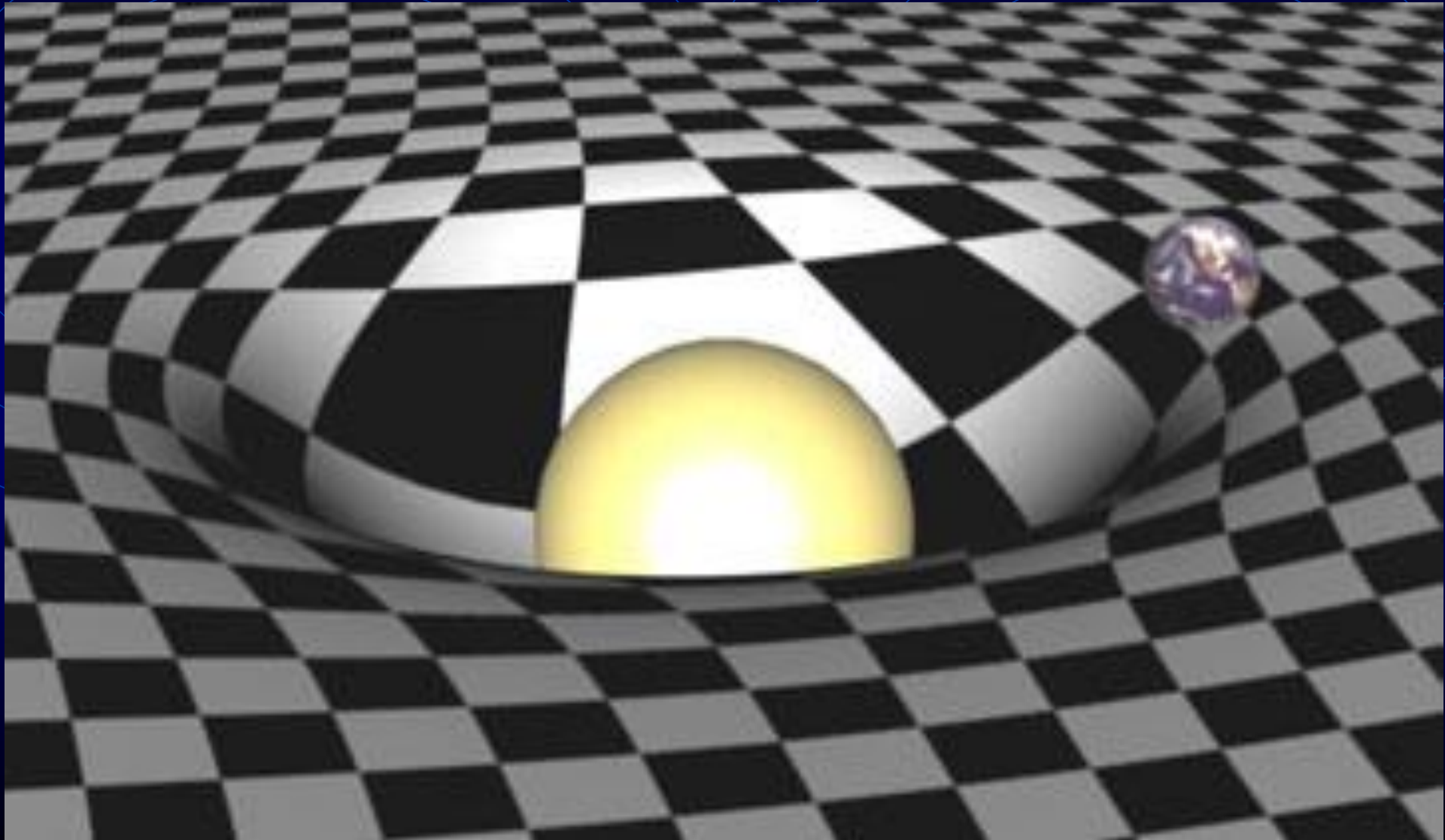
(d)

Q：如果太陽塌縮成了黑洞，  
對地球有何影響？

- 會被吸進去嗎？
- 不再有春、夏、秋、冬？
- 從此適合天文觀測？



如果物質夠緻密（在一定的體積內有太多物質）  
→ 造成周圍空間彎曲



# Space and Time

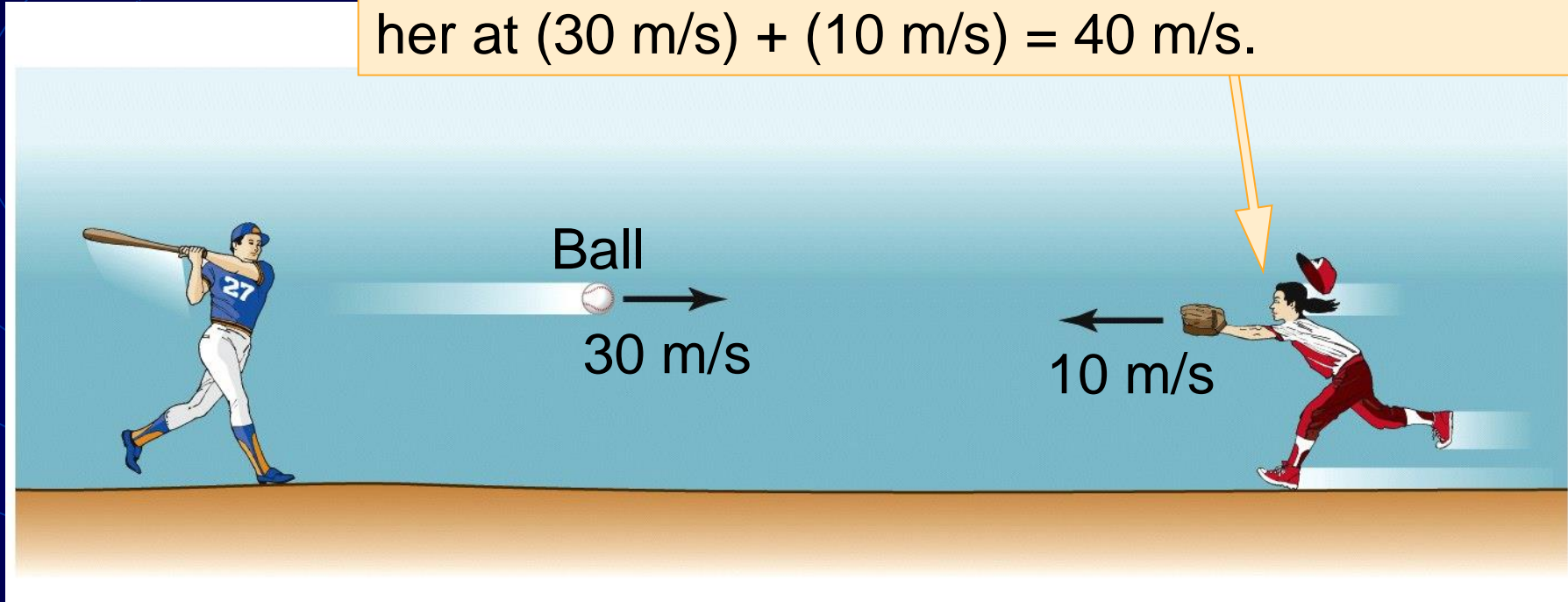
**Special Theory of Relativity** --- how motion affects our measurements of distance and time (special, because gravity is not included)

與古典牛頓力學不同，特殊相對論中，時間與空間沒有絕對參考座標系統

1. 只要等速直線運動，看到的事情都一樣
2. 任何等速直線運動觀察者量到相同光速

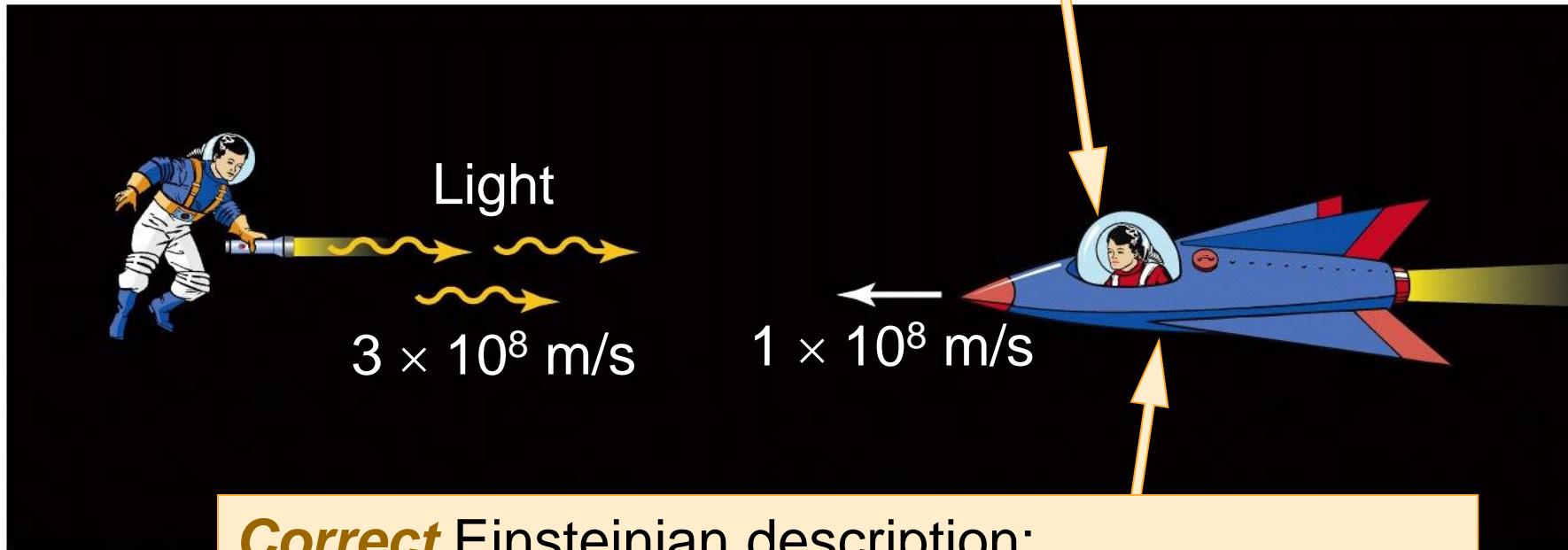
**Spacetime as a 4-dimensional entity**

As seen by the outfielder, the ball is approaching her at  $(30 \text{ m/s}) + (10 \text{ m/s}) = 40 \text{ m/s}$ .



**Incorrect** Newtonian description:

As seen by the astronaut in the spaceship, the light is approaching her at  $(3 \times 10^8 \text{ m/s}) + (1 \times 10^8 \text{ m/s}) = 4 \times 10^8 \text{ m/s}$ .



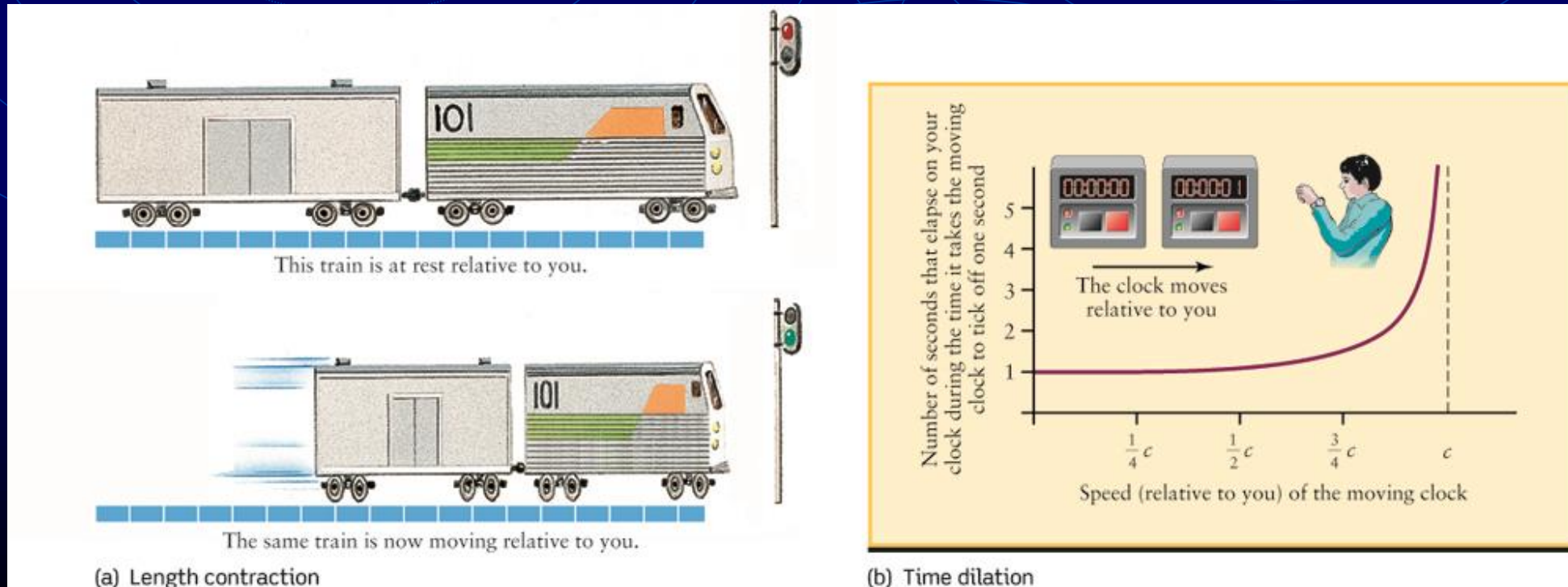
**Correct** Einsteinian description:

As seen by the astronaut in the spaceship, the light is approaching her at  $3 \times 10^8 \text{ m/s}$ .

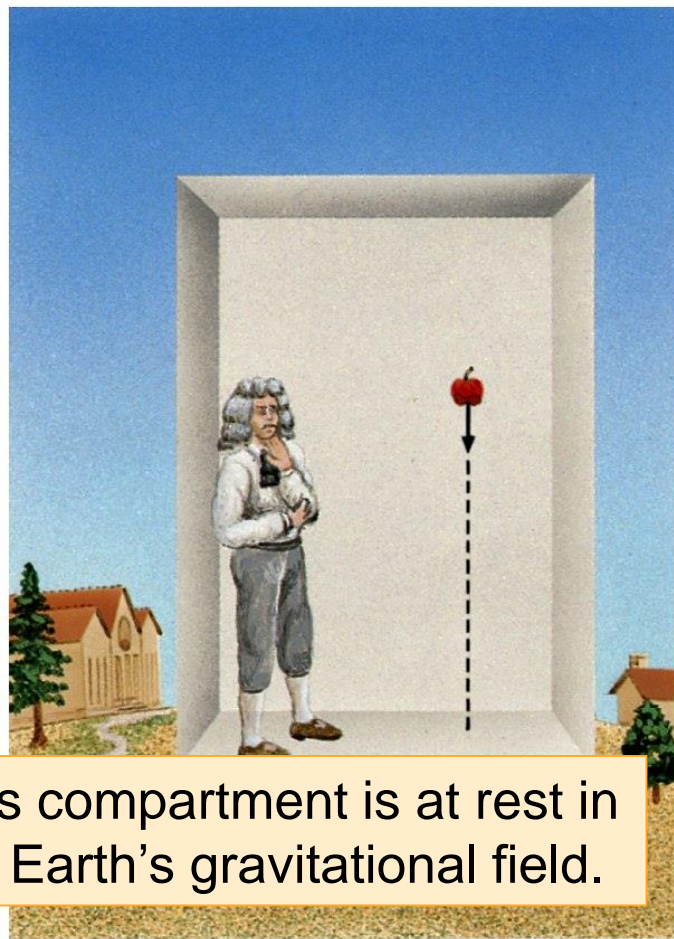
# 特殊相對論

- **Length Contraction**  
速度越快者長度越短
- **Time Dilation**  
速度越快者時間越慢

- **Mass Increase**  
速度越快者質量越大

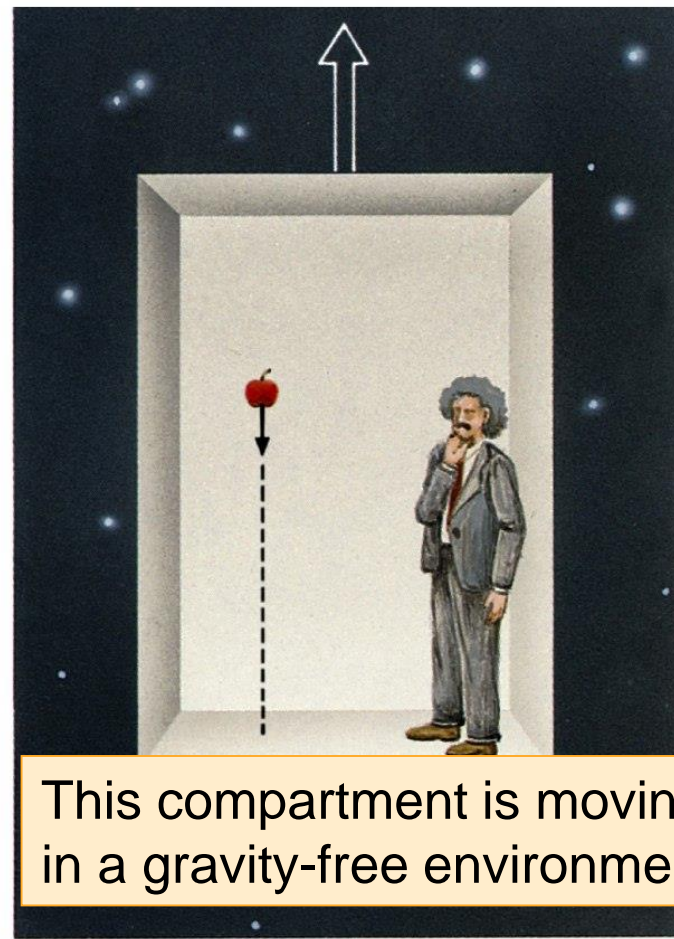


# 廣義相對論



This compartment is at rest in the Earth's gravitational field.

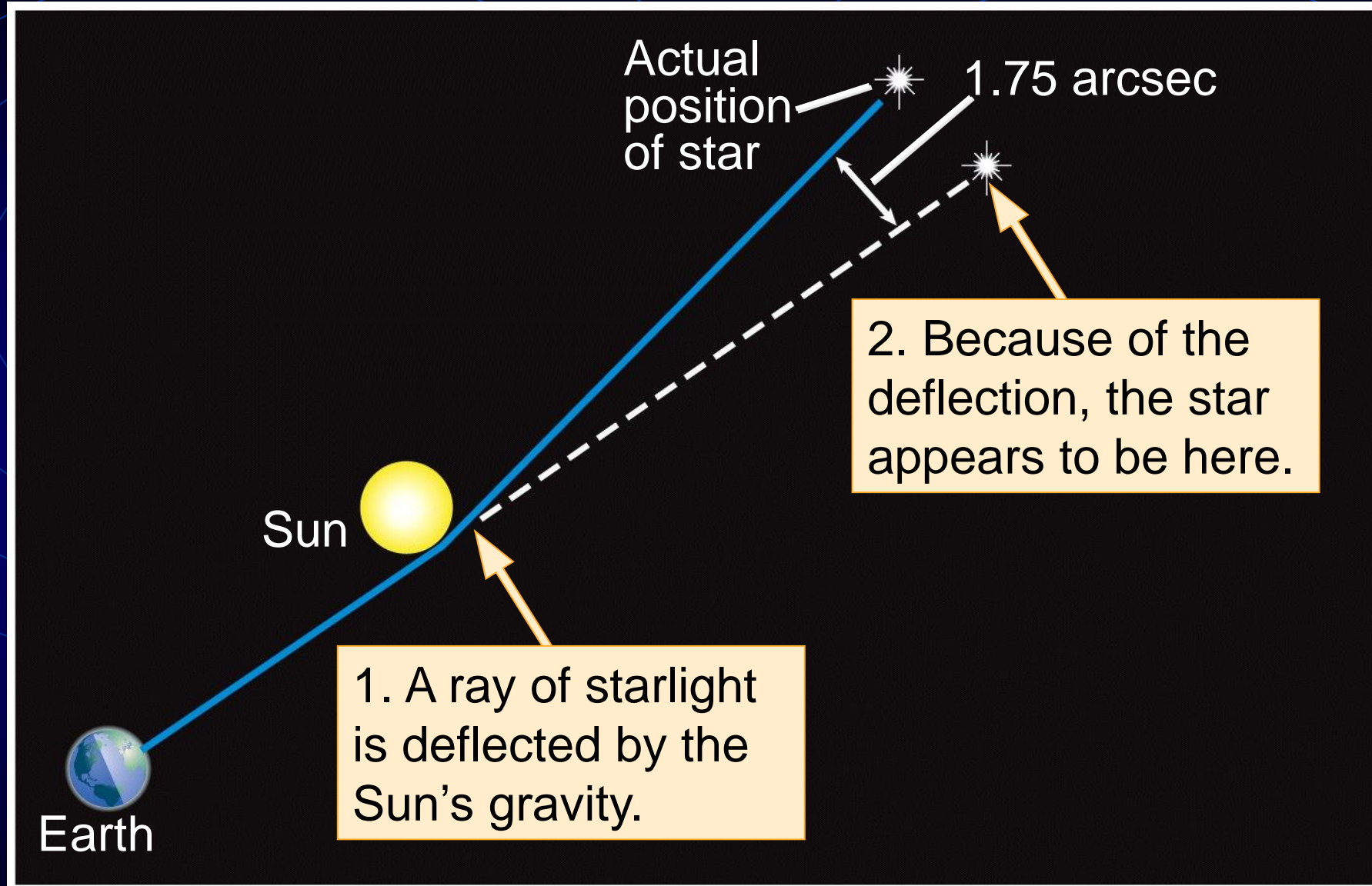
(a) The apple hits the floor of the compartment because Earth's gravity accelerates the apple downward.



This compartment is moving in a gravity-free environment.

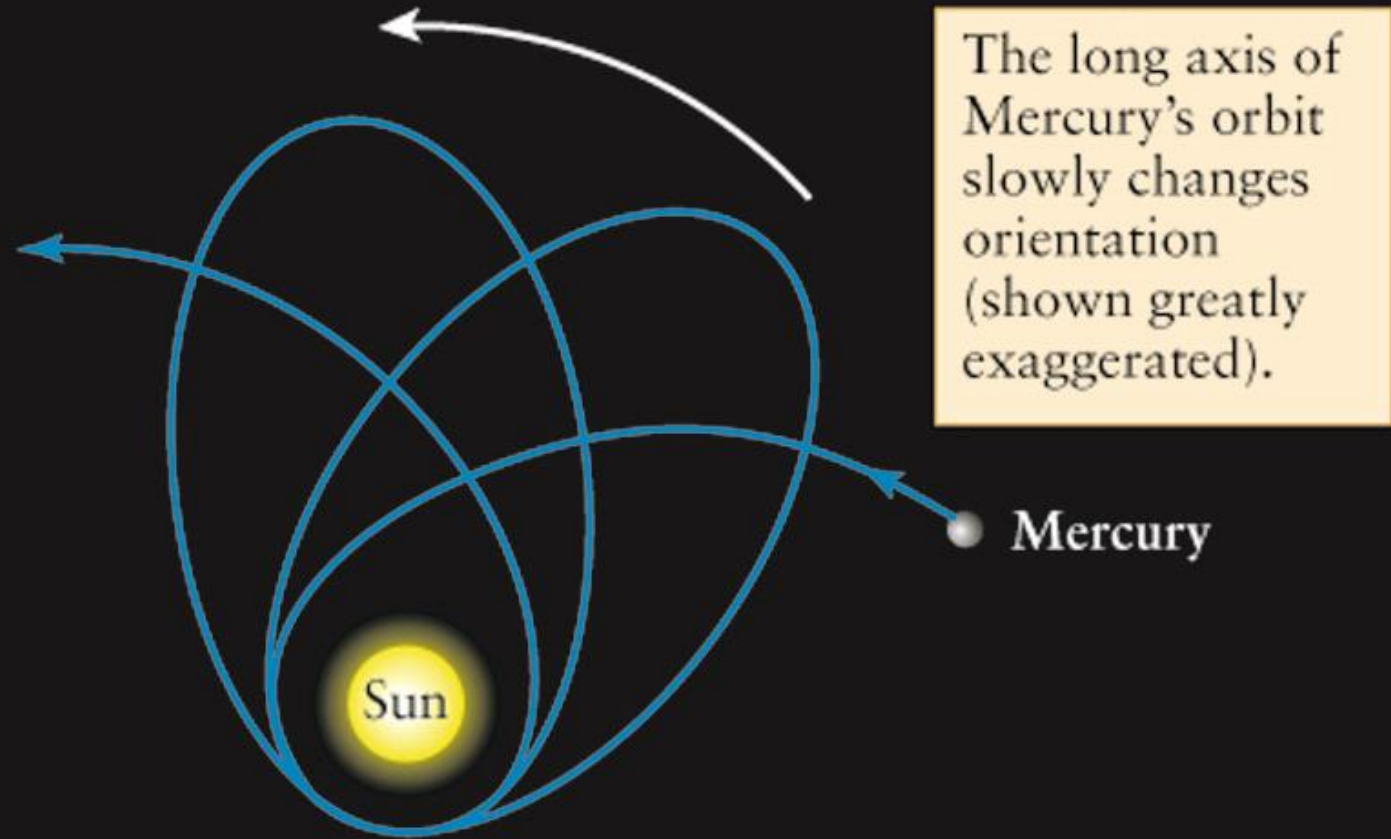
(b) The apple hits the floor of the compartment because the compartment accelerates upward.

# Testing General Relativity I



Confirmed during a solar eclipse in 1919.

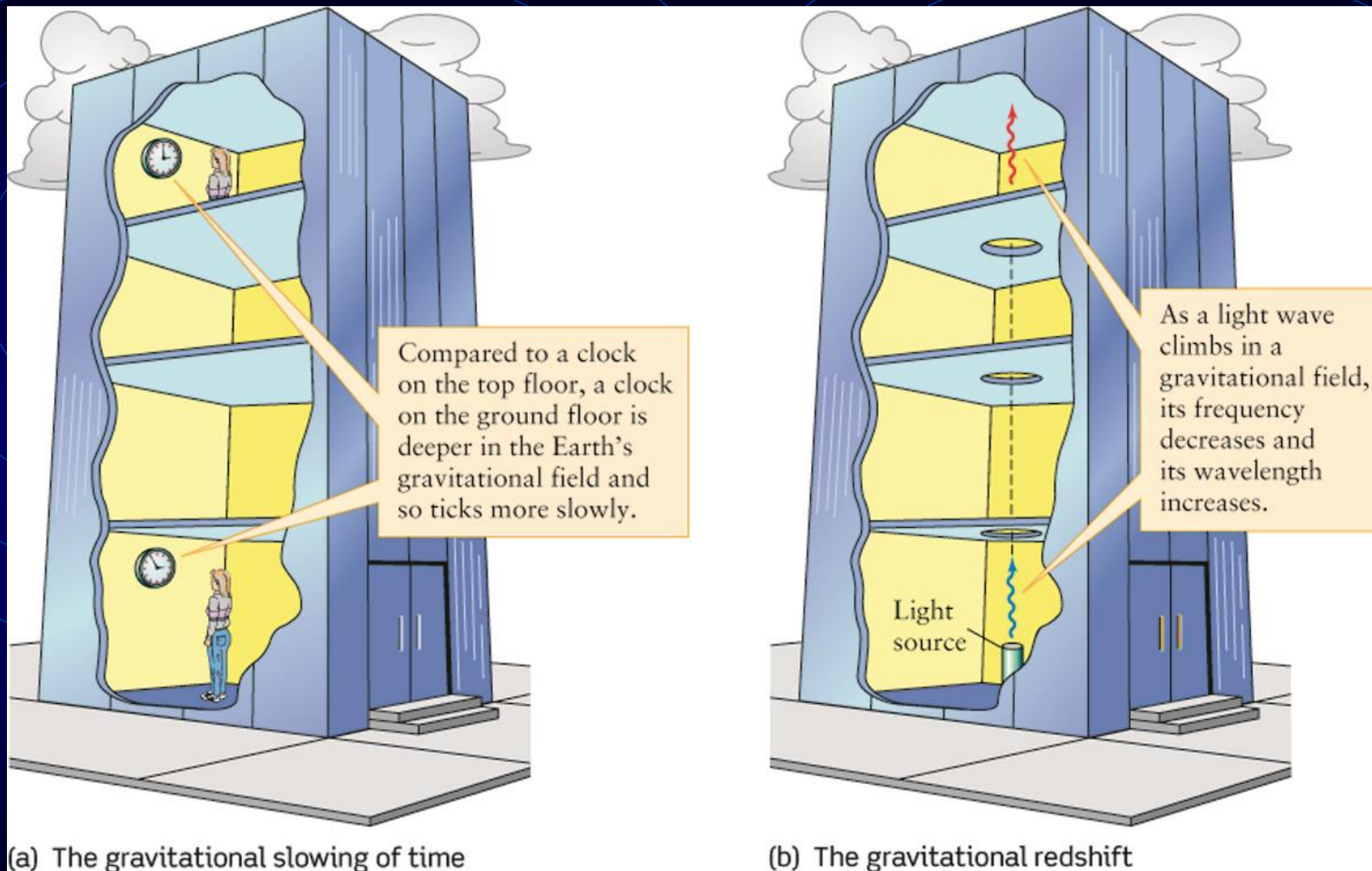
## Testing General Relativity II



Precession caused by pull of planets →  
531"/century, but 574"/century observed



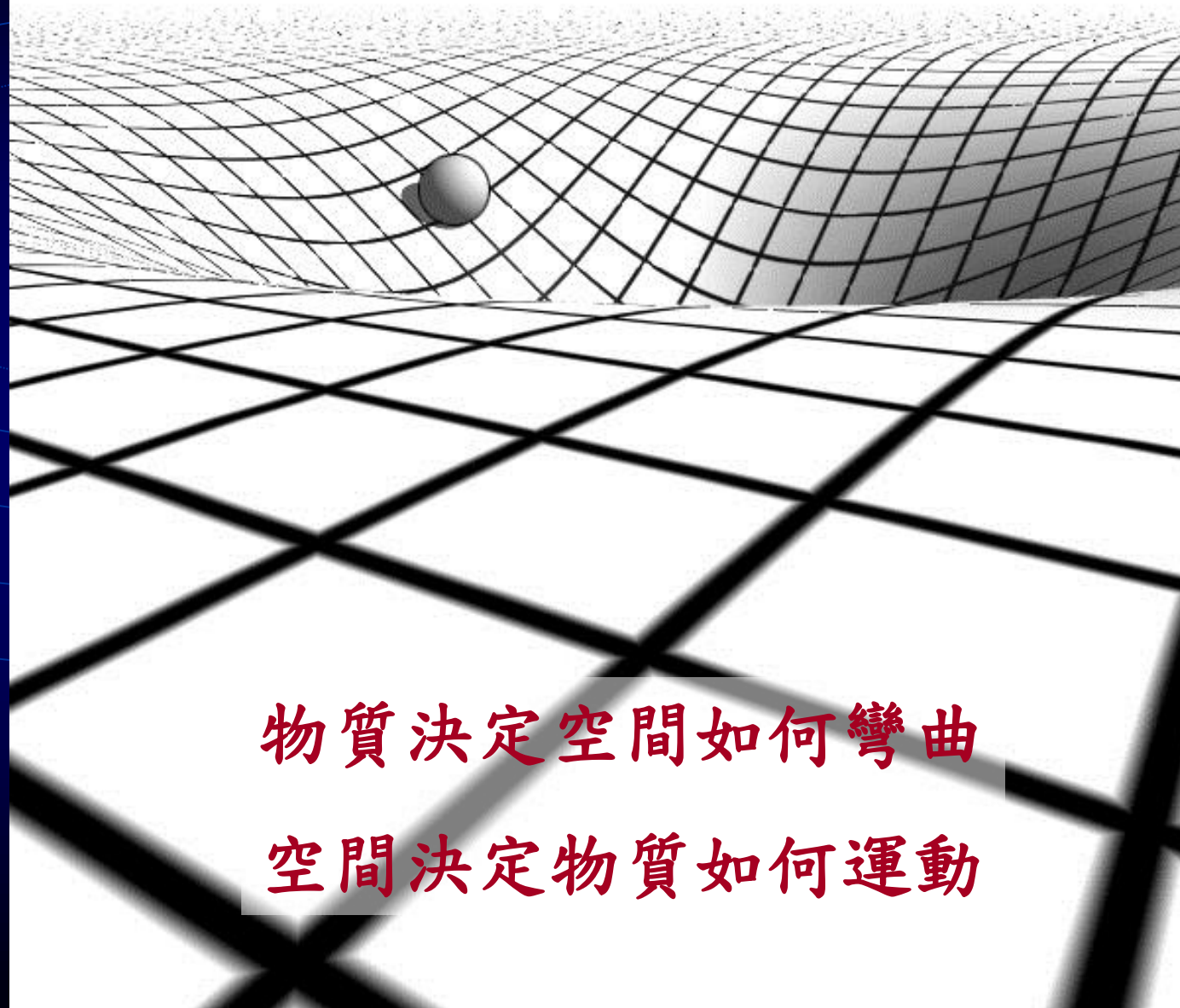
# Testing General Relativity III



樓下的鐘比較慢

脫離重力場的光子損失能量

黑洞：物質壓縮，密度無限大  
→ 時空奇異點 (spacetime singularity)



物質決定空間如何彎曲

空間決定物質如何運動

- 決定某個天體逃脫速度的因素包括它的質量以及大小（直徑）

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$

- 若某天體相對於體積，質量很大，逃脫速度等於光速  
→ 黑洞

- 該半徑稱為 **Schwarzschild radius 史瓦茲半徑** )  
該球面稱為 **事件地平面 (event horizon)**  
其內的訊息跑不出來

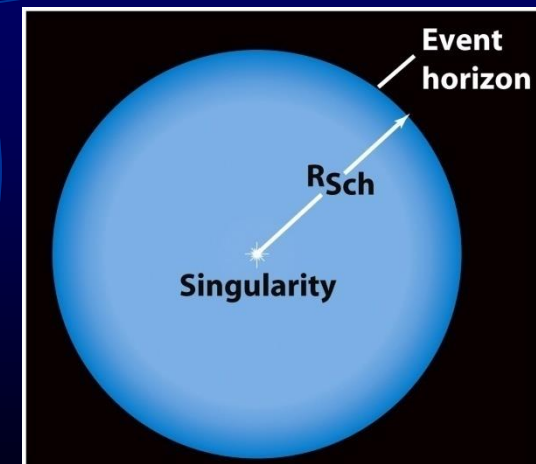


Figure 14-10  
Discovering the Universe, Seventh Edition  
© 2006 W. H. Freeman and Company

# 史瓦茲半徑

$$R_{Sch} \approx 2GM/c^2 \approx 3(\mathcal{M}/\mathcal{M}_{\odot}) [\text{km}]$$

其中  $\mathcal{M}$  代表黑洞的質量， $\mathcal{M}_{\odot}$  為太陽質量

- 如果太陽成為黑洞，半徑約為 3 公里
- 如果地球成為黑洞，半徑約為 1 公分
- 如果喜馬拉雅山變成黑洞，大小有多大？
- 質量為太陽質量 1 億倍的黑洞，其大小為...

這相當於多大？（和地日距離比） 密度呢？

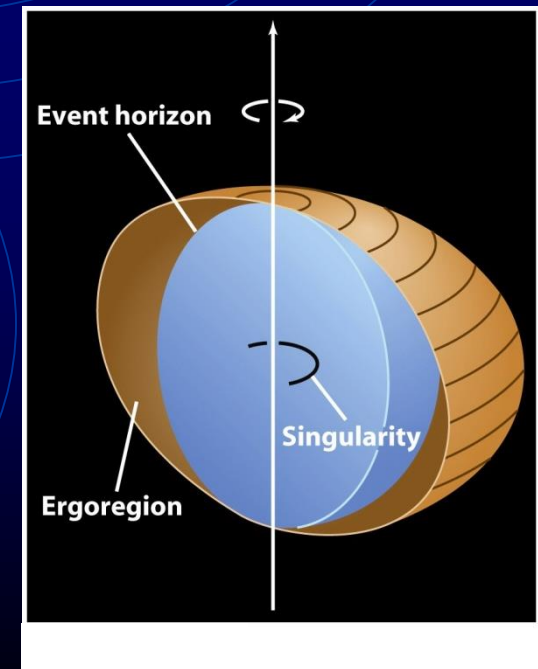
# 黑洞的種類

- **Schwarzschild black holes**

沒有自轉的黑洞（原來的物質就沒有自轉），  
物質全集中在奇異點，該處密度無窮大

- **Kerr black holes**

自轉黑洞，每秒達數千轉。  
Beyond the event horizon  
**ergoregion**（動區），  
被黑洞帶著一起旋轉

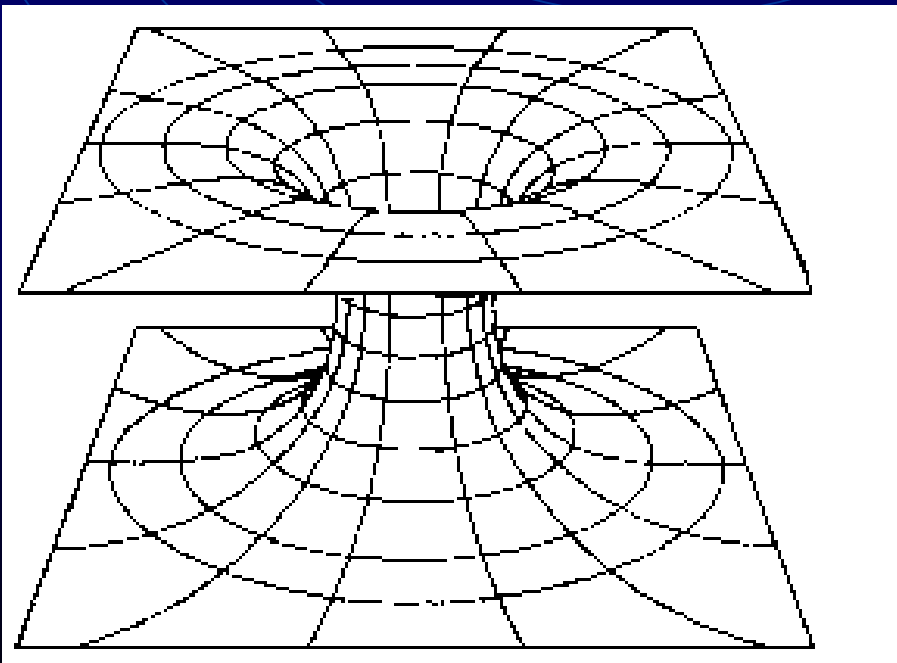




	<b>Non-rotating</b>	<b>Rotating</b>
<b>Uncharged</b>	<b>Schwarzschild</b>	<b>Kerr</b>
<b>Charged</b>	<b>Reissner–Nordström</b>	<b>Kerr–Newman</b>

時空扭曲可以成為  
「捷徑」，通往宇宙  
其他角落？

時空旅行？

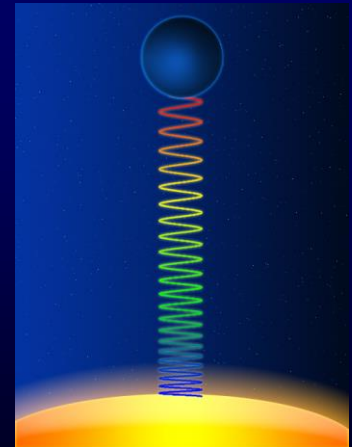


# Traveling to a black hole

Which kind of a black hole? A supermassive black hole is drastically different from a stellar black hole.

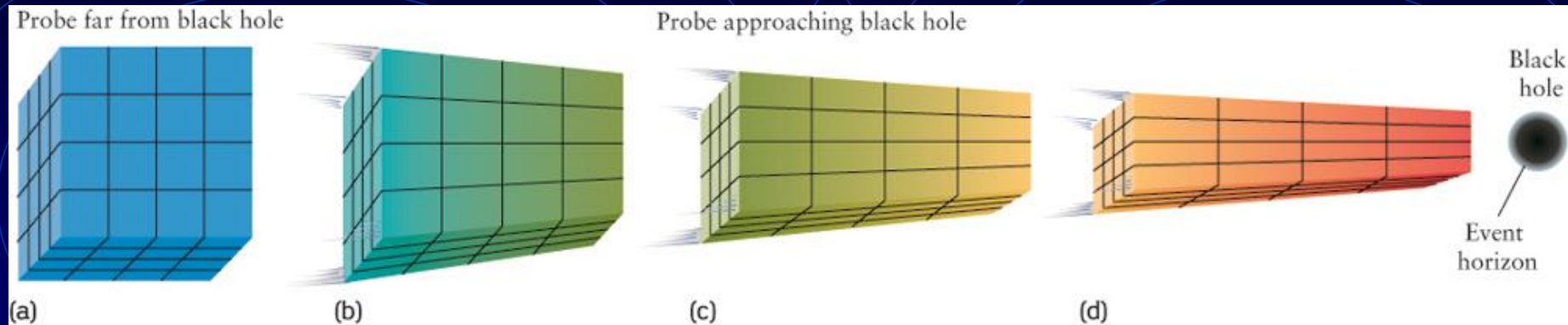
The experience/view is very different for the traveler from that for a distant observer.

- ◆ Strong gravitation force/tidal disruption
- ◆ Special Relativity: Length contraction, Time dilation, Relativistic mass
- ◆ Gravitational redshift





# 黑洞附近奇特的時空現象



For a distant observer:

Gravitational redshift

Tidal force  $\rightarrow$  stretching, stripping apart

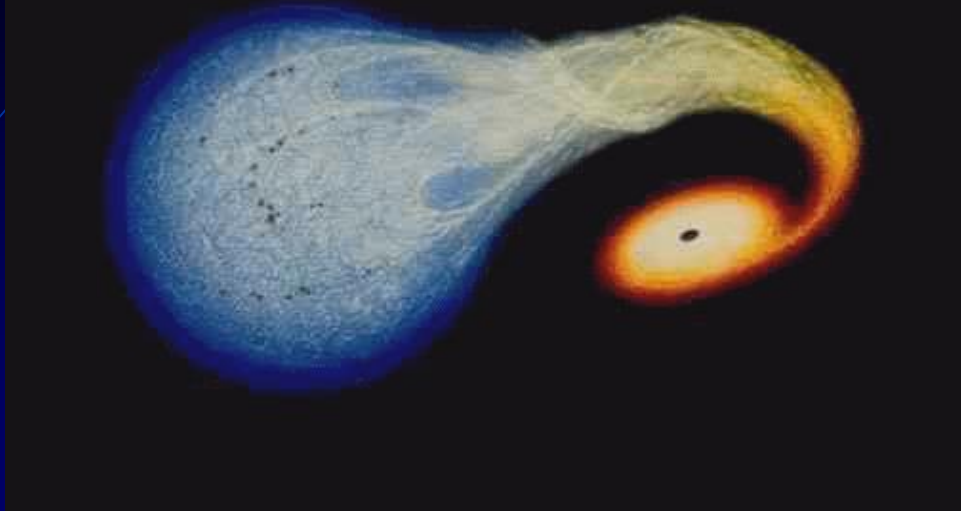
Time dilation  $\rightarrow$  Probe slowing down

For a passenger on the probe toward the BH:

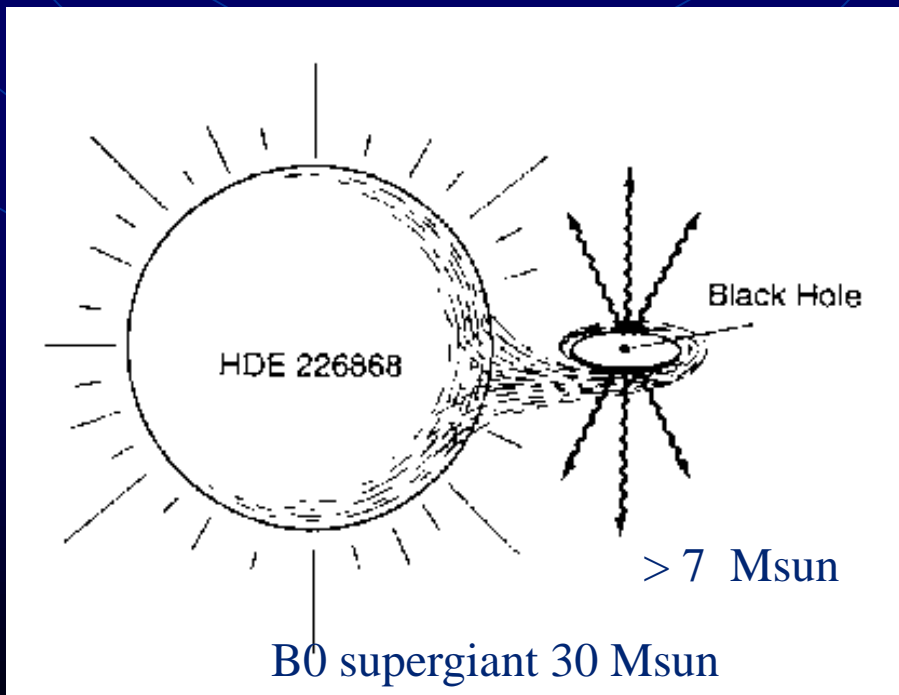
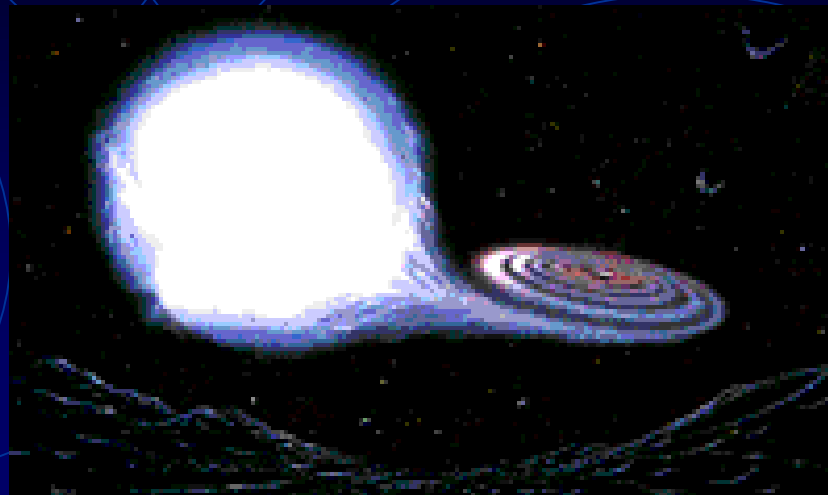
No slowing down

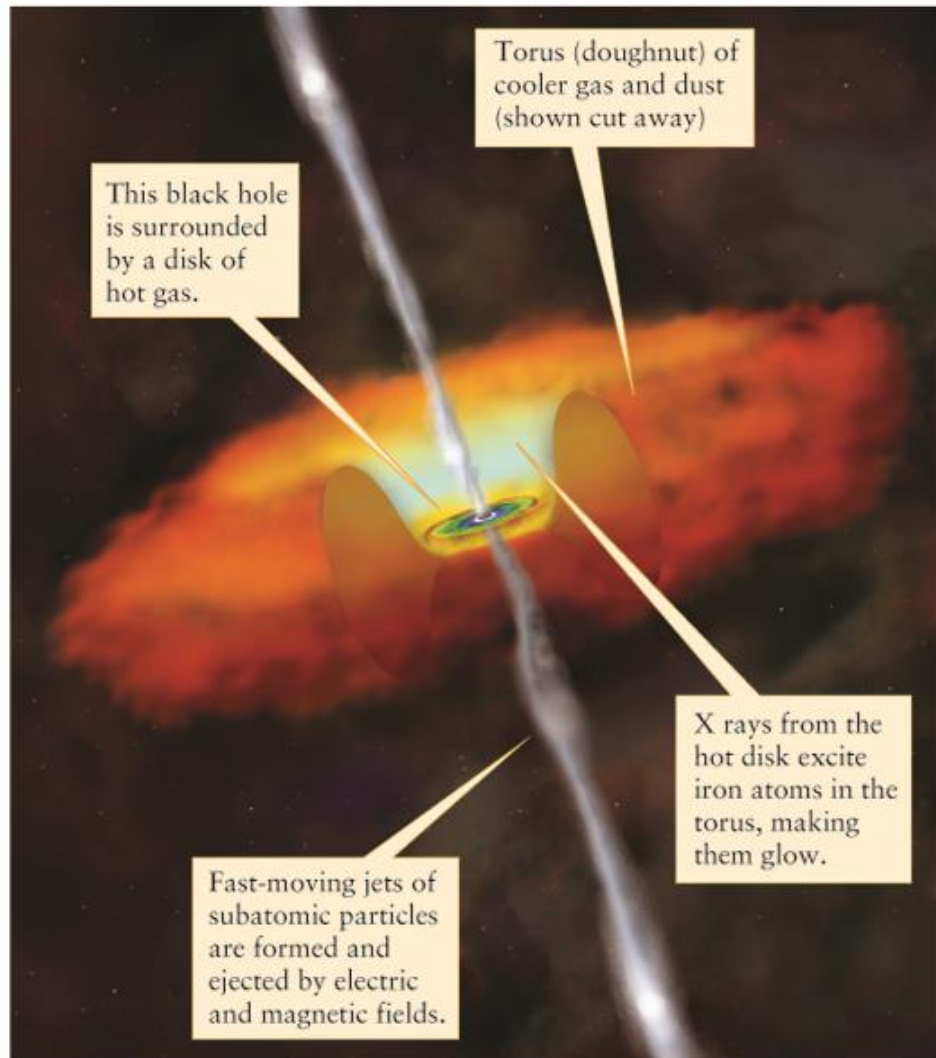
Cygnus X-1 black hole  
C2

© 1993, Loch Ness Productions  
markpet@scicom.alphacdc.com  
Artist: Tim W. Kuzniar



# 利用黑洞對伴星的引力作用，藉以發現黑洞





## Environment of an accretion black hole

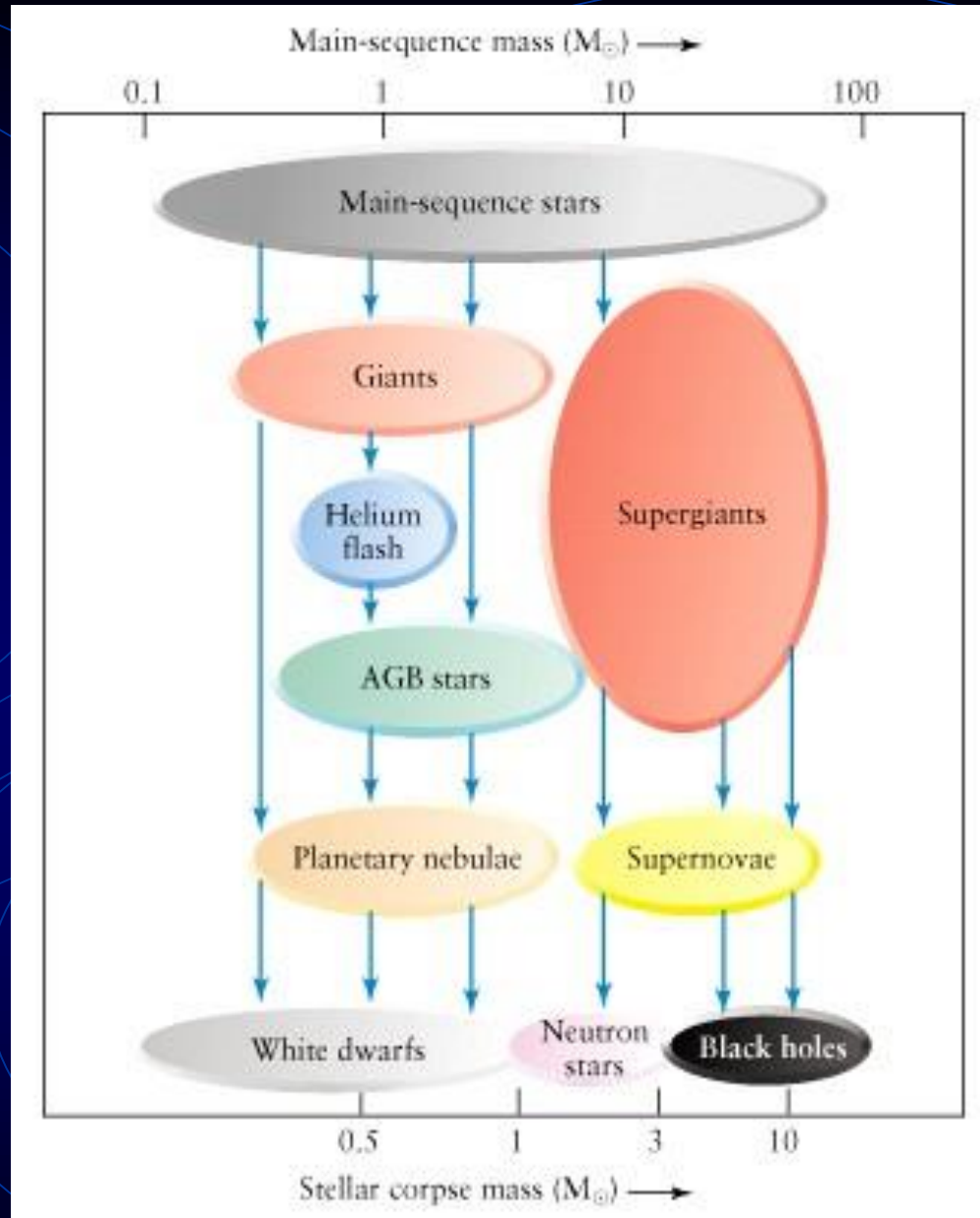
BH rotates → electric and magnetic fields

Channeling accreted material and accelerating as jets along rotation axis

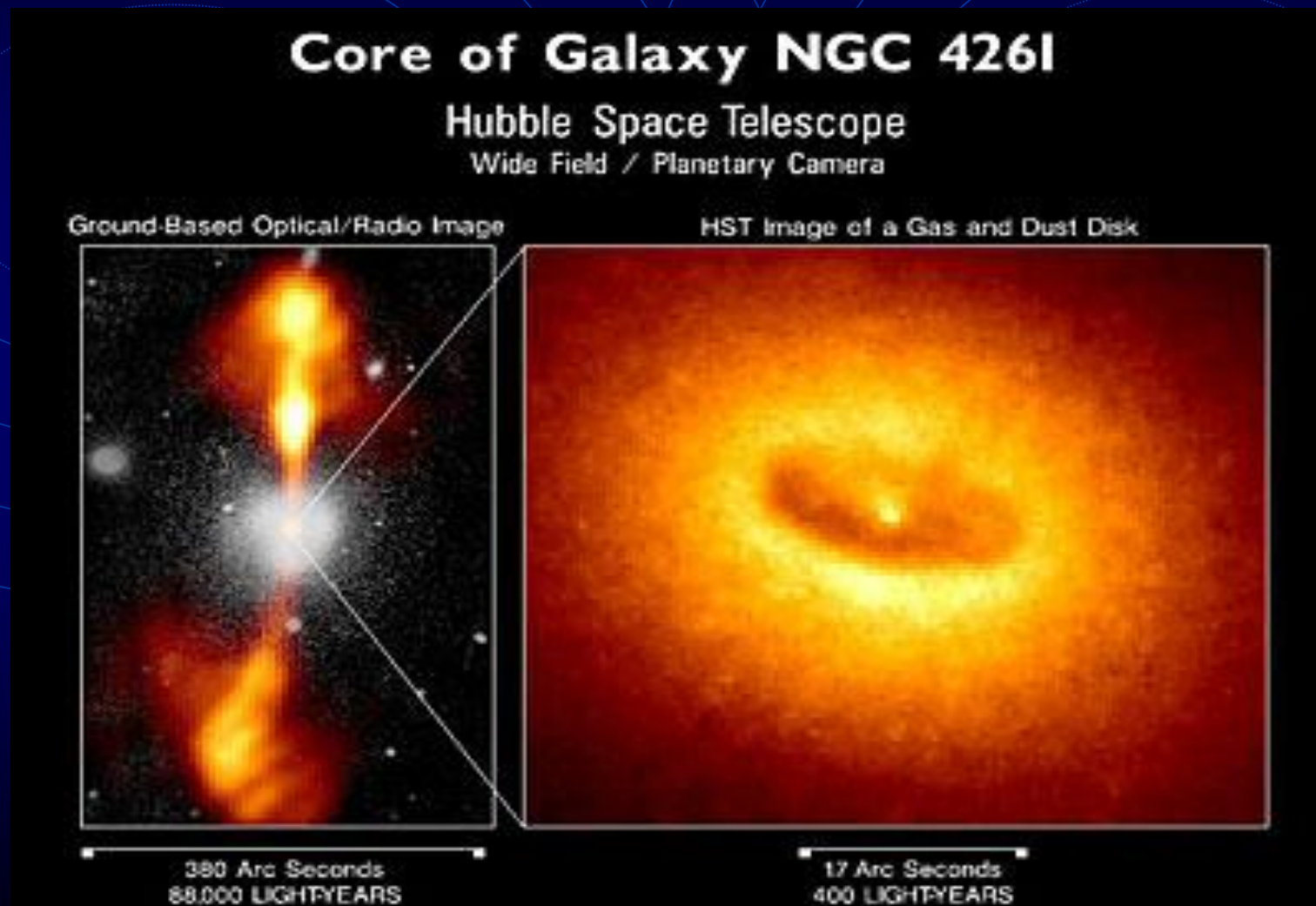
恆星在主序時的質量

質量流失

恆星死亡時的質量

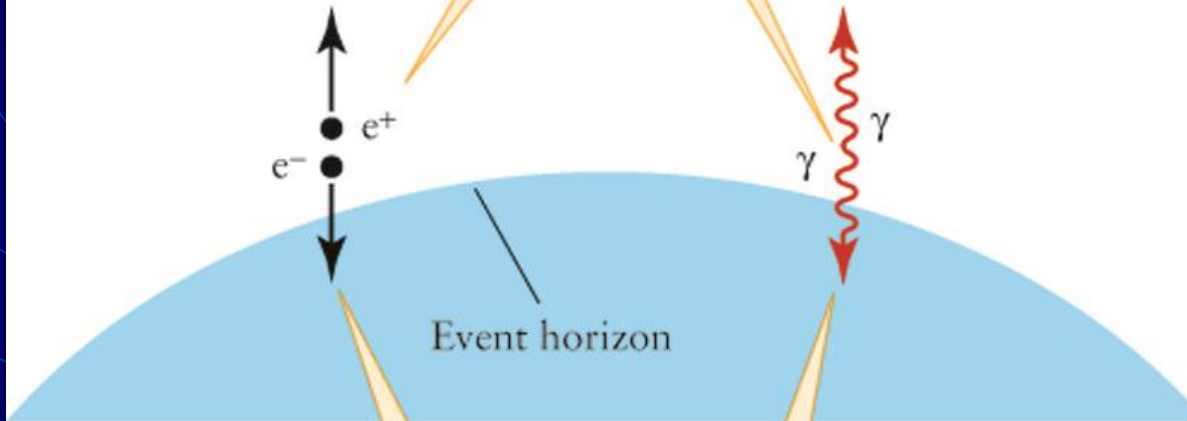


一些星系的核​​心可能也有  
**超大質量黑洞 (supermassive black holes)**



1. Pairs of virtual particles spontaneously appear and annihilate everywhere in the universe.

2. If a pair appears just outside a black hole's event horizon, tidal forces can pull the pair apart, preventing them from annihilating each other.



3. If one member of the pair crosses the event horizon, the other can escape into space, carrying energy away from the black hole.

A  $10^{10}$  kg BH will take 15 billion years to evaporate.

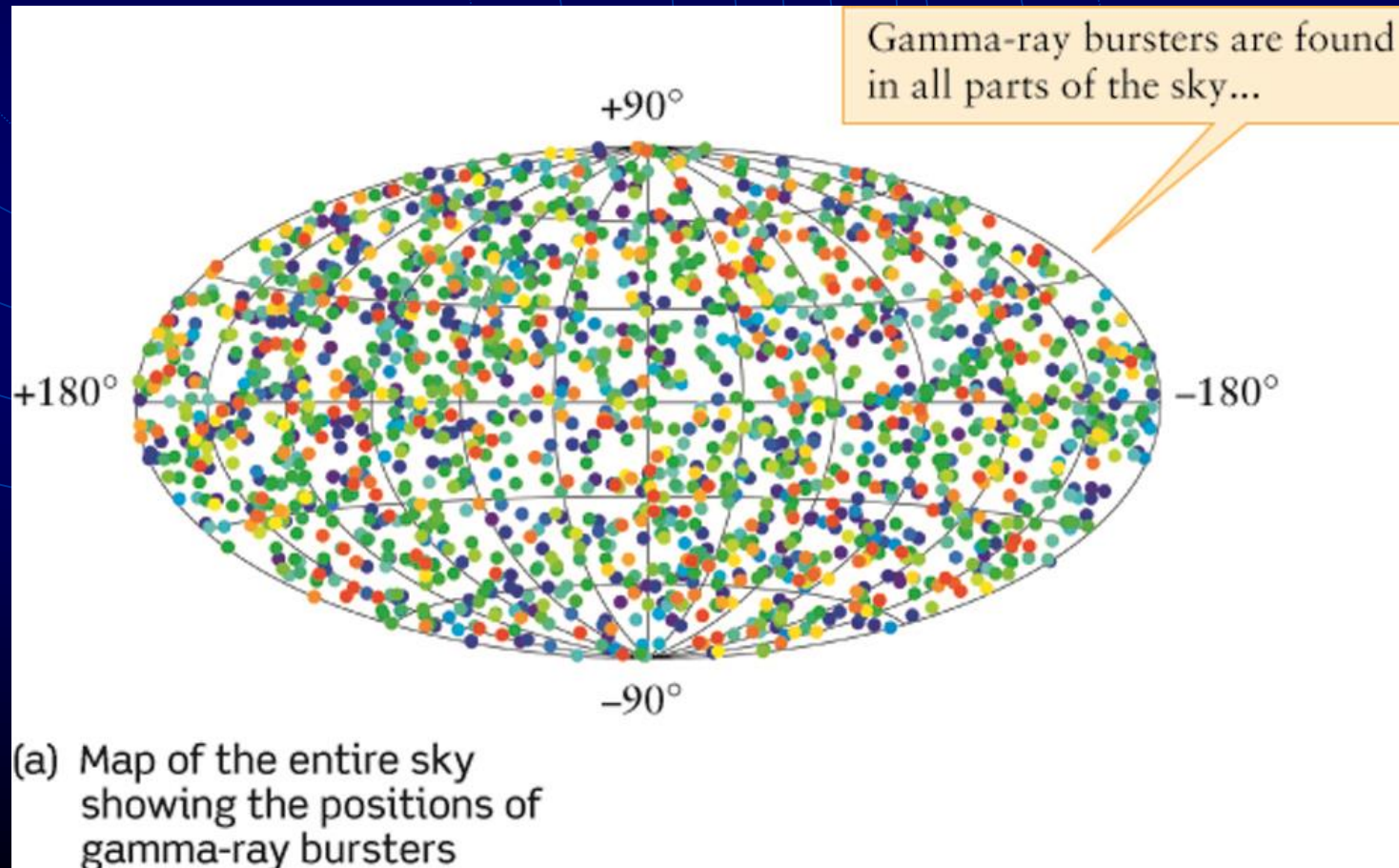
A  $5 M_{\odot}$  BH will take  $10^{62}$  years.

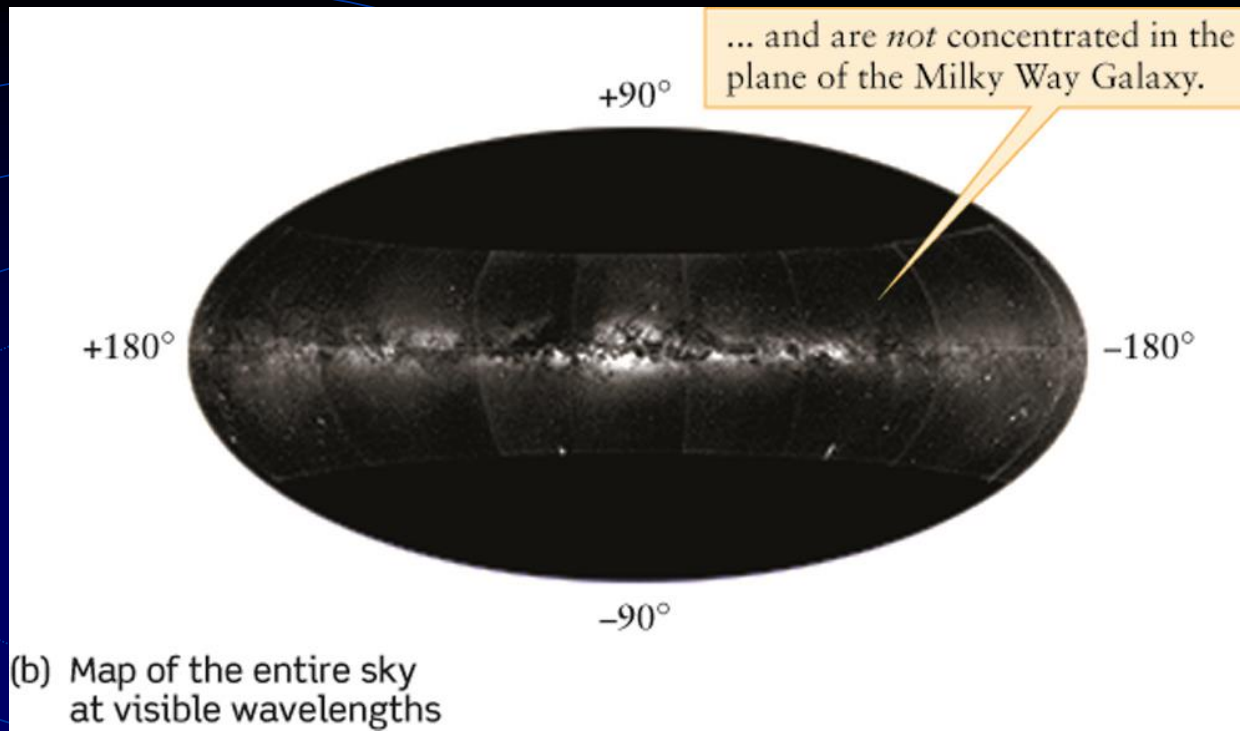
A supermassive (5 million  $M_{\odot}$ ) BH will take  $10^{80}$  years.

黑洞會「蒸發」，逐漸消失 (Hawking process)

# Gamma-ray Burst

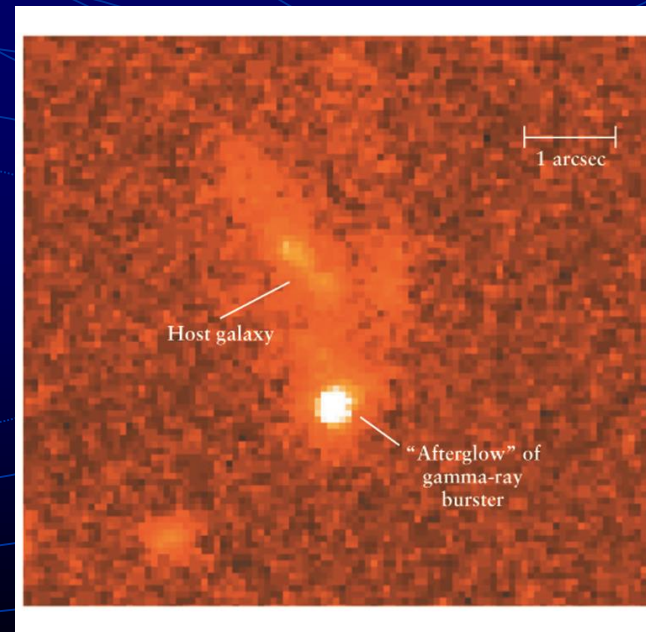
Incredibly intensive gamma-ray flashes  
→ gamma-ray bursters 伽碼線爆源 (GBRs)





This means GRBs are of extragalactic origin  
→ formation of BHs?

Hosting galaxies have been identified in some cases.





# 奇怪的天體

- 太空中還可能有更奇怪的天體
- 白矮星的質量上限 → Chandrasekhar limit
- 中子星也有質量上限 → **Oppenheimer-Volkov limit**  
大約是  $3 M_{\odot}$
- 很多基本粒子（包括質子、中子，但不包括電子）由夸克組成
- 如果星體壓力太大，中子分解成 quarks → **quark stars**  
性質仍不清楚，但質量可能與 Oppenheimer-Volkov 相當
- **strange matter** → **a strange star**

# *Dawn* spacecraft zooms in to Ceres ...

