Collisional Processes

- Long range interaction --- between ions/electrons and ions/electrons; Coulomb $1/r$
- Intermediate range interaction --- between ions/electrons and neutral atoms/molecules; Induced dipole $1/r^4$
- Short range interaction --- between neutrals, $1/r^6$

In general, for a two-body collision,

$A + B \rightarrow$ Products,

the reaction rate per unit volume = $n_A \, n_B \, <\sigma \, v>_{AB}$,

where the rate coefficient is

$$< \sigma v >_{AB} = \int_0^\infty \sigma_{AB} \, v \, f(v) \, dv \quad [\mathrm{cm}^3 \, \mathrm{s}^{-1}]$$

v = relative velocity between A and B

$\sigma_{AB}(v)$ = reaction cross section; velocity depedent

$f(v)$ = velocity distribution function

In thermal equilibrium

$$f_v\,dv = 4\pi(\frac{\mu}{2\pi kT})^{3/2} e^{-\mu v^2/2kT}\, v^2\, dv$$

In terms of energy

$$< \sigma v >_{AB} = (\frac{8kT}{\pi\mu})^{1/2} \int_0^\infty \sigma_{AB}(E) \frac{E}{kT} e^{-E/kT} \frac{dE}{kT}$$

If the density is high, e.g., in the Earth's atmosphere, three-body collision may become important,

A + B + C → Products,

the reaction rate per unit volume = $k_{ABC}\, n_A\, n_B\, n_C$ where k_{ABC} is the three-body collisional rate coefficient [$cm^6\ s^{-1}$]

Elastic scattering by an inverse-square force, e.g., Rutherford scattering

Exact solutions complicated; use the "**impact approximation**", i.e., motion in a straight line

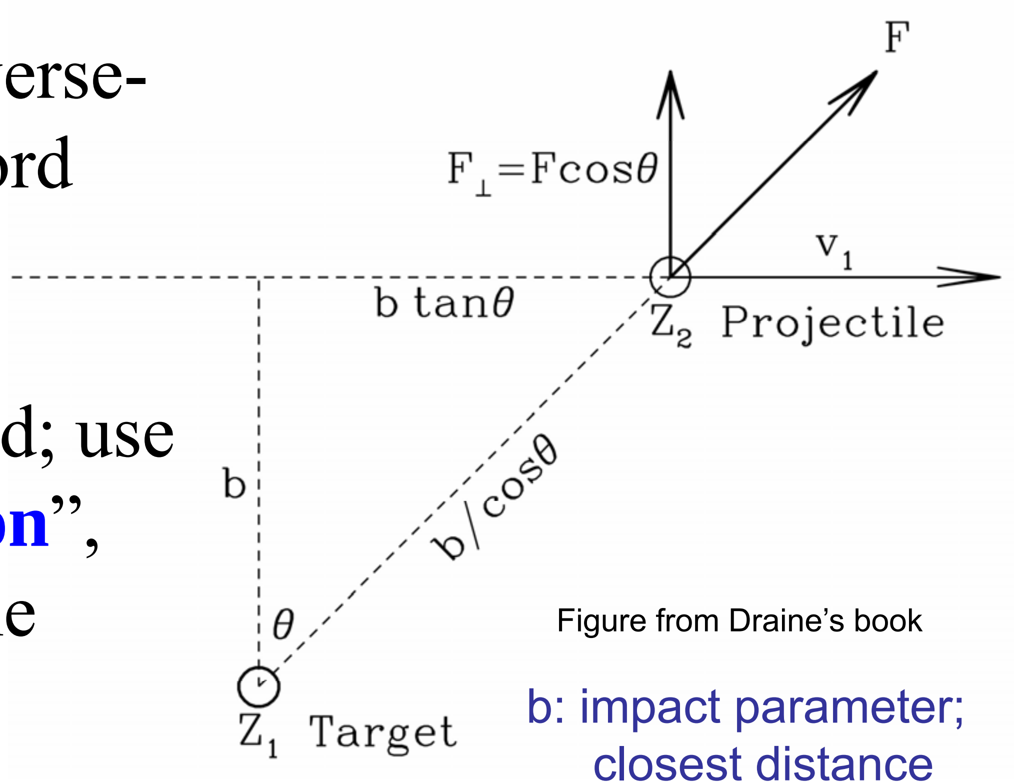


Figure from Draine's book

b: impact parameter; closest distance
v_1: relative velocity

Assumption: constant velocity during the encounter between the target and the projectile

Question: How much momentum is transferred ($\perp$ direction)?

Impact Approximation

Coulomb force

$$F_\perp = \frac{Z_1 e Z_2 e}{(b/\cos\theta)^2}\cos\theta = \frac{Z_1 Z_2 e^2}{b^2}\cos^3\theta$$

Interaction time scale $dt = \frac{d(b\tan\theta)}{v_1} = \frac{b}{v_1}\frac{d\theta}{\cos^2\theta}$

Total momentum transfer is

$$\begin{aligned}\Delta p_\perp &= \int_{-\infty}^{\infty} F_\perp\, dt = \frac{Z_1 Z_2 e^2}{b v_1}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta \\ &= \frac{2 Z_1 Z_2 e^2}{b v_1} \approx \underbrace{\frac{Z_1 Z_2 e^2}{b^2}}_{\text{Force at closest distance}}\underbrace{\frac{b}{v_1}}_{\text{Time scale}}\end{aligned}$$

In a collisional ionization, there must be enough momentum transfer. ($E=p^2/2m$)

Fast moving
→ $(1/2)\, m_e v^2 >> E_I$

e^-

⊕

$$(\Delta P_\perp)^2 > 2mE_I \Rightarrow \left(\frac{2Z_1 Z_2 e^2}{b v_1}\right)^2 > 2mE_I$$

So,

$$b^2 < b_{\text{max}}^2(v) = \frac{(2Z_1 Z_2 e^2)^2}{v_1^2 \cdot 2mE_I} = \frac{2Z_p^2 e^4}{m_e v^2 E_I}$$

and the ionization cross section becomes

$$\sigma(v) \approx \pi b_{\text{max}}^2 = \frac{2\pi Z_p^2 e^4}{m_e v^2 E_I}$$

This is ok if v↑↑.

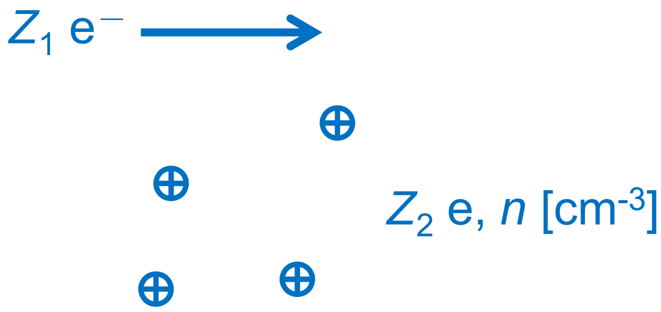
For $v_{\min} = \frac{1}{2} m_e v_{\min}^2 = E_I$

$$
\begin{aligned}
< \sigma v > &= \int \sigma(v) v f(v)\, dv \\
&= \int_{v_{\min}}^{\infty} \frac{2\pi Z_p^2 e^4}{m_e v^2 E_I}\, v\, 4\pi \left(\frac{m_e}{2\pi kT}\right)^{3/2} v^2\, e^{-m_e v^2/2kT}\, dv \\
&= Z_p^2 \left(\frac{8\pi}{m_e kT}\right)^{1/2} \frac{e^4}{E_I}\, e^{-E_I/kT}
\end{aligned}
$$

For an H atom at level n, $E_I = 13.6$ [eV]/n^2, so for a large n, e.g., n~100, and T~10^4 K, E_I ↓↓ (<< kT)
→ in radio frequencies.

$< \sigma v > \propto \frac{1}{E_I} \propto n^2$, so is very large.

Deflection Timescale



Net momentum transfer

$$< \frac{d}{dt}(\Delta P_\perp)^2 >$$

There must be a range of distance, for which

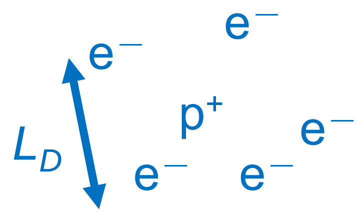
$b_{\min} = Z_1 Z_2 e^2/\text{Energy}$, and

$b_{\max} \approx L_D$ (Debye length)

In plasma, the distributions of ions and electrons are correlated because of charge neutrality.

Near a proton → more electrons than protons

➔ the proton is "shielded"



Average charge within a region <Q(L_D)> = -e

$$L_D = \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = 690\ T_4^{1/2}\left[\frac{n_e}{\text{cm}^{-3}}\right]^{-1/2} [\text{cm}]$$

$$< \frac{d}{dt}(\Delta P_\perp)^2 > \propto \frac{n_2}{v_1} \, \ln \Lambda$$

So Λ is large, $\equiv b_{\text{max}}/b_{\text{min}} \approx 20 - 35$ in ISM conditions.

$\Rightarrow$ For elastic scattering of electrons by ions, weak distant encounters ($>>$ atomic scales) more important than close encounters.

If an electron comes in in $\approx$ atomic dimensions,
the atom is
suddenly perturbed $\rightarrow$ transision
$\rightarrow$ deexcitation $\rightarrow$ line radiation

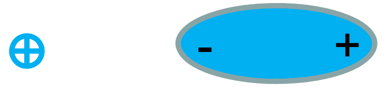
$$< \sigma v >_{10} = \frac{8.629 \times 10{-}8}{\sqrt{T_4}} \frac{\Omega_{10}}{g_1} \; [\mathrm{cm}^3\,\mathrm{s}^{-1}]$$

where Ω_{10} is **collision strength**.

(i) almost independent of T for $T < T_4 = 10^4$ K
(ii) $1 < \Omega_{10} < 10$

Ion-Neutral Collisions

Neutral is polarized.



Dipole moment $\vec{P} = \alpha_N \vec{E}$

Interaction potential

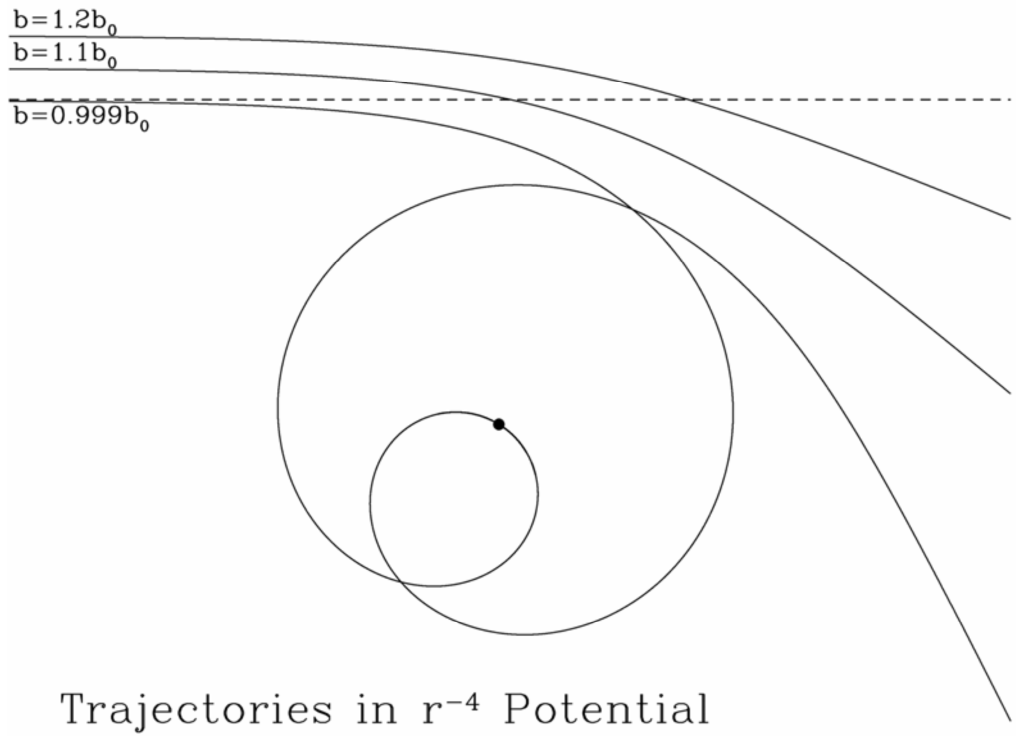
$$U(r) = -\frac{1}{2}\alpha_N \frac{Z^2 e^2}{r^4} \propto r^{-4}$$

where α_N is **polarizability** $\approx$ a few a_0.

$a_0 =$ Bohr radius $\equiv \frac{\hbar^2}{m_e e^2} = 5.292 \times 10^{-9}$ [cm]

For such a potential, if $b < b_0$, the deflection cross section is large.
$\sigma = \pi b_0^2 \propto 1/v$, and the rate coefficient $\langle \sigma v \rangle \not\leftrightarrow T$

Usually if T ↑, σ ↓



Trajectories in r^{-4} Potential

The ion-neutral reactions are important in <u>cool</u> ISM.

Electron-Neutral Collisions

In low-ionization ISM (e.g., protoplanetary disks) ions are rare. e^--neutral (H_2, He) scattering is important.

e^--H_2 scattering

(1) If $E < 0.044$ eV → pure elastic scattering

(2) If $E > 0.044$ eV → rotational excitation possible

(3) If $E > 0.5$ eV → vibrational excitation possible

(4) If $E > 11$ eV → electronic excitation

By experiment $\sigma \simeq 7.3 \times 10^{-6} \left(\frac{E}{0.01\mathrm{eV}}\right)$ [cm^2]

and $< \sigma v > \simeq 4.8 \times 10^{-9} \left(\frac{T}{10^2\mathrm{K}}\right)^{0.68}$ [cm^3 s^{-1}]

Neutral-Neutral Collisions

Repulsive if distance ↓
Weakly attractive if distance ↑ ∵ van der Waals interactions (mutual induced electric dipole)

$$U(r) \propto r^{-6}$$

Hard sphere OK; radii R ~ 1Å
$b < R_1 + R_2$ $\sigma = \pi\,(R_1 + R_2)^2 \sim 1.2\times\,10^{-15}$ [cm^2]

$$< \sigma v >= 1.81\times10^{-10}\,(\frac{T}{10^2\mathrm{K}})^{1/2}\,(\frac{m_H}{\mu})^{1/2}\,(\frac{R_1 + R_2}{2\mathring{A}})^2\ [\mathrm{cm}^3\,\mathrm{s}^{-1}]$$

Collision

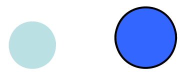
Gas (hydrogen atoms) root-mean-squared speed

$$m_{\rm H}\sqrt{< v^2 >} = 3kT$$

For H I regions, $T \sim 100$ K, $< v >_{\rm HI} \sim 1$ km s^{-1}

For e^-, $< v >_{e^-} \sim 50$ km s^{-1}

Cross sections σ



- Hard sphere OK for neutral atoms, $\sigma = \pi(a_1 + a_2)^2$
  i.e., 'physical' cross section

$$\sigma_{\rm HI,HI} \leftarrow a \sim 5.6 \times 10^{-9} \text{ cm}$$

$$\text{c.f., Bohr radius (first orbit)} = 5.3 \times 10^{-9} \text{ cm}$$

Cross sections σ

- For free e$^-$, p$^+$

$\sigma >> \sigma_{\rm physical}$ because of Coulomb force, need QM

$$a \sim \frac{2.5\times10^{-2}}{v^2} \text{ cm } (v \text{ in km})$$

If $v_{e^-} \sim 50$ km s^{-1}, $a \sim 10^{-5}$ cm for e^--e^- collision

$T = 3 \times 10^4$ K, $< v > \sim 10^3$ km s^{-1}

→ $a \sim 2.5 \times 10^{-8}$ cm

c.f., classical electron radius $\sim 2.8 \times 10^{-13}$ cm

Conventional unit for cross section

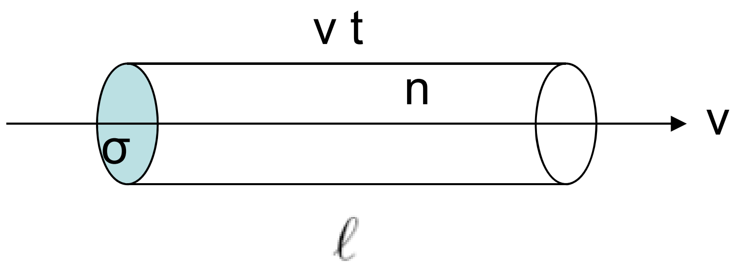
1 barn $= 10^{-24}$ cm^2

$\sigma_{\rm HI,HI} =\sim 10^8$ barns ($\sim 10^{-16}$ cm^2)

$$\frac{e^2}{r_0} = m\,c^2$$

$$r_0 = \frac{e^2}{m\,c^2} \sim 2.8 \times 10^{-13} \text{ cm}$$

Collision



# of collisions = # of particles in the (moving) volume

$$N = n\sigma\, v\, t$$

# of collisions per unit time = $N/t = n\,\sigma\, v$

Time (mean-free time) between 2 consecutive collisions (N=1) = $t_{\text{collision}} = \frac{1}{n\,\sigma\, v}$

Mean-free path $\ell = v t_{\text{collision}}$, i.e.,$\ell = \dfrac{1}{n\,\sigma}$

Ex 1 $n_{\rm HI} \sim 10~{\rm cm}^{-3};~v_{\rm HI} \sim 1~{\rm km~s}^{-1};~\sigma_{\rm HI,HI} \sim 10^{-16}~{\rm cm}^2$

$$t_{\rm HI,HI} \sim 10^{10}~{\rm s} \sim 300~{\rm years}$$

$$\ell \sim 10^{15}~{\rm cm} \sim 100~{\rm AU}$$

∴ Collisions are indeed very rare.

Ex 2 $\sigma_{\rm e^-,HI} \sim 10^{-15}~{\rm cm}^2~({\rm polarization})$

$$t_{\rm e^-,HI} \sim \frac{1}{10\times10^{-15}\times10^5} \sim 30~{\rm years}$$

Ex 3 $\sigma_{\rm e^-,e^-} \sim 10^{-12}~{\rm cm}^2;~n_e \sim 0.2~{\rm cm}^{-3}$

$$t_{\rm e^-,e^-} \sim \frac{1}{0.2\times10^{-12}\times50\times10^5} \sim 10~{\rm days}$$