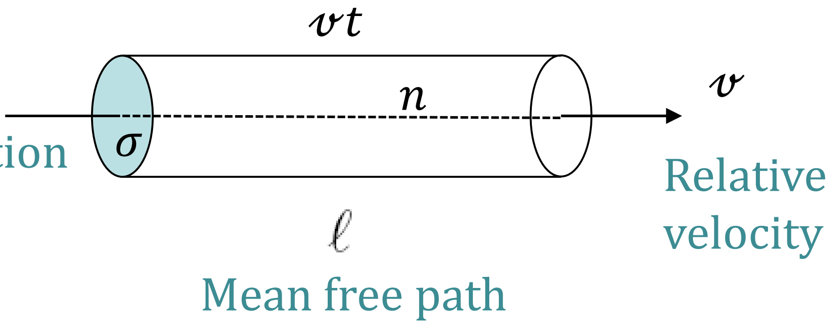
Collisional Processes

- Long range interaction
  --- between charges (ions, electrons); Coulomb $1/r^2$
- Intermediate range interaction
  --- between charges (ions) and neutrals (atoms/molecules);
  induced dipole, $1/r^4$
- Short range interaction
  --- between neutrals, $1/r^6$

Collision



n:(Volume) number density
t: Mean free time

A two-body encounter,

[# of collisions] = [total # of particles in the (moving) volume]

so $N = n\,(\sigma v t)$

✓ # of collisions per unit time $= {}^{N}/_{t} = n\,\sigma v$

σv: collisional rate

✓ Time (interval) between 2 consecutive collisions,

mean free time ($N = 1$), $t_{\mathrm{col}} = 1/\,(n\,\sigma v)$

✓ Mean free path $\ell = v t_{\mathrm{col}} = 1/(n\,\sigma)$

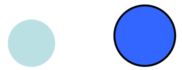
Thermal Motion

Gas (mostly H atoms), the root-mean-squared speed

$$\frac{1}{2} m_H \left(\sqrt{\langle v^2 \rangle}\right)^2 = \frac{3}{2} k_B T$$

e.g., in H I regions, $T \sim 100$ K, $v_{\mathrm{rms,HI}} \sim 1\ \mathrm{km\ s^{-1}}$, $v_{\mathrm{rms},e^-} \sim 50\ \mathrm{km\ s^{-1}}$

Cross Section

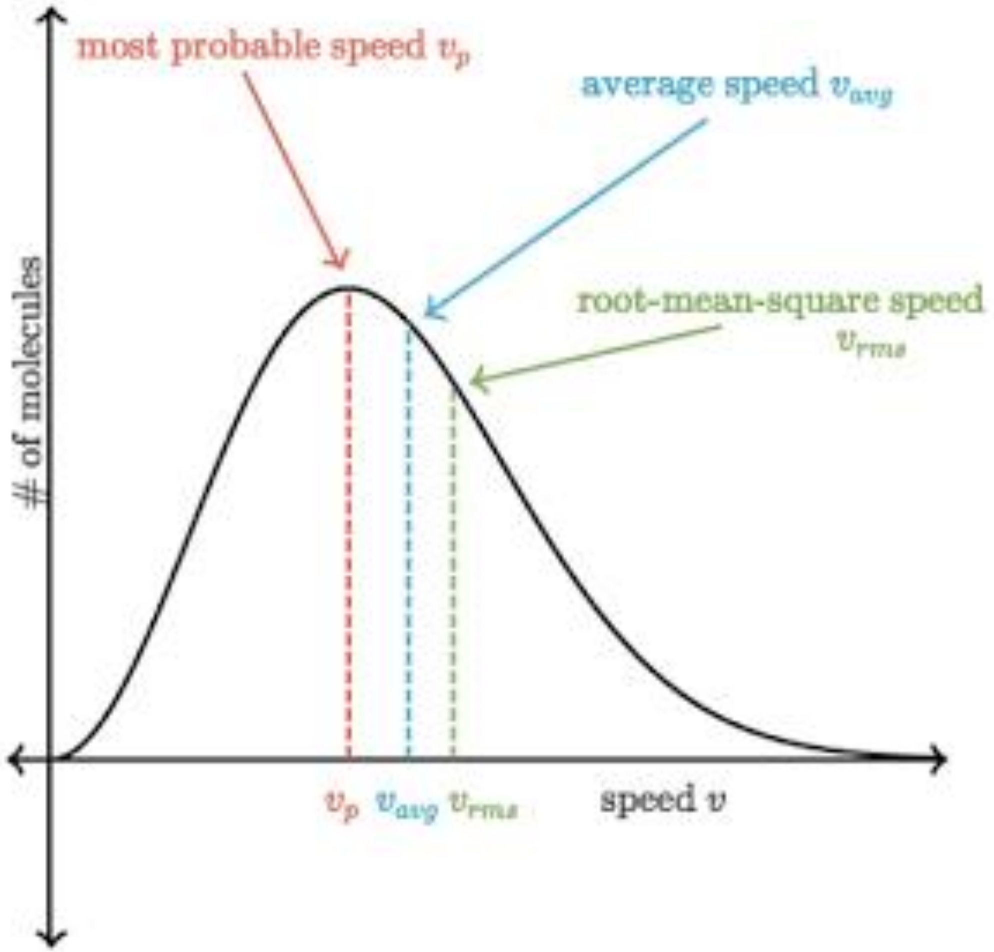


$$\sigma = \pi (a_1 + a_2)^2$$

For neutrals, hard spheres (physical cross section) OK,

$$\sigma_{\mathrm{HI,HI}} \leftarrow a \sim 5.6 \times 10^{-9}\ \mathrm{cm}$$

This is to be compared with the Bohr radius of the first orbit of $a_0 = 5.3 \times 10^{-9}$ cm



$$v_{\mathrm{mp}} = \sqrt{\frac{2\,k_B\,T}{m}} \qquad \langle v \rangle = \sqrt{\frac{8\,k_B\,T}{\pi\,m}} \qquad v_{\mathrm{rms}} = \sqrt{\frac{3\,k_B\,T}{m}}$$

In an HI cloud, $n_{HI} \sim 10\ \mathrm{cm}^{-3}$; $v_{HI} \sim 1\ \mathrm{km\ s}^{-1}$; $\sigma_{HI,HI} \sim 10^{-16}\ \mathrm{cm}^2$

$$t_{HI,HI} \sim 10^{10}\ \mathrm{s} \sim 300\ \mathrm{years};\ \ell \sim 10^{15}\ \mathrm{cm} \sim 100\ \mathrm{au}$$

∴ Collisions are indeed very rare in ISM.

$$\sigma_{HI,\,e^-} \sim 10^{-15}\ \mathrm{cm}^2\ \text{(polarization)}$$

$$t_{HI,\,e^-} \sim (10 \times 10^{-15} \times 10^{5})^{-1} \sim 10^{10}\ \mathrm{s} \sim 30\ \mathrm{years}$$

$$\sigma_{e^-,\,e^-} \sim 10^{-12}\ \mathrm{cm}^2;\ n_e \sim 0.2\ \mathrm{cm}^{-3}$$

$$t_{HI,\,e^-} \sim 10^{10}\ \mathrm{s} \sim 10\ \mathrm{days}$$

Cross Section (*cont.*)

For free e^- and p^+, $\sigma \gg \sigma_{\text{physical,}}$ because of Coulomb force

Need QM, $a \sim 2.5 \times 10^{-2}/v_{\text{km/s}}^2$ [cm]

If $v_{e^-} \sim 50$ km s^{-1}, $a \sim 10^{-5}$ cm for e^--e^- encounters

If $T = 3 \times 10^4$ K, $\langle v \rangle \sim 10^3$ km s^{-1} $\longrightarrow$ $a \sim 2.5 \times 10^{-8}$ [cm]

c.f., the classical electron radius $r_e = \dfrac{e^2}{m_e\, c^2} = 2.82 \times 10^{-13}$ [cm]

$r_{\text{proton}} \approx ?\, 0.8$ fm $\approx\ 0.8 \times 10^{-13}$ cm

---

Conventional unit: 1 barn $= 10^{-24}$ [cm^2]

$\sigma_{HI,HI} \sim 10^{-16}$ cm$^2 \sim 10^8$ barns

In general, for a two-body collision,

$$A + B \rightarrow \text{Products}$$

[reaction rate per unit volume] $= n_A n_B \langle \sigma v \rangle_{AB}$,
where the **collisional rate coefficient** is

$$\langle \sigma v \rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f(v)\, dv \quad [\text{cm}^3\ \text{s}^{-1}]$$

and

v: relative velocity between A and B
$\sigma_{AB}(v)$: reaction cross section; v dependent
$f(v)$: velocity distribution function (probability)

In thermal equilibrium → Maxwellian velocity distribution

$$f_v \, dv = 4\pi \left(\frac{\mu}{2\pi\, kT}\right)^{3/2} e^{-\mu v^2/2kT} \, v^2 \, dv$$

$\mu \equiv m_A \, m_B/(m_A + m_B)$ is the reduced mass

In terms of energy,

$$\langle \sigma v \rangle_{AB} = \left(\frac{8\, kT}{\pi\mu}\right)^{1/2} \int_0^\infty \sigma_{AB}(E) \, \frac{E}{kT} e^{-E/kT} \, \frac{dE}{kT}$$

If the density is high, e.g., in the Earth's atmosphere, three-body collisions may become important,

$$A + B + C \rightarrow \text{Products},$$

The reaction rate per unit volume is then $\kappa_{ABC} \, n_A \, n_B \, n_C$, where κ_{ABC} is the three-body collisional rate coefficient [$\mathrm{cm^6 \, s^{-1}}$]

Elastic scattering by an **inverse-square force**, e.g., Rutherford scattering

Exact solutions complicated; so use the "**impact approximation**", i.e., motion in a straight line (little deflection)

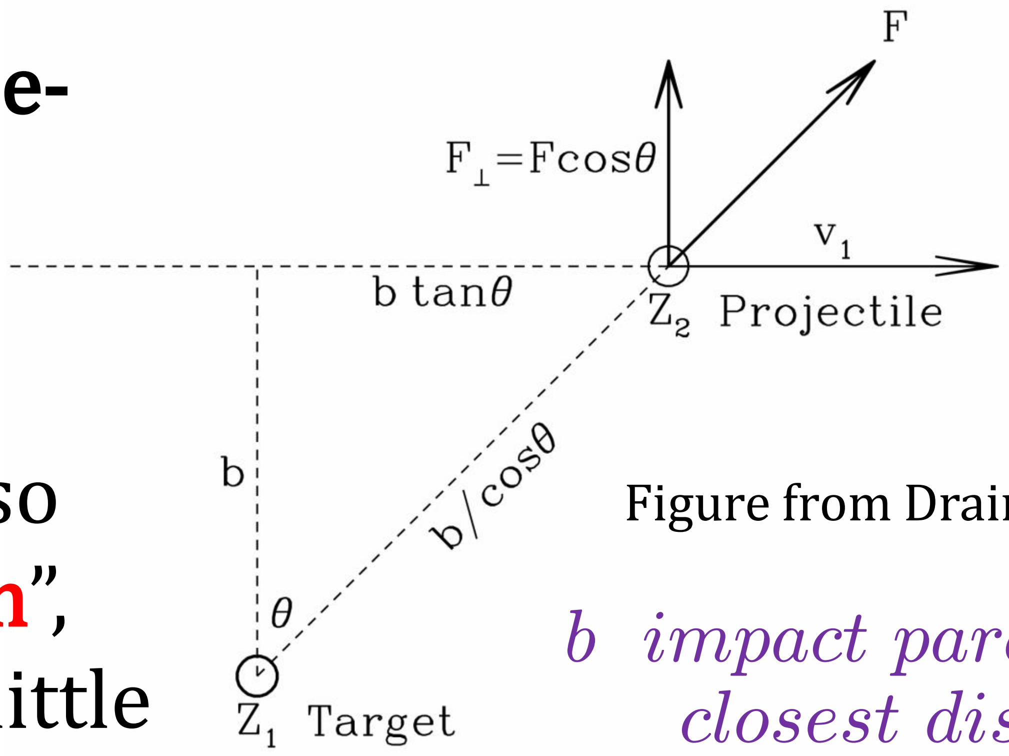


Figure from Draine's book

b impact parameter, closest distance

v_1 relative velocity

<u>Assumption</u>: constant velocity during the encounter between the target and the projectile

Question: How much momentum is transferred (⊥ direction)?

Impact Approximation

Coulomb force

$$F_\perp = \frac{Z_1 e Z_2 e}{(b/\cos\theta)^2}\cos\theta = \frac{Z_1 Z_2 e^2}{b^2}\cos^3\theta$$

Interaction time scale $dt = \frac{d(b\tan\theta)}{v_1} = \frac{b}{v_1}\frac{d\theta}{\cos^2\theta}$

Total momentum transfer is

$$\Delta p_\perp = \int_{-\infty}^{\infty} F_\perp\, dt = \frac{Z_1 Z_2 e^2}{b v_1}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

$$= \frac{2 Z_1 Z_2 e^2}{b v_1} \approx \underbrace{\frac{Z_1 Z_2 e^2}{b^2}}_{\text{Force at closest distance}}\ \underbrace{\frac{b}{v_1}}_{\text{Time scale}}$$

Collisional ionization: an electron energetic/fast enough ($E = p^2/2m$) to ionize an atom or ion of ionization energy E_I

Fast moving
→ (1/2) $m_e v^2 >> E_I$

⊕ e^-

$$(\Delta P_\perp)^2 > 2mE_I \Rightarrow \left(\frac{2Z_1 Z_2 e^2}{b v_1}\right)^2 > 2mE_I$$

So,

$$b^2 < b^2_{\mathrm{max}}(v) = \frac{(2Z_1 Z_2 e^2)^2}{v_1^2 \cdot 2mE_I} = \frac{2Z_p^2 e^4}{m_e v^2 E_I}$$

and the ionization cross section becomes

$$\sigma(v) \approx \pi b^2_{\mathrm{max}} = \frac{2\pi Z_p^2 e^4}{m_e v^2 E_I}$$

This is ok if v↑↑.

For minimum velocity, $v_{min} = (2I/m_e)^{1/2}$

$$
\begin{aligned}
< \sigma v > &= \int \sigma(v) v f(v)\, dv \\
&= \int_{v_{\rm min}}^{\infty} \frac{2\pi Z_p^2 e^4}{m_e v^2 E_I} v\, 4\pi \left(\frac{m_e}{2\pi kT}\right)^{3/2} v^2\, e^{-m_e v^2/2kT}\, dv \\
&= Z_p^2 \left(\frac{8\pi}{m_e kT}\right)^{1/2} \frac{e^4}{E_I} e^{-E_I/kT}
\end{aligned}
$$

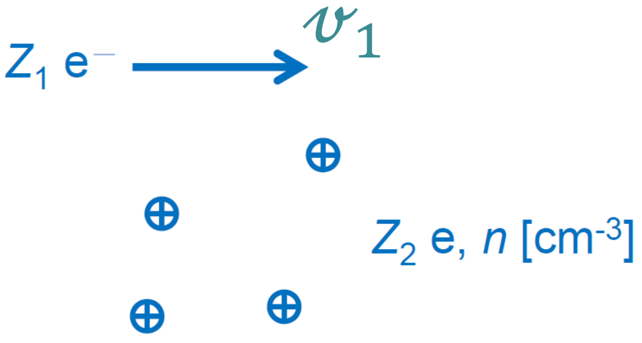
For an H atom at level n, $E_I = 13.6\ [\mathrm{eV}]/n^2$, so for a large n, e.g., $n\sim100$, and $T\sim10^4$ K, $E_I \downarrow\downarrow$ ($<< kT$)

→ in radio frequencies.

$< \sigma v > \propto \frac{1}{E_I} \propto n^2$, so is very large.

For large n (highly excited), the collisional ionization rate is high (i.e., easy to happen)

Deflection Timescale



Net momentum transfer

$$< \frac{d}{dt}(\Delta P_\perp)^2 >$$

There must be a range of distance, for which

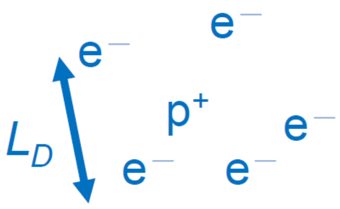
$b_{\rm min} = Z_1 Z_2 e^2/$Energy, and

$b_{\rm max} \approx L_D$ (Debye length) The effective range of the $\vec{E}$ field of a charge

In plasma, the distributions of ions and electrons are correlated because of charge neutrality.

Near a proton → more electrons than protons
→ the proton is "shielded"

Average charge within a region $\langle Q(L_D) \rangle = -e$



$$L_D = (\frac{kT}{4\pi n_e e^2})^{1/2} = 690\ T_4^{1/2}\ [\frac{n_e}{\rm cm^{-3}}]^{-1/2}\ [\rm cm]$$

$$< \frac{d}{dt}(\Delta P_\perp)^2 > \propto \frac{n_2}{v_1} \ln \Lambda$$

$\Lambda \equiv b_{max}/b_{min}$ = relative importance of distant encounters to close encounters

$$\ln \Lambda = 22.1 + \ln\left[E_{\mathrm{kT}}\, T_4^{3/2} n_\mathrm{e}^{-1}\right]$$

generally very large; in ISM, $\ln \Lambda \approx 20 - 35$

$\Rightarrow$ For elastic scattering of electrons by ions, weak distant encounters ($>>$ atomic scales) more important than close encounters.

Distant encounters dominate, so impact approximation OK

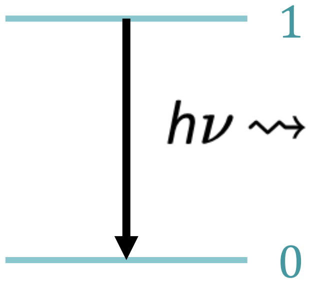
Electron-Ion Inelastic Scattering

Draine Chap 2.3

An ion originally in state 1, with degeneracy g_1, is deexcited to state 0.

When an electron comes in about the atomic dimensions, the atom is suddenly perturbed

→ transition → deexcitation → line radiation



$$\langle \sigma v \rangle_{1\to 0} \equiv \gamma_{10} = \frac{8.629 \times 10^{-8}}{\sqrt{T_4}} \frac{\Omega_{10}(T)}{g_1} \ [\mathrm{cm^3 s^{-1}}]$$

where $\Omega_{10}(T)$ is **collision strength**.

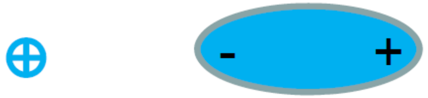
- Ω_{10} is dimensionless; almost independent of T for $T \lesssim 10^4$ K
- Typically $1 \lesssim \Omega_{10}(T) \lesssim 10$.

Ω_{10}, $\Omega_{u\ell}$, or Ω_{ji}

Ion-Neutral Collisions

Draine Chap 2.4

Neutral is polarized.



Dipole moment $\vec{P} = \alpha_N \vec{E}$

Interaction potential

$$U(r) = -\frac{1}{2}\alpha_N \frac{Z^2 e^2}{r^4} \propto r^{-4}$$

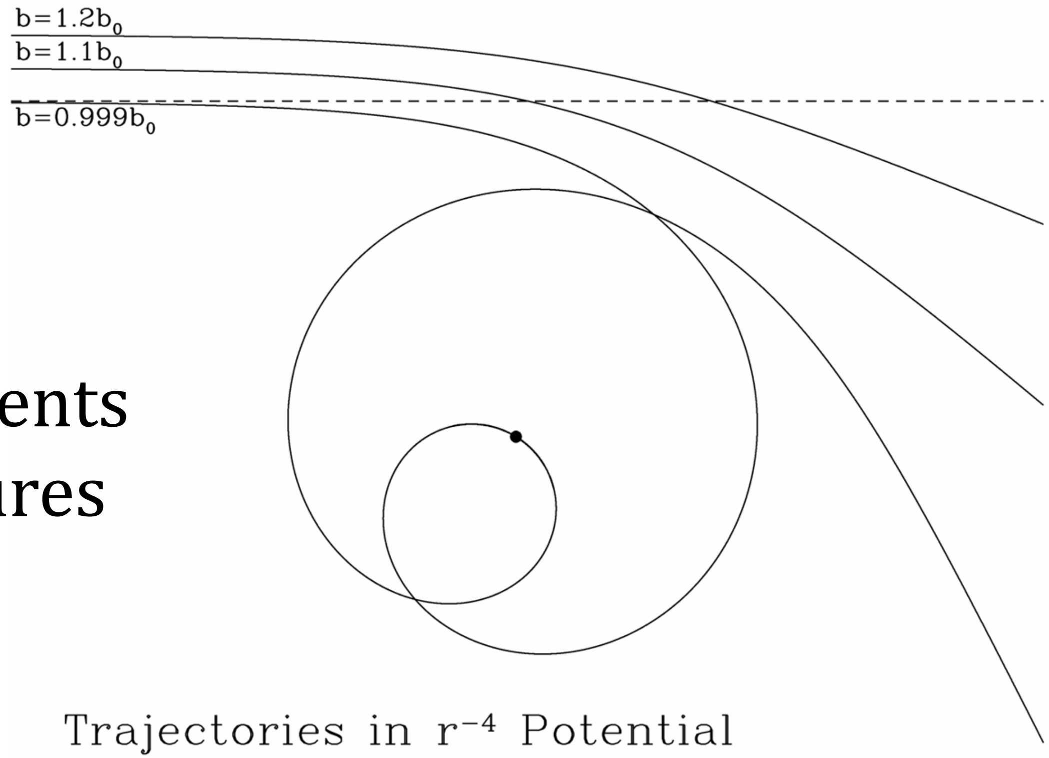
where α_N is **polarizability** $\approx$ a few a_0.

a_0 = Bohr radius $\equiv \frac{\hbar^2}{m_e e^2} = 5.292 \times 10^{-9}$ [cm]

For such a potential, if $b < b_0$, the deflection cross section is large.

$\sigma = \pi b_0^2 \propto 1/v$, and the rate coefficient $\langle \sigma v \rangle \not\leftrightarrow T$ ... independent of temperature

Usually if T ↑, σ ↓



Trajectories in r^{-4} Potential

... so the collisional rate coefficients are large even at low temperatures

The ion-neutral reactions are important in cool ISM.

Electron-Neutral Collisions

Draine Chap 2.5

In low-ionization ISM (e.g., protoplanetary disks) ions are rare. e^--neutral (H_2, He) scattering is important.

e^--H_2 scattering

(1) If E < 0.044 eV → pure elastic scattering
(2) If E > 0.044 eV → rotational excitation possible
(3) If E > 0.5 eV → vibrational excitation possible
(4) If E > 11 eV → electronic excitation

By experiment $\sigma \simeq 7.3 \times 10^{-6} \left(\frac{E}{0.01\mathrm{eV}}\right) \; [\mathrm{cm}^2]$

and $< \sigma v > \simeq 4.8 \times 10^{-9} \left(\frac{T}{10^2\mathrm{K}}\right)^{0.68} \; [\mathrm{cm}^3\,\mathrm{s}^{-1}]$

Neutral-Neutral Collisions

Draine Chap 2.6

Repulsive at short distances, and weakly attractive at longer distances due to van der Waals interaction (mutually induced electron dipole moment)

Hard sphere OK; each with a radius ≈ 1 Å; impact parameter $b < R_1 + R_2, \sigma = \pi(R_1 + R_2)^2 \approx 1.2 \times 10^{-15}\ \mathrm{cm}^2$

$$\langle \sigma v \rangle = 1.81 \times 10^{-10} \left(\frac{\mathrm{T}}{10^2\ \mathrm{K}}\right)^{1/2} \left(\frac{\mathrm{m_H}}{\mu}\right)^{1/2} \left(\frac{R_1 + R_2}{2\text{Å}}\right)^{2}\ [\mathrm{cm}^3\ \mathrm{s}^{-1}]$$

For $T \lesssim 100$ K, the rate coefficient for neutral-neutral scattering is an order smaller than that for ion-neutral scattering.

The Interstellar Medium --- HW20250313

due in two weeks

1. Dark molecular clouds have a typical temperature of 20 K, whereas the intergalactic gas of a galaxy cluster has 10 million K. Estimate in each case the wavelength around which the blackbody radiation has the strongest intensity. Briefly describe the principle of the detector technology at that wavelength.

2. Calculate the Doppler FWHM for a gas of H atoms radiating at 100 nm with a temperature of 300 K. Show that the collisional broadening in such a gas will not be important until the number density reaches approximately $10^{21}\ \mathrm{cm}^{-3}$. Assume a geometric cross-section for hydrogen-atom collisions.

3. In one measurement of the Orion Complex, the core of M 42 was estimated to have an angular size (FWHM) of 2.33' and a root-mean-square electron density of $4.9 \times 10^3\ \mathrm{cm}^{-3}$. Estimate the mass of the ionized hydrogen, assuming a pure hydrogen gas, dust-free cloud.