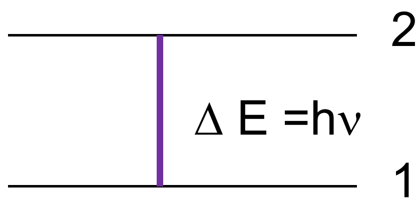
Emission and Absorption

Two ways to decay from an excited state

1. $X_2 \rightarrow X_1 + h\nu$

   spontaneous emission

   occurrence rate $\leftrightarrow$ atomic properties



2. $X_2 + h\nu \rightarrow X_1 + 2\,h\nu$

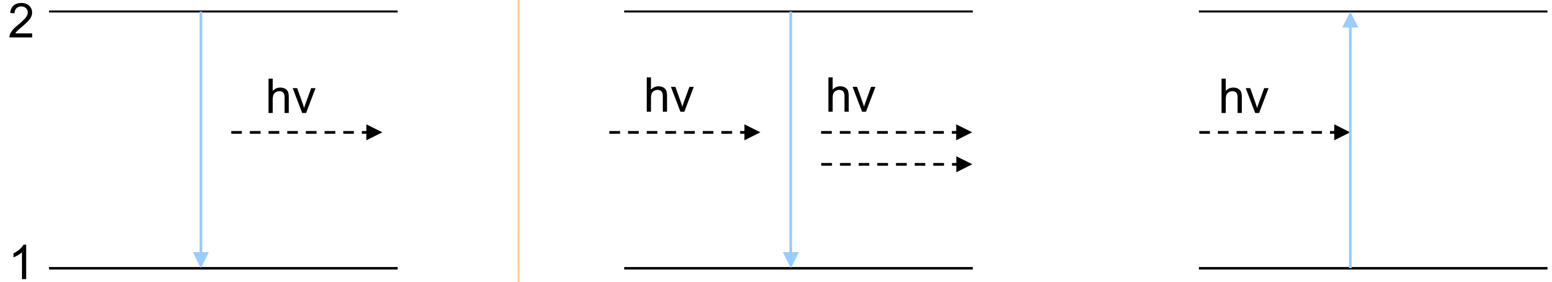
   stimulated emission

   occurrence rate $\leftrightarrow$ density of incoming photons of the same ν, polarization, and direction of propagation

Einstein Coefficients

by Einstein (1917) regarding radiation probabilities

Spontaneous emission | Stimulated (induced) emission | Stimulated absorption



j | | *k*

A_{21} | B_{21} | B_{12}

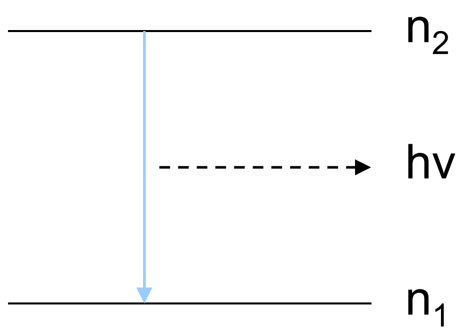
$[s^{-1}]$

$[cm^{3}\ ergs^{-1}\ s^{-1}\ Hz^{-1}]$

A21 --- probability

$B\ u_{\nu}$ --- probability

Transition Probability



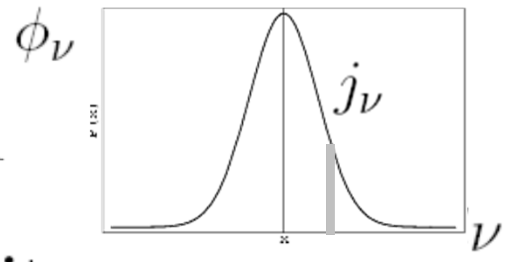
Considering a 2-level system, we want to calculate the emission arising from this transition,

Assuming $j_\nu \not\leftrightarrow (\theta, \phi)$

$$j_\nu \ [\mathrm{ergs\ s^{-1}\ cm^{-3}\ ster^{-1}\ Hz^{-1}}]$$

j_ν is governed by a distribution function (line profile)

$$j_\nu = \frac{h\nu\, A_{21}\, n_2\, \phi_\nu}{4\pi}$$



$$j = \int j_\nu\, d\nu, \ \int \phi_\nu\, d\nu = 1$$

$$\longrightarrow \quad j = \frac{h\nu\, A_{21}\, n_2}{4\pi}$$ **volume emissivity**

A_{21}: transition probability (per unit time) $\simeq 10^{-15}\ \mathrm{s}^{-1}$ for H I 21 cm line

Energy absorbed in a line [ergs s^{-1} cm^{-3} $ster^{-1}$]

$$\int \kappa_\nu \, I_\nu \, d\nu \simeq I_\nu \int \kappa_\nu \, d\nu$$

This is valid for a sharp line, i.e., $\kappa_\nu \approx \delta$ function

Emission probability: A

Absorption probability: $B \, u_\nu = B \, \frac{I_\nu}{c}$

$$\kappa_\nu = \frac{h\nu(n_1 B_{12} - n_2 B_{21})}{c} \, \phi_\nu \longrightarrow \int \kappa_\nu \, d\nu = \frac{h\nu(n_1 B_{12} - n_2 B_{21})}{c}$$

In equilibrium, **detailed balance** (equal probabilities) gives

ups=downs

$$\frac{h\nu n_2 A_{21}}{4\pi} = \frac{h\nu I_\nu}{c}(n_1 B_{12} - n_2 B_{21})$$

$$\frac{n_2 A_{21}}{4\pi} = \frac{n_1 B_{12} - n_2 B_{21}}{c}\,\frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

$$\frac{n_2}{n_1}\frac{A_{21}}{4\pi} = \frac{B_{12} - \frac{n_2}{n_1}B_{21}}{c}\,\frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

In case of TE, $\frac{n_2}{n_1} = \frac{g_2}{g_1}e^{-h\nu/kT}$

$$\begin{aligned}
\frac{g_2}{g_1}e^{-h\nu/kT}\frac{A_{21}}{4\pi} &= \frac{B_{12} - \frac{g_2}{g_1}e^{-h\nu/kT}B_{21}}{c}\,\frac{2h\nu^3}{c^2}\,\frac{1}{e^{h\nu/kT}-1} \\
&= \frac{2h\nu^3}{c^3}\left[B_{12} - \frac{g_2}{g_1}e^{-h\nu/kT}\cdot B_{21}\right]\frac{1}{e^{h\nu/kT}-1} \\
&= \frac{2h\nu^3}{c^3}e^{-h\nu/kT}\left[B_{12}e^{h\nu/kT} - \frac{g_2}{g_1}\cdot B_{21}\right]\frac{1}{e^{h\nu/kT}-1}
\end{aligned}$$

If this is to hold true for any T, the $[e^{h\nu/kT} - 1]$ term must cancel out, i.e.,

$$g_1 \, B_{12} = g_2 \, B_{21}$$

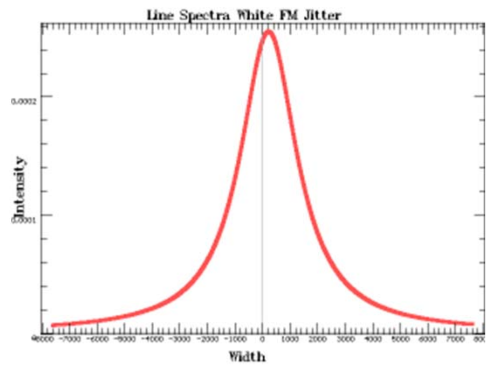
and

$$A_{21} = \frac{8\pi \, h\nu^3}{c^3} B21$$

In fact, the Einstein coefficients are properties of atoms, so the relations hold whether it is TE or not.

Line Shapes

- **Natural broadening**: uncertainty principle
- **Doppler broadening**: random thermal velocities of particles
- **Pressure broadening**: interruption of radiation train (usually not important in ISM)
- **Opacity broadening**: photons at the line wings have smaller reabsorption probabilities than those near the line center (line-of-sight effect)

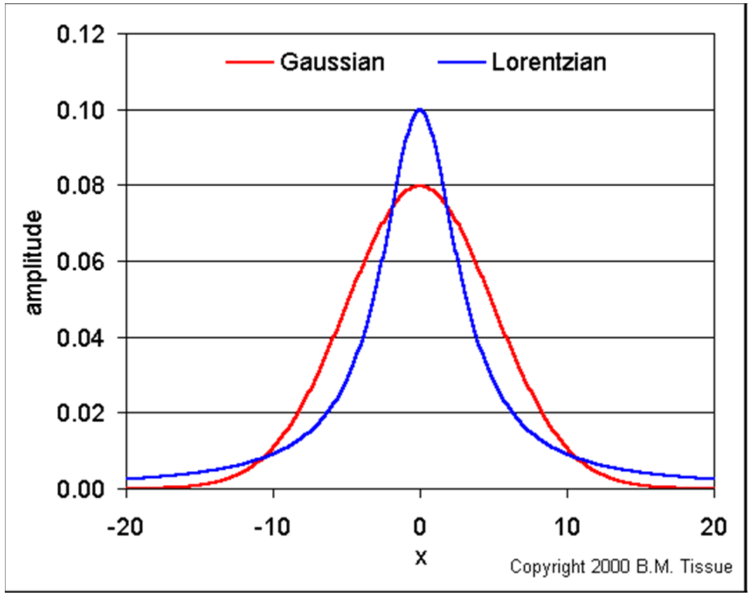


Lorentzian Profile

$$I_\nu \propto \frac{1}{(\nu-\nu_0)^2 + (\frac{\gamma}{4\pi})^2}$$

Peaked at ν_0 and width measured by γ.

$$(\Delta\nu)^{\text{intrinsic}}_{\text{FWHM}} = \frac{\gamma_{21}}{2\pi}$$



Also known as the Cauchy probability distribution; the solution of the differential equation describing forced resonance.

The Lorentzian profile = an accurate approximation to the actual line profile; more accurately by the Kramers-Heisenberg formula.

Doppler Profile

Emitted light ν_0 shifted to ν due to v_z

$$\frac{\nu - \nu_0}{\nu_0} = \frac{v_z}{c}$$

range of frequencies $\leftrightarrow$ range of velocities

For a Maxwellian distribution, $I_\nu \propto e^{-(\nu-\nu_0)^2/2\sigma^2}$, where $\sigma^2 = (\nu_0^2\, k\, T_{\rm kin})/(m\, c^2)$

T↑ ➔ velocity dispersion↑

$$(\Delta v)_{\rm FWHM} = \sqrt{8 \ln 2}\, \sigma_v$$

In general in a turbulent medium,

$$2\sigma^2 = \frac{\nu_0^2}{c^2}\left(\frac{2kT_{\rm kin}}{m} + v_{\rm tub}^2 + \cdots\right)$$

Natural Broadening

Described by the Lorentzian profile

Line very narrow; usually an order of magnitude or less than other effects, e.g., intensity drops to 2% of peak at 0.003 Å from line center

$2\,\gamma \leftrightarrow A$, where $1/A$ = time in the upper level
(uncertainty principle $\Delta E\, \Delta t < h$, i.e., $h\Delta\nu\,(1/A) < h$)

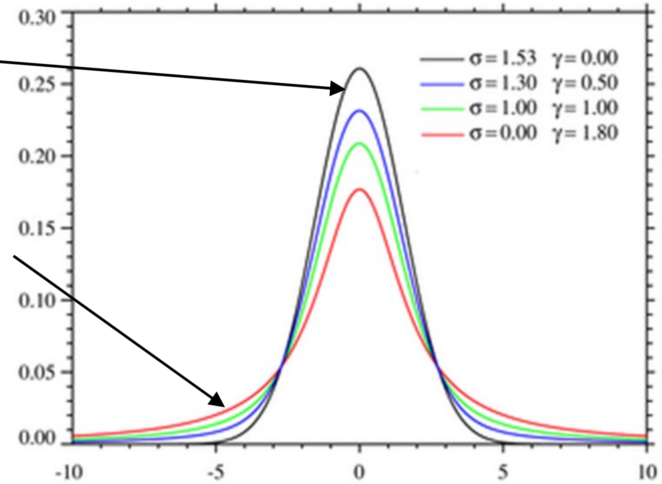
At optical frequencies ($\nu \sim 10^{15}$) a typical strong line has
$A \sim 10^{8}\ \mathrm{s}^{-1} \quad \rightarrow \Delta\nu/\nu \sim 10^{-7}$

The nature width can be expressed in terms of the line-of-sight velocity, so as to compare with Doppler width, for example:

$$(\Delta v)^{\text{intrinsic}}_{\text{FWHM}} = c\,\frac{(\Delta \nu)^{\text{intrinsic}}_{\text{FWHM}}}{\nu_{21}} = \frac{\lambda_{21}\gamma_{21}}{2\pi}$$

the natural width for H Ly α, $h\nu$ = (3/4) 13.6 eV, f_{12}=0.4162, g_1/g_2=2/6, so the intrinsic (Δ v) =0.0121 km s^{-1}

Voigt Profile



$$V(x;\sigma,\gamma) = \int_{-\infty}^{\infty} D(x';\sigma)L(x-x';\gamma)\,dx'$$

$$D(x;\sigma) \equiv \frac{e^{-x^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \qquad L(x;\gamma) \equiv \frac{\gamma}{\pi(x^2+\gamma^2)}.$$

- Convolution of a Doppler profile (= Gaussian) and a Lorentzian profile (Gaussian core + Lorentzian wings)
- The Doppler profile is more strongly peaked
- Away from the line center (i.e., $|\nu-\nu_0|$ ↑ → wings), Lorentzian ↑
- Gaussian core: FWHM=2.3556 σ

Pressure (collisional) Broadening

- Profile similar to Lorentzian, with width $1/\tau_0$ where τ_0 is the mean time interval between collisions, which in the ISM is about 1000 years

→ narrower even than the natural broadening

- Pressure broadening therefore is not important in the ISM but important in stellar atmosphere where collisions are frequent.

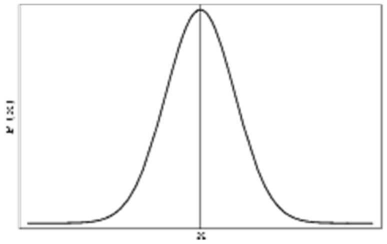
The damping profile

$$\kappa_\nu = \frac{h\nu\,(n_1 B_{12} - n_2 B_{21})}{c}\,\phi_\nu$$

$$= \frac{h\nu}{c} n_1 B_{12}\,(1 - \frac{n_2}{n_1}\frac{B_{21}}{B_{12}})\,\phi_\nu$$

$$= \frac{h\nu}{c} n_1 B_{12}\,(1 - \frac{n_2}{n_1}\frac{g_1}{g_2})\,\phi_\nu$$

ϕ_ν



So, if in TE $\kappa_\nu = \frac{h\nu n_1 B_{12}}{c}(1 - e^{-h\nu/kT})\phi_\nu$

If non-TE, then define $b_j \equiv n_j(\mathrm{actural})/n_j(\mathrm{inTE})$

***b*:departure coefficient**

In LTE, $b_j = 1$ $\quad \frac{n_2}{n_1} = \frac{b_2}{b_1}(\frac{n_2}{n_1})_{\mathrm{eq}} = \frac{b_2}{b_1}\frac{g_2}{g_1}e^{-h\nu/kT}$

So in general $\boxed{\kappa_\nu = \frac{h\nu n_1 B_{12}}{c}(1 - \frac{b_2}{b_1}e^{-h\nu/KT})\phi_\nu}$

Computation of b's is not trivial.

If $h\nu >> kT$ stimulating emission is negligible.

Define σ_ν, the absorption cross section per particle

$$\kappa_\nu \ [\text{cm}^{-1} \ \text{Hz}^{-1}] = n \ [\text{cm}^{-3}] \quad \sigma_\nu \ [\text{cm}^2 \ \text{Hz}^{-1}]$$

$$\sigma_\nu = \sigma_0 [1 - \frac{b_2}{b_1} e^{h\nu/kT}] \ \phi_\nu$$

where $\sigma_0 = h\nu B_{12}/c$

$$f_{12} \equiv \frac{m_e c}{\pi e^2} \int \sigma_{12}(\nu) \, d\nu$$

Classically $\sigma_\nu = \frac{\pi e^2}{mc} f \, \phi_\nu$ and $\sigma_0 = \frac{\pi e^2}{mc} f$ where f is **oscillator strength**, is the effective number of electrons per atom. so, $\frac{\pi e^2}{mc} f_{12} = \frac{h\nu B_{12}}{c} = \frac{h\nu}{c} \frac{g_2}{g_1} \frac{c^3}{8\pi h\nu^3} A_{12}$

$$f_{12} = \frac{mc^3}{8\pi\nu^2 e^2} \frac{g_2}{g_1} \ A_{21} \qquad A_{12} = \frac{0.6670 \ \text{cm}^2 \, \text{s}^{-1}}{\lambda_{12}^2} \frac{g_1}{g_2} f_{12}$$

$$\sigma_0 = \frac{c^2}{8\pi\nu^2} \frac{g_2}{g_1} \ A_{21} \qquad g_1 f_{12} = -g_2 f_{21}$$

- Under some conditions, the upper level may be "pumped" (by collision or by radiative excitation of a higher level followed by a decay).
- If pumping more rapid than depopulation → $n_2 / n_1 > g_2 / g_1$ (excitation temperature, $T_{21} < 0$)
- This is called a **population inversion** → stimulated emission is stronger than absorption → radiation **amplified**. laser=Light Amplification by the Stimulated Emission of Radiation
- Such inversions have been observed in **microwave** transitions of H I, OH, and SiO (**maser**) (Elitzur 1992 ARAA)
- Maser sources can be very bright → motion measured by interferometry, e.g., Galactic SFRs, or supermassive black hole in NGC 4258 (Herrnstein et al 1999)