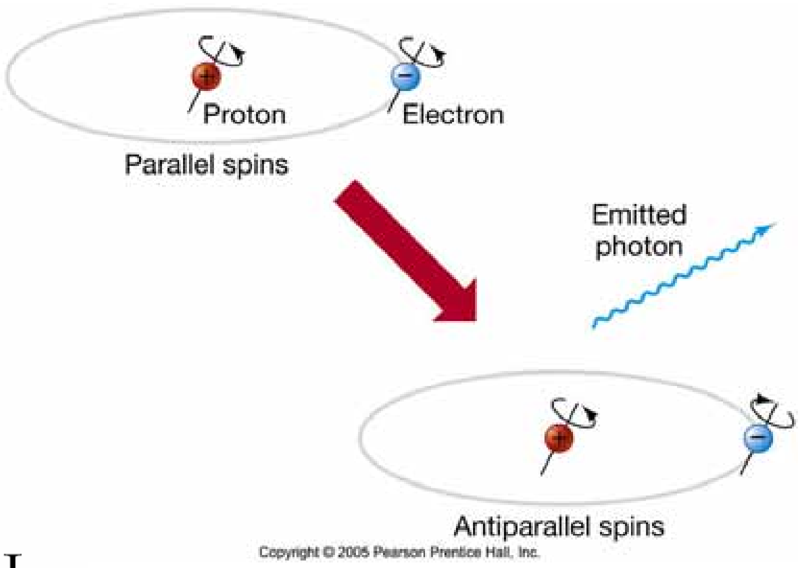
# 21 cm Line

Collisional excitation




Spontaneous
de-excitation

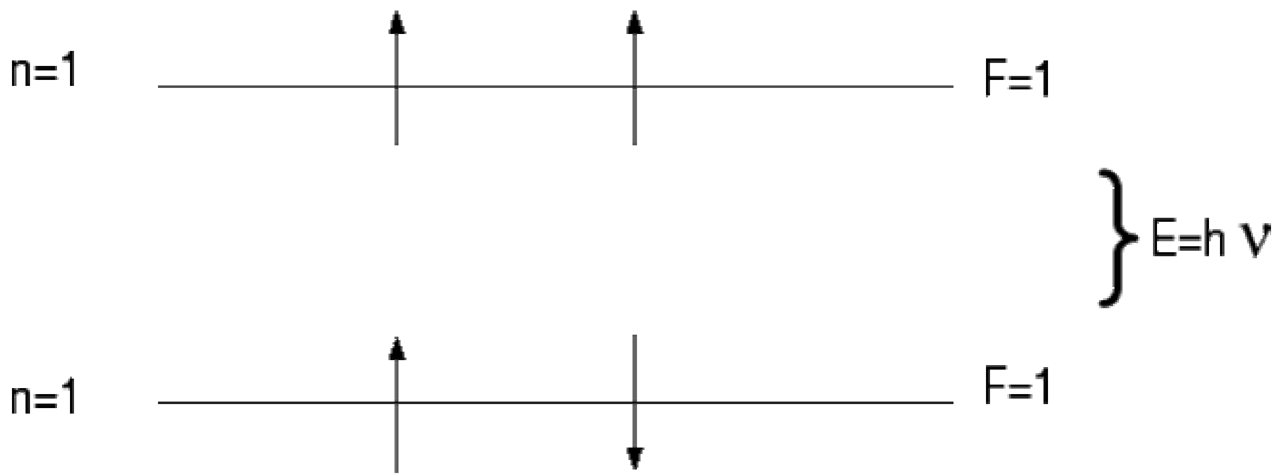
F=1 to F=0

$A_{10} = 2.869 \times 10^{-15}\ s^{-1}$

$$\lambda \approx 21\ \mathrm{cm};\ \nu \approx 1420.40575179\ \mathrm{MHz}$$

Magnetic moment of p+ ←→ of e charge
←→ of e orbit

➔ **hyperfine structure**
i.e., splitting of the ground state of H I




http://tesla.phys.unm.edu/phy537/8/node2.html

21 cm line is

- a magnetic dipole transition
- a forbidden transition

$$\tau_{HI,HI}(\text{collision}) \approx \frac{1}{n\sigma v} \approx \frac{1}{1 \times 10^{-16} \times 10^{5}} \approx 10^{11}\ \text{s} << 1/A_{10}$$

So LTE ok, $n_1/n_0 = (g_1/g_0)\ \exp^{-\chi/kT}$,
where $g_0 = 1, g_1 = 3, \chi/k \approx 70$ mK

Population ratio determined by

- collisions
- cosmic background radiation
- Ly alpha pumping →

Field (1959) ApJ, 129, 551: Ly alpha radiation could excite 21 cm line via transitions involving the n=2 level as an intermediate state

$$T_B = T_{bg}\, e^{-\tau} + T_s(1 - e^{-\tau})$$

In reality we measure the spectrum with respect to the continuum (beam-switching)

$$\begin{aligned} \Delta T_B &= T_B - T_{bg} \\ &= T_{bg}\, e^{-\tau} + T_s - T_s\, e^{-\tau} - T_{bg} \\ &= (T_s - T_{bg})(1 - e^{-\tau}) \end{aligned}$$

One can show that

$$\tau(v) = \frac{N(v)}{C \cdot T_s}$$

$v$ [km s$^{-1}$]; $C = 1.83 \times 10^{18}$ cm$^{-2}$ K$^{-1}$ (cm s$^{-1}$)$^{-1}$; $N$: column density [# cm$^{-2}$]

For $T_{bg} << T_s$

$$N_H = 1.82 \times 10^{18} \int_0^{\infty} T_s \, \tau_\nu(v) \, dv$$

$v$ in $[\mathrm{km\ s^{-1}}]$

(Mihalas & Binney)

- Optically thin ($\tau << 1$)

  $T_B = T_s \, \tau = \frac{N(v)}{C \cdot T_s}$

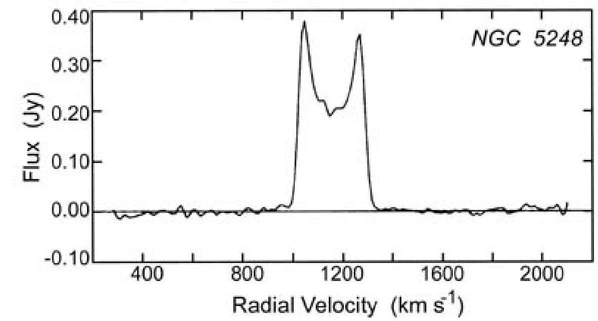
  That is, measured brightness temperature $\propto$ column density per unit velocity

- Optically thick ($\tau >> 1$)

  $T_B = T_s$

  That is, photons emitted within the cloud get absorbed inside the cloud; only photons emitted within $\tau < 1$ of the front surface manage to escape.

- $T_s > T_{bg} \rightarrow$ 21 cm in emission
- $T_s < T_{bg} \rightarrow$ 21 cm in absorption



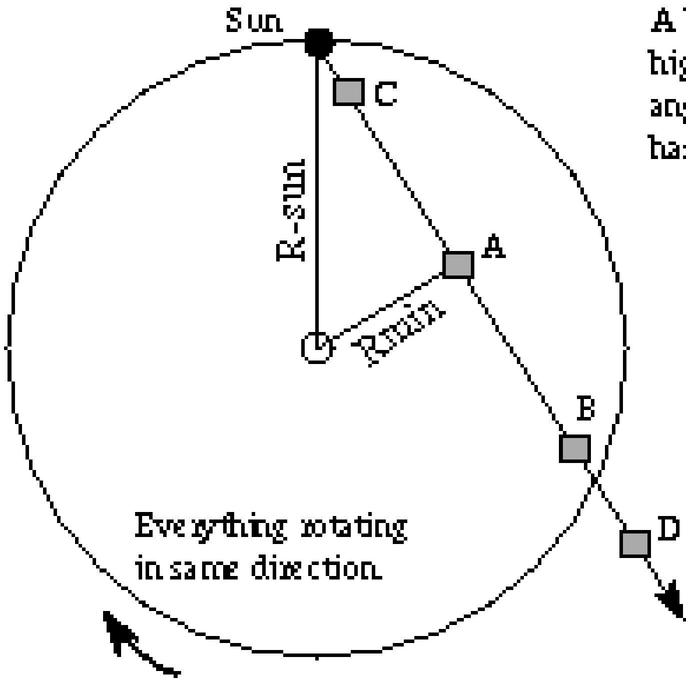

If $T_s = 2.73$ K, the cloud is not detectable. For any Galactic H I cloud, $T_s > 2.73$ K.

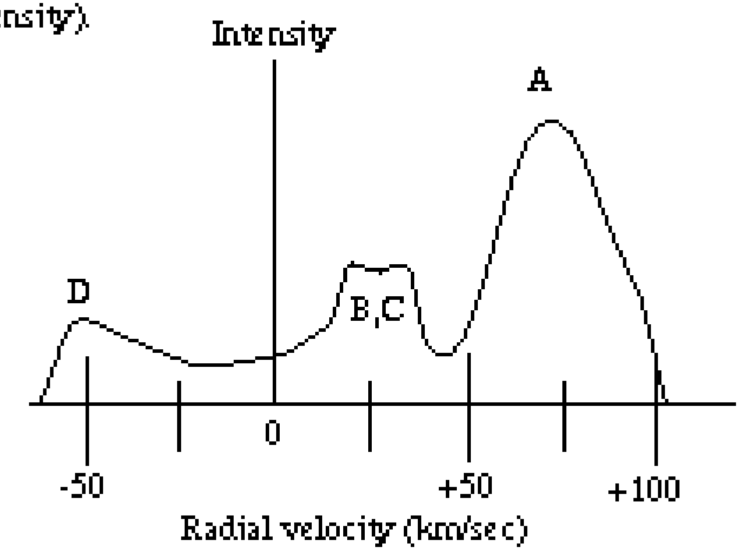
21 cm emission has been used extensively to map out the distribution of Galactic H I.

Spiral arms are prominent. Two or more arms toward the same direction can be separated by different velocities.

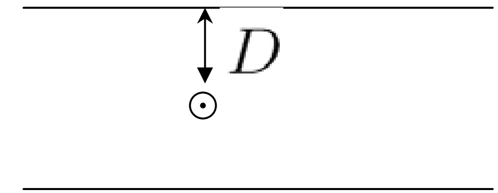
With strong distant radio sources (e.g., QSOs), one can observe H I in absorption thus study of the warm IS component.

A has greatest angular speed and moving fastest away from sun. A has higher density of H. B & C moving at about same angular speed > sun's angular speed. D is outside solar distance–slower angular speed and has less material (density).



$$N_H = 1.82 \times 10^{18} \int_0^{\infty} T_s \, \tau_\nu(v) \, dv$$




$T_B \sim 10$ K, $\Delta v \sim$1–10 km s$^{-1}$, so $N_H \sim 10^{20}$ cm$^{-2}$

$D \sim$ 100–200 pc

Assuming $N_H = \int_0^{\infty} n_H \, dx \approx n_H \, D$, so $n_H \sim 1$ cm$^{-3}$

$$\int I \, ds = \int h\nu \, \alpha \, n_e \, n_p \, ds$$

where EM = Emission Measure = $\int_0^L n_e \, n_p \, ds$ along the line of sight

If $n_e \approx n_p$, EM = $\int_0^L n_e^2 \, ds$

Typically EM$\sim 10^2$–$10^3$ [pc cm$^{-6}$] for Galactic clouds

If assuming [depth]$\sim$[width] $\rightarrow$ get $n_e$

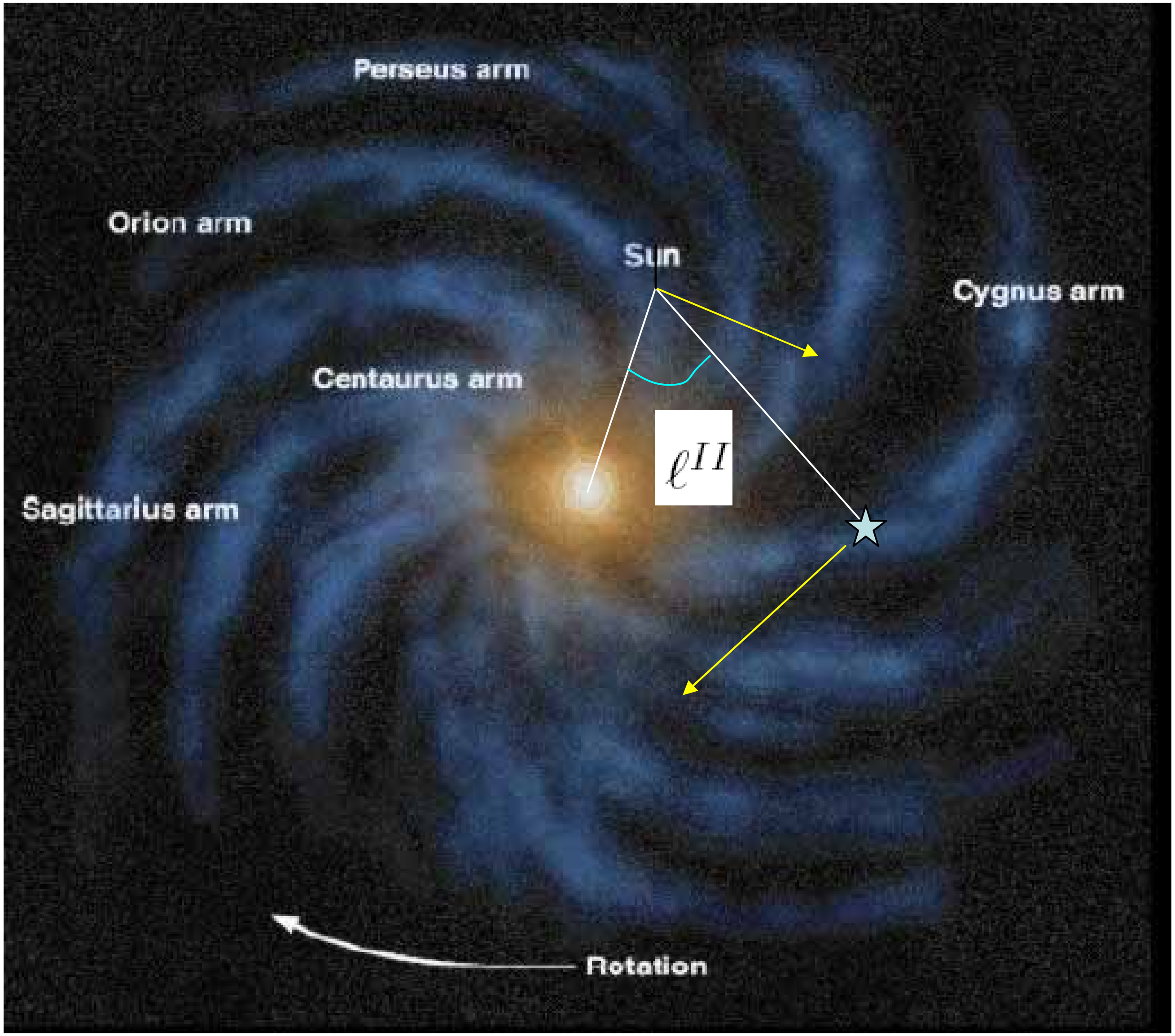
## Emission Measure

A measure of the "amount of material" of a plasma available to produce the observed flux, the product of the square of the electron number density and the volume of emission, with units [cm-3]. Often, because observations are carried out along a line of sight, the cross-section area is taken out of the expression and the units become [cm-5].

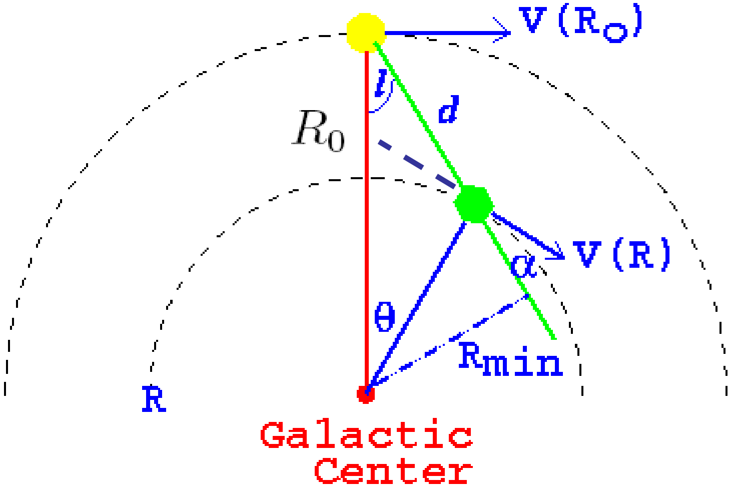
From Astro Jargon for Statisticians
http://hea-www.harvard.edu/AstroStat/astrojargon.html

# Galactic Rotation

$\ell^{II}$: Galactic longitude



$v_r$: radial velocity (line of sight)




$$v_r = v(R)\cos\alpha - v(R_0)\sin\ell$$

$$v_r = R\ \omega\cos\alpha - R_0\ \omega_0\sin\ell$$

$$\frac{\sin\ell}{R} = \frac{\sin(90+\alpha)}{R_0} = \frac{\cos\alpha}{R_0}$$

$$v_r = (\omega - \omega_0)\ R_0\ \sin\ell$$

This is general; only under the assumption of circular motion.

One can show that the relative tangential velocity

$$v_t = (\omega - \omega_0)\ R_0\cos\ell - \omega\ d$$

## $1^{st}$ Quadrant

Along a certain line of sight, variation of the radial velocity

- Near the Sun

  $d \uparrow \rightarrow R \downarrow \rightarrow \omega \uparrow \rightarrow v_r \uparrow$

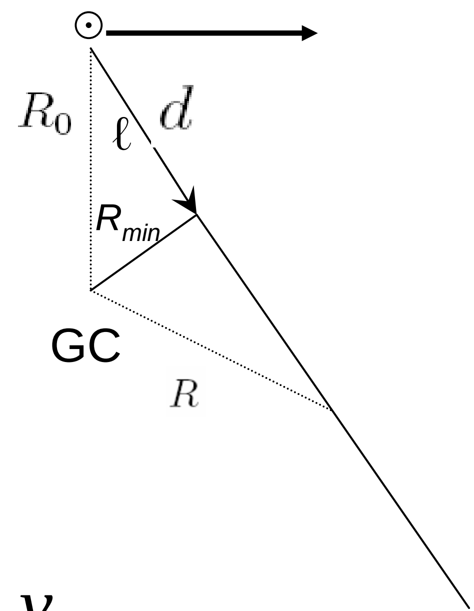
- At D, $R_{\min} = R_0 \sin \ell$

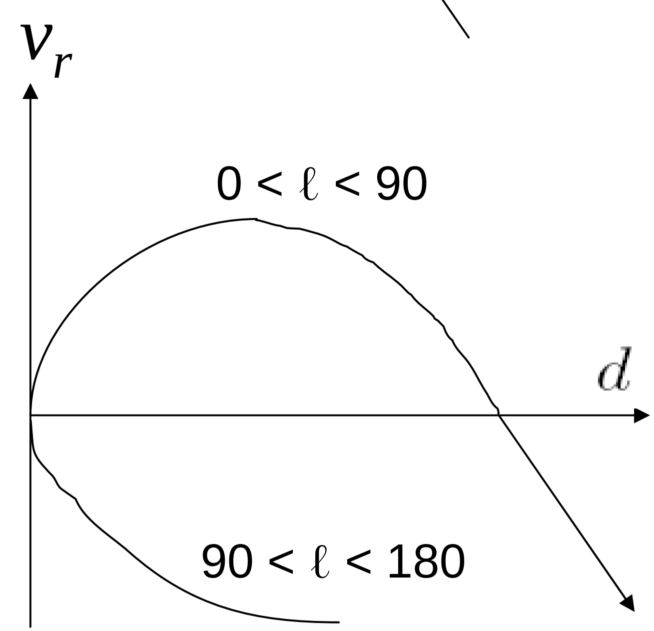
- Further away, $R \uparrow \rightarrow v_r \downarrow$

- When $R = R_0 \rightarrow v_* = v_\odot \rightarrow v_r = 0$

- Even further away, $R > R_0$

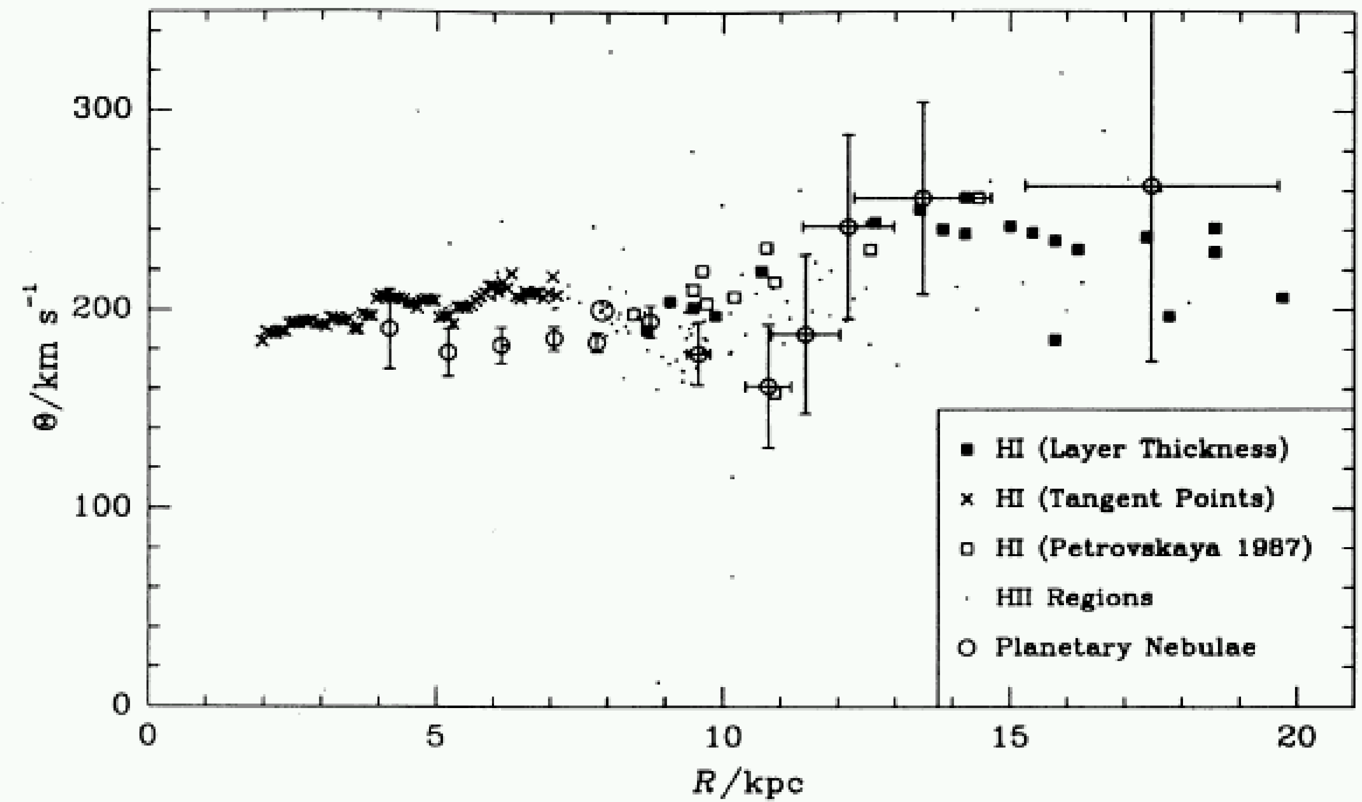
  $\rightarrow \omega(R) < \omega(R_0) \rightarrow v_r < 0$




## $2^{nd}$ Quadrant

$R > R_0 \rightarrow \omega(R) < \omega(R_0) \rightarrow v_r < 0$



300
200
100
0
θ/km s$^{-1}$
0
5
10
15
20
R/kpc
HI (Layer Thickness)
HI (Tangent Points)
HI (Petrovskaya 1987)
HII Regions
Planetary Nebulae

$$N_H = 1.82 \times 10^{18} \int_0^{\infty} T_s \, \tau_\nu(v) \, dv$$

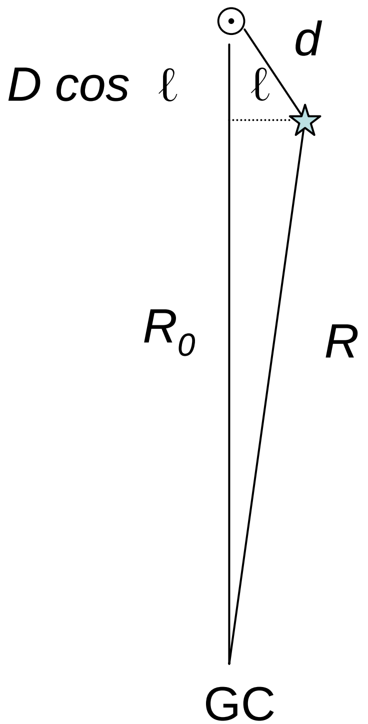
$$N_H = 1.82 \times 10^{18} \int_V T_b \, dV \qquad (172)$$

Lecture 9 in
http://tesla.phys.unm.edu/phy537/

# Oort's Formulae

$$v_r = (\omega - \omega_0)\ R_0\ \sin\ell$$

For small distances (< 2 kpc)




$$\begin{aligned}(\omega - \omega_0) &= (\frac{d\omega}{dR})_{R_0}\ (R - R_0) \\ &= [\frac{d}{dR}(\frac{v}{R})]_{R_0}\ (R - R_0) \\ &= [\frac{1}{R}\frac{dv}{dR} - \frac{v}{R^2}]_{R_0}\ (R - R_0) \\ &= \frac{1}{R_0}\ [(\frac{dv}{dR})_{R_0} - \frac{v_0}{R_0}]\ (R - R_0)\end{aligned}$$

$$v_r = [(\frac{dv}{dR})_{R_0} - \frac{v_0}{R_0}]\ (R - R_0)\ \sin\ell$$

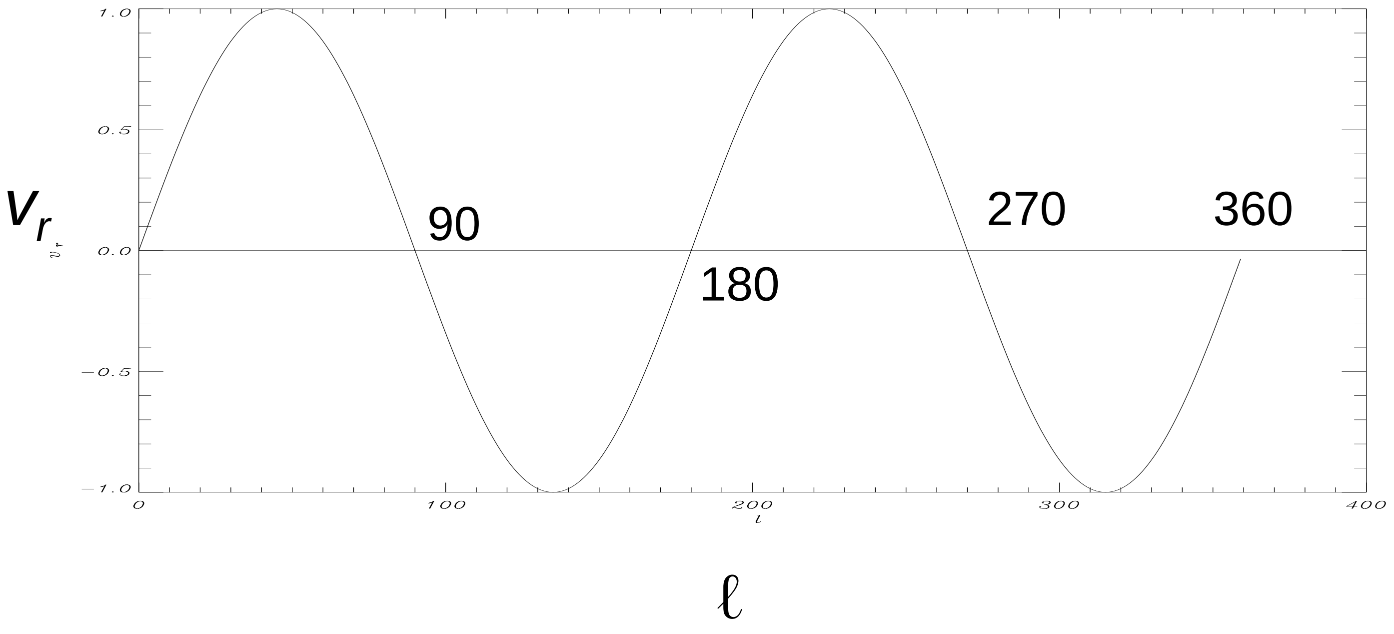
$$(R_0 - R) \approx d\cos\ell,\ \ d \ll R_0$$

$$v_r \approx -[(\frac{dv}{dR})_{R_0} - \frac{v_0}{R_0}]\, d\, \cos\ell \sin\ell$$

$$= \frac{1}{2}[\frac{v_0}{R_0} - (\frac{dv}{dR})_{R_0}]\, d\, \sin 2\ell$$

Oort's constant A

For $d \ll R_0$, $v_r = A\, d\, \sin 2\ell$



This can be further simplified.

$$
\begin{aligned}
A &= \frac{1}{2}\,[\frac{v_0}{R_0} - (\frac{dv}{dR})_{R_0}] \\
&= \frac{1}{2}[\omega_0 - (\frac{d(R\omega)}{dR})_{R_0}] \\
&= \frac{1}{2}[-R_0(\frac{d\omega}{dR})_{R_0}] \\
&= -\frac{1}{2}\,R_0\,(\frac{d\omega}{dR})_{R_0}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
v_t &= [(\frac{dv}{dR})_{R_0} - \frac{v_0}{R_0}]\,(R - R_0)\ \cos\ell - \omega_0\ d \\
&\approx [\frac{v_0}{R_0} - (\frac{dv}{dR})_{R_0}]\,d\ \cos^2\ell - \frac{v_0}{R_0}\,d \\
&\quad (\text{Because } \cos^2\ell = \frac{1}{2}(1+\cos 2\ell)) \\
&= \frac{1}{2}[\frac{v_0}{R_0} - (\frac{dv}{dR})_{R_0}]\,d\ \cos 2\ell \boxed{- \frac{1}{2}\,[\frac{v_0}{R_0} + (\frac{dv}{dR})_R]}\,d
\end{aligned}
$$

Oort's constant B

$$
v_t = d\,(A\cos 2\ell + B)
$$

# Oort's constants

$$\begin{aligned} A &= \frac{1}{2}\,[\frac{v_0}{R_0} - (\frac{dv}{dR})_{R_0}] \\ &\approx 14.5 \pm 1.5 \text{ km s}^{-1} \text{ kpc}^{-1} \end{aligned}$$

$$\begin{aligned} B &= -\frac{1}{2}\,[\frac{v_0}{R_0} + (\frac{dv}{dR})_{R}] \\ &\approx -12 \pm 3 \text{ km s}^{-1} \text{ kpc}^{-1} \end{aligned}$$