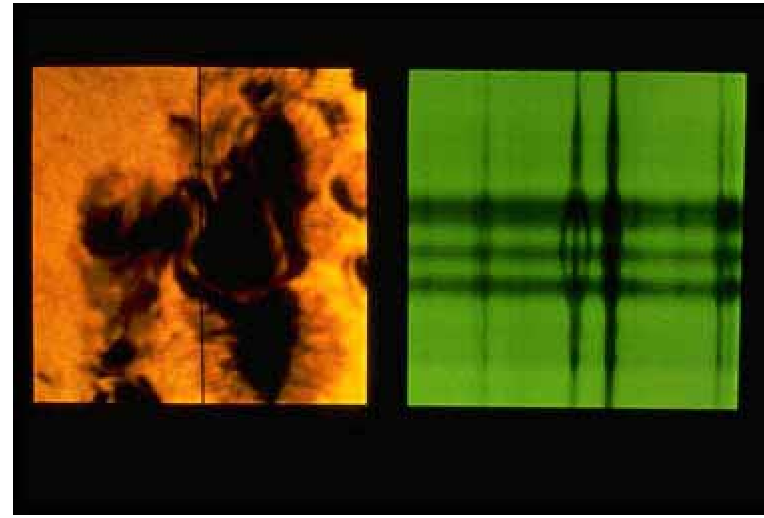
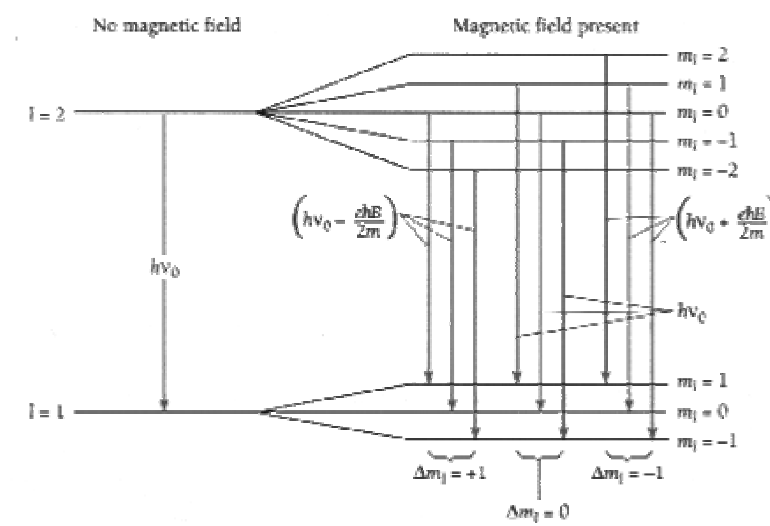
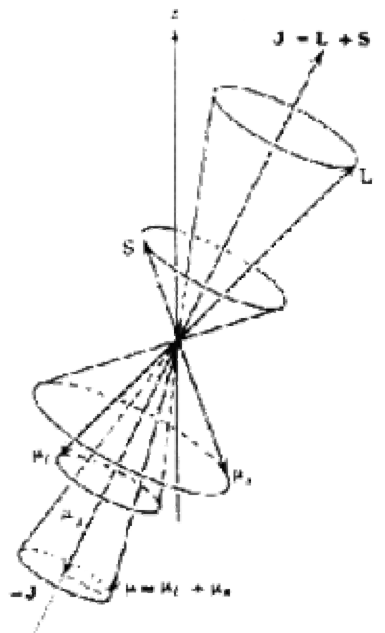
Zeeman Effect

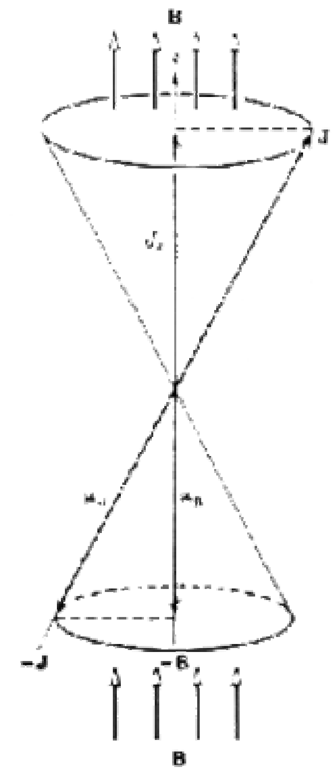


The **Zeeman effect** is the split of a spectral line into several components in the presence of a **magnetic field**. It is analogous to the **Stark effect**, the splitting of a spectral line into several components in the presence of an **electric field**.

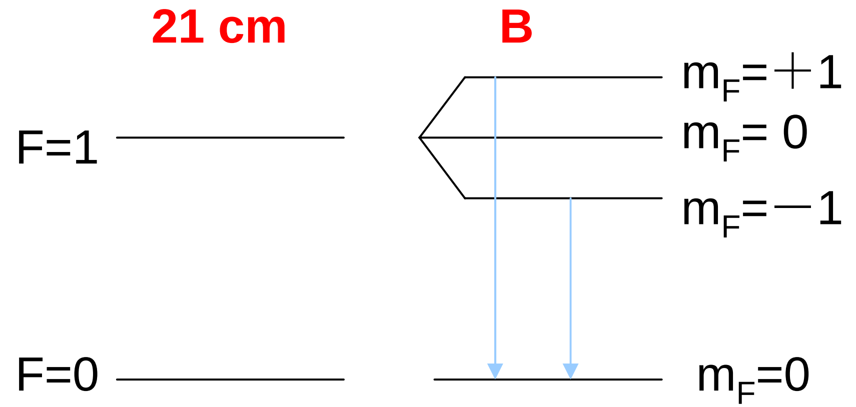


http://www.pha.jhu.edu/~rt19/hydro/node10.html





Selection Rule: $\Delta m_F = 0, +/- 1$;
a level with $m_F=0$ cannot combine with another $m_F=0$



Singlet line $\xrightarrow{\text{split}}$ σ , π , σ

$$-\frac{eB}{4\pi m_e c}$$

Elliptical polarization

$$\nu_{mn}$$

Plane polarization

$$+\frac{eB}{4\pi m_e c}$$

Elliptical polarization

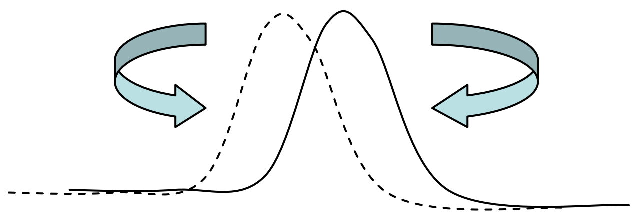
$$\frac{eB}{4\pi m_e c} = 1.4 \times 10^6 \; B_{\text{Gauss}} \; [\text{Hz}]$$ Larmor frequency of precession

Along $\vec{B}$, $\pi = 0$, σs are circularly polarized in opposite directions

Total splitting $\Delta\nu = 2.80 \times 10^6 \ B_{\rm Gauss} \ [\rm Hz]$

Typically in ISM, $B \sim 10^{-6}$ Gauss, so $\Delta\nu \sim$ a few hertzs

very difficult to detect ($\ll$ Doppler width)



$< B_{\rm Galactic} > \sim 4 \ \mu\rm G$

Zeeman splitting first detected in 21-cm absorption (Verschuur 1969); later seen in emission, too (Heiles 1982)

Also has been observed in OH 18 cm line; 6 cm H_2CO
➔ to derive *B* and *n* (HI)

$B \propto n^p$ for H I clouds $\quad p \sim 2/3$–$1/3$

Note: For an isotropically contracting cloud with a "frozen-in" magnetic field,

$$B \propto \frac{1}{R^2}, \text{ and } \rho \propto \frac{1}{R^3} \quad \Longrightarrow \quad \boxed{B \propto n_H^{2/3}}$$

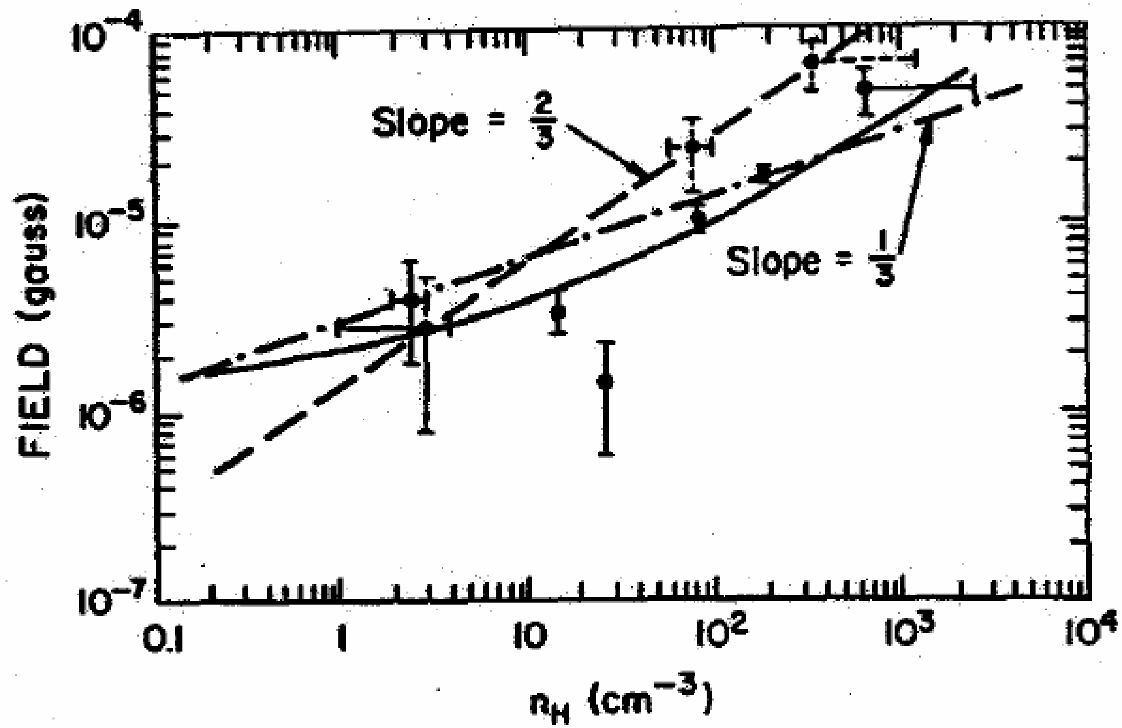


Fig. 1. Magnetic field strength in H I clouds as a function of gas density.

Mouschovias (1983) Adv. Space Res. 2, 71

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INTERSTELLAR MAGNETIC FIELD STRENGTHS AND GAS DENSITIES: OBSERVATIONAL AND THEORETICAL PERSPECTIVES

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ABSTRACT

We present an updated compilation of observational data concerning the relationship between the interstellar magnetic field strength and the gas density. Pulsar and Zeeman-effect data provide the only reliable information about the (B, n) relationship, and they now span nearly six orders of magnitude in gas density. Field strengths show no evidence of increase over the density range $0.1 - \sim 100\ cm^{-3}$. At higher densities, a modest increase in field strength is observed in some regions, in line with theoretical expectations for self-gravitating clouds. In two regions of the interstellar medium, the magnetic field is unusually high; however, these are not locales where self-gravitation is important. Despite the consistency between observations and theory, questions still exist about how the magnetic field strength remains constant for densities up to $\sim 100\ cm^{-3}$. Further Zeeman effect studies and a better theoretical understanding of the formation of interstellar clouds and complexes will be necessary to answer these questions.

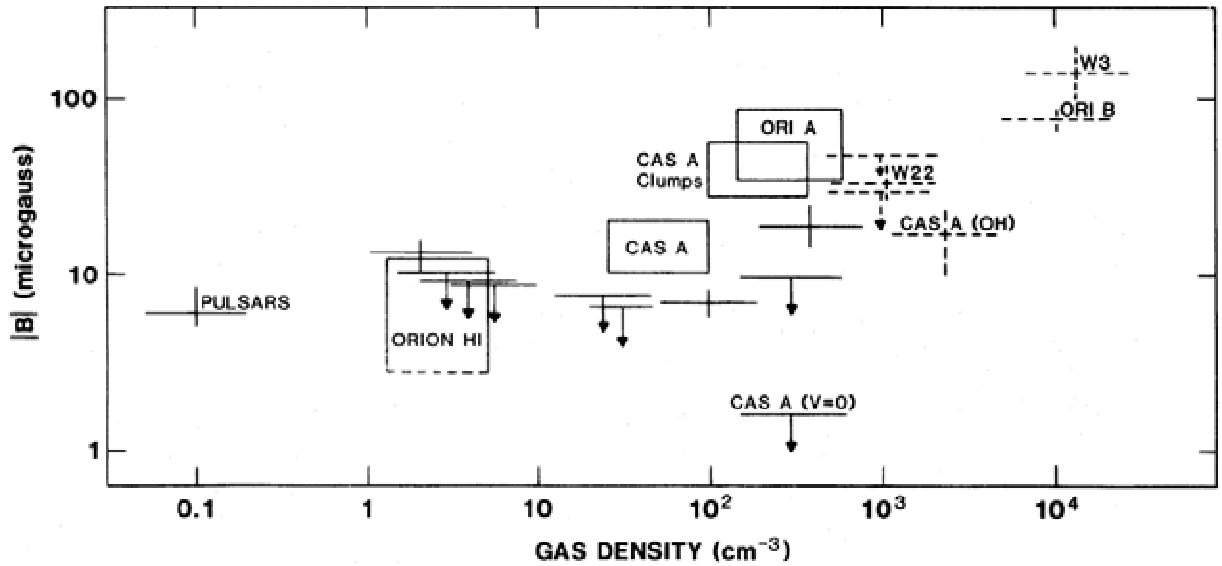

FIG. 1.—Observed magnetic field strengths as a function of estimated volume density. All results come from measurements of the H I (*solid lines*) and OH (*dashed lines*) Zeeman effect, except for the point labeled "pulsars." This point is derived from pulsar rotation and dispersion measures. Rectangular boxes represent ranges of field strengths encountered in Zeeman effect maps made either with a single-dish or with aperture synthesis instruments. See § II for further details.

Troland & Heiles (1986) ApJ, 301, 339

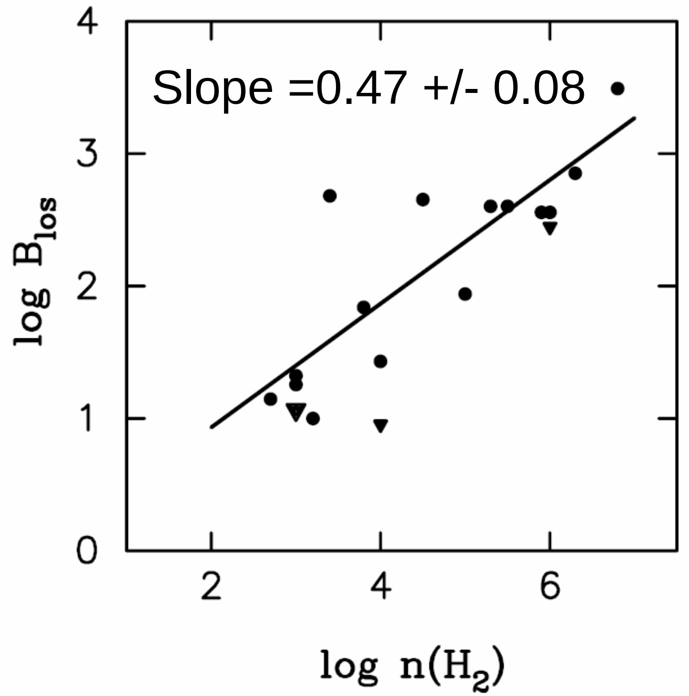

FIG. 1.—Plot of log B_{los} vs. log $n(H_2)$. Inverted triangles are the upper limits for undetected clouds; the averaged limit for all of the dark clouds with log $n(H_2) = 3$ is plotted as a single large inverted triangle. The line is the fit to detected clouds.

Crutcher 1999, ApJ, 520, 706

MAGNETIC FIELDS IN MOLECULAR CLOUDS: OBSERVATIONS CONFRONT THEORY

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ABSTRACT

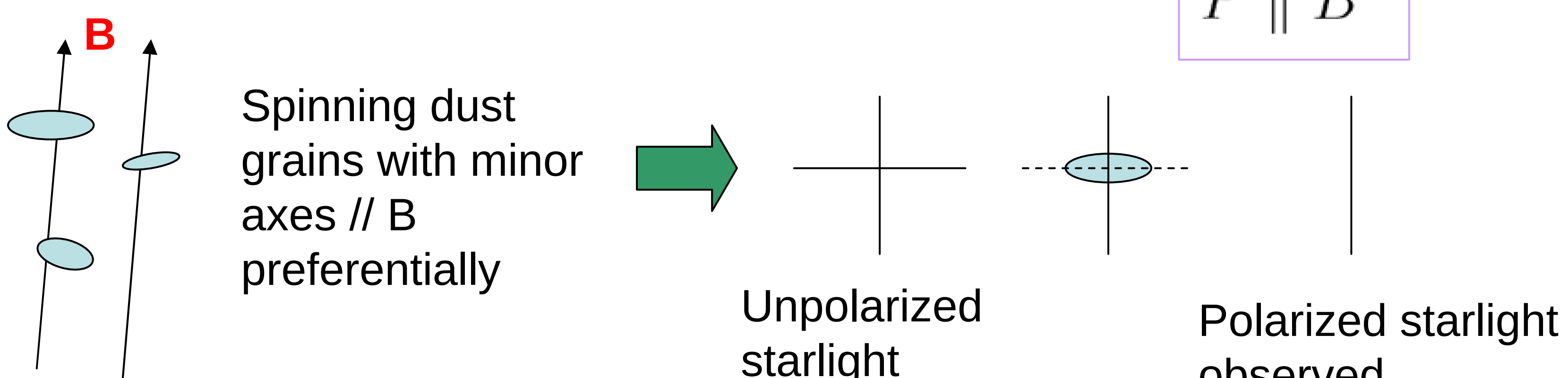
This paper presents a summary of all 27 available sensitive Zeeman measurements of magnetic field strengths in molecular clouds together with other relevant physical parameters. From these data input parameters to magnetic star formation theory are calculated, and predictions of theory are compared with observations. Results for this cloud sample are the following: (1) Internal motions are supersonic but approximately equal to the Alfvén speed, which suggests that supersonic motions are likely MHD waves. (2) The ratio of thermal to magnetic pressures $\beta_p \approx 0.04$, implying that magnetic fields are important in the physics of molecular clouds. (3) The mass–to–magnetic flux ratio is about twice critical, which suggests but does not require that static magnetic fields alone are insufficient to support clouds against gravity. (4) Kinetic and magnetic energies are approximately equal, which suggests that static magnetic fields and MHD waves are roughly equally important in cloud energetics. (5) Magnetic field strengths scale with gas densities as $|B| \propto \rho^\kappa$ with $\kappa \approx 0.47$; this agrees with the prediction of ambipolar diffusion driven star formation, but this scaling may also be predicted simply by Alfvénic motions. The measurements of magnetic field strengths in molecular clouds make it clear that magnetic fields are a crucial component of the physics governing cloud evolution and star formation.

Polarized Starlight

Magnetic field in the ISM first discerned by linearly polarized starlight (~1%) (Hiltner 1949 and Hall 1949)

It is thought that the partial polization of starlight is produced by elongated dust grains aligned by magnetic fields in the ISM (see a review by Lazarian astro-ph 0003314 "*Physics of Grain Alignment*")

$$\vec{P} \parallel \vec{B}$$



Chandrashkhar → atmosphere of early type stars should produce polarization due to electron scattering

Polarization of Light From Distant Stars by Interstellar Medium

W. A. Hiltner

Yerkes Observatory, University of Chicago

IN THE COURSE OF PHOTOELECTRIC OBSERVATIONS made last summer with the 82-inch telescope of the McDonald Observatory (University of Texas) the writer found that the light from distant galactic stars is polarized. Polarizations as high as 12 percent were found. The plane of polarization appears to be close to the galactic plane in the cases examined. More recently control measures were made at the Lick Observatory, thanks to the courtesy of Director Shane and Dr. G. Kron; and during December the work at the McDonald Observatory was extended to different regions of the Milky Way.

In view of the unexpected nature of this result the circumstances leading to its discovery are recorded. Photometric observations for the detection of partially polarized radiation from eclipsing binary stars have been in progress at the Yerkes Observatory for several years with a view to establishing observationally the effect pointed out by Chandrasekhar that the continuous radiation of early-type stars should be polarized (*1, 2*). On the assumption that the opacity of early-type stars is due to scattering by electrons, the continuous radiation emerging from a star should be polarized with a maximum of polarization of 11 percent at the limb. Since the presence of this polarization can be detected only when the early-type star is partially eclipsed by a larger-type companion of the system, the effect is masked by radiation from this companion so that the expected maximum observable effect was only of the order of 1.2 percent in one case investigated (RY Persei).

At this stage Dr. John Hall, of Amherst College, proposed to the writer a program of collaboration whereby Dr. Hall would construct a "flicker" photometer which was to be tested jointly at the McDonald Observatory. Independently the writer was developing his own equipment which used polaroids. Dr. Hall's equipment was tested in August 1947, during a short session at the McDonald Observatory, but no dependable results were obtained and it was found that the equipment had to be remodeled. Unfortunately, Dr. Hall was unable to come for a second trial period, scheduled for August 1948.

Meanwhile the writer's own equipment was completed and put to use during the summer of 1948 and was found satisfactory. Certain Wolf Rayet stars which were known or suspected to be eclipsing binaries were examined for polarization. Fairly large polarizations were found, but *they did not appear to depend on the phase of the binary motion.* The possibility of instrumental polarization was considered, of course, but ruled out by control measures on check stars. The Wolf Rayet stars give the following results:

Star	*Polarization* %	Position angle
CQ Cep	10.0	62°
BD 55°2721	8.0	44
WN Anon*	12.5	44

*Coordinates: $22^h08^m + 57°26'$ (1945); 12.5 magnitude.

The control stars had similar color and brightness, but showed no polarization except for one object, BD 55°2723, which gave 3 percent. This star, however, is a giant and more distant than the other control stars. Similar observations made on a group of Wolf Rayet stars in Cygnus showed no appreciable polarization, while two stars in Scutum gave positive results. Other regions, such as the double cluster in Perseus, also show polarization with values ranging up to 12 percent.

We conclude from the positive and negative results quoted that the measured polarization does not arise in the atmospheres of these stars but must have been introduced by the intervening interstellar medium. If this conclusion is accepted, a new factor in the study of interstellar clouds is introduced. Further observations are in progress for relating this phenomenon with other observable characteristics of interstellar medium. As has been stated, the results already at hand indicate that the plane of polarization approximates the plane of the galaxy.

References

1. CHANDRASEKHAR, S. *Astrophys. J.*, 1946, **103**, 365.
2. HILTNER, W. A. *Astrophys. J.*, 1947, **106**, 231.

Polarization of eclipsing binary WR stars → polarization does not change with orbital phase

Polarization therefore must be of ISM origin

Observations of the Polarized Light From Stars

John S. Hall

U. S. Naval Observatory, Washington, D. C.

PHOTOELECTRIC OBSERVATIONS of the polarization of starlight made during the period November 1948 to January 1949 with the 40-inch reflector at Washington substantiate the hypothesis of W. A. Hiltner (*2*) that this effect is produced by interstellar matter. Furthermore, the percentage of polarization appears to be independent of wavelength; and the plane of polarization (plane containing the magnetic vector and the line of sight) appears to have no one preferential orientation.

The observations were obtained with a photoelectric polarizing photometer (*1*) built at Amherst College in 1946 with the aid of a grant from the Research Corporation of New York. The light from a star is collimated and directed through a cover glass, which serves as a calibrating device, and then through a Glan-Thompson prism rotated at 15 cycles per second to a 1P21 photomultiplier. The 30-cycle voltage developed by the polarized component of the light is selectively amplified and mixed with a phasing voltage in such a way that the d-c output can be impressed as a sine wave on a Brown recorder. The amplitude of this wave is proportional to the intensity of the polarized light, and the phase of maximum defines the plane of polarization. Records of two stars showing large and small percentages of polarized light are shown in Figs. 1 and 2. The vertical lines represent two-minute intervals. The trace during interval *S* is produced by polarized light from the star. During interval *D* a quartz depolarizer is placed in the light path, and *C* is the result when the cover glass is tilted 20° about an axis whose position angle is arbitrarily set at 94°. The starlight was already depolarized during the interval *C*. The plane of polarization is defined by the direction of the light and the axis about which the glass is tilted. A 20° tilt corresponds to 1.4% polarization.

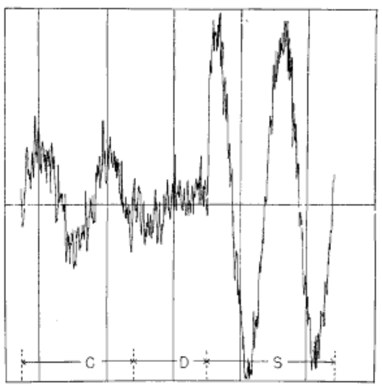


FIG. 1. Star HD 19820, a reddened star of spectral type O8. Polarization, 5.0%; position angle of the plane of polarization, +30°.

The percentages of polarization of the light from 27 early-type stars are shown in Fig. 3 as a function of the color excesses determined by Stebbins and Huffer (*3*). A strong correlation is obvious; the scatter, however, is much greater than the accidental errors of the observations.

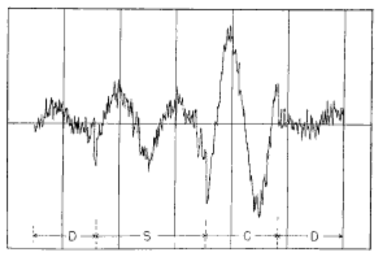


FIG. 2. Star HD 24760, ε Persei. This bright star is not generally considered to be a reddened B-type star. Polarization, 0.5%; position angle, −68°.

The dependence of polarization on color was determined on three nights from observations of ζ Persei using Schott filters UG1 and BG14 for the ultraviolet region and RG1 or a Wratten yellow filter for the red region. The effective wavelengths of the two spectral regions were near 3,700 A and 6,200 A. The observed percentages with the ultraviolet filter were 2.0, 1.6, and 1.8; and with the red filter, 1.6, 1.0, and 2.2. The average value obtained when no filter was used is 1.8 percent. A second star, HD 33,203, was observed on one night, the result being 1.8 in the ultraviolet, 2.2 in the red, and 1.8 with no filter. No definite variation of the orientation of the plane of polarization with color is indicated by these observations.

Fig. 4 shows the observed planes of polarization for 28 early-type stars. The amount of polarization and the orientation of the plane of each is indicated by the length and direction of the line, whose midpoint represents the position of a star. The group of seven stars near the middle of the diagram exhibit a remarkable similarity in percentages of polarization and orientations of the planes, which may be a consequence of the relative homogeneity of the obscuring material in the direction from which their light comes.

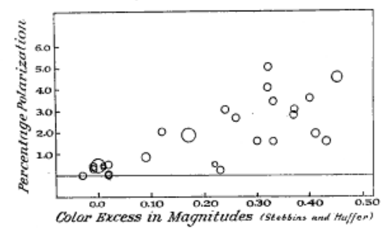


FIG. 3. Observational evidence of a correlation between color excess and percentage of polarization for early-type stars. The size of the circle indicates the weight of the observations.

I have obtained these preliminary results from a project initiated in collaboration with W. A. Hiltner. My grateful appreciation is expressed to Dr. Hiltner and to the Yerkes and McDonald Observatories for the use of the 82-inch reflector for a period of two weeks during the summer of 1947. Despite very unfavorable weather conditions and some difficulty with a new type of photometer, we obtained some evidence of polarization in the light from one star, CQ Cephei. Accordingly it was planned to make a second trial at McDonald during the summer of 1948 with improved equipment, but a second trial could not be made because of other obligations incurred by my transfer from Amherst to Washington on September 1, 1948. With the improved photometer, however, it was easily possible to detect polarization in the light from CQ Cephei with the 18-inch refractor at the Amherst College Observatory. Furthermore, these observations, made during the summer of 1948, showed little if any change in the amount of polarization with the phase of this eclipsing binary star. Meanwhile, Dr. Hiltner's independent work presumably had progressed so far that he did not feel justified in accepting my proposal, made in November 1948, to prepare a joint paper on our work.

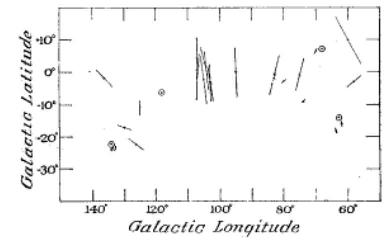
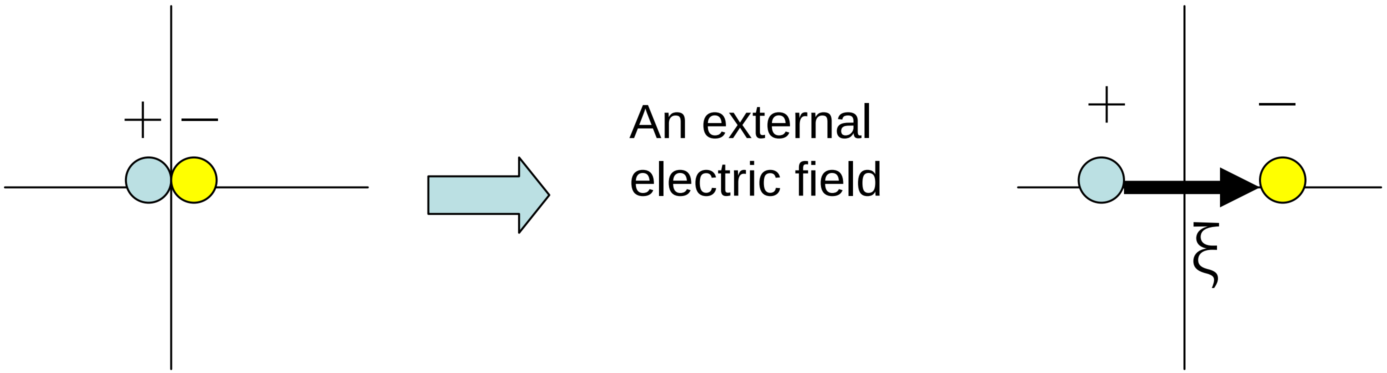


FIG. 4. Observational evidence that there is no one preferential orientation of the plane of polarization. Stars showing no polarization are represented by circles.

References

1. HALL, JOHN S. *Astronom. J.*, 1948, **54**, 39.
2. HILTNER, W. A. *Science*, 1949, **109**, 165.
3. STEBBINS, J., and HUFFER, C. M. *Publ. Washburn Observ.*, 1934, **15**, 5.

Plasma Frequency



The dipole moment $\vec{P} = -n\,e\,\vec{\xi}$

Because of no net charge, $\vec{D} = \vec{E} + 4\pi\vec{P} = 0 \longrightarrow \vec{E} = 4\pi n\,e\,\vec{\xi}$

In 1-D, $E_x = 4\pi\,n\,e\,\xi$

The force is $F_x = -e\,E_x$ $\quad m\dfrac{d^2\xi}{dt^2} = -eE_x \quad m\dfrac{d^2\xi}{dt^2} = -4\pi n\,e^2\,\xi$

(Taking $\frac{\hat{d}}{dt} = j\omega$) $\longrightarrow -m\omega^2 = -4\pi\,n\,e^2$

$$\omega_p = \sqrt{\frac{4\pi n e^2}{m}} = 5.6 \times 10^4 \sqrt{n} \text{ rad s}^{-1}$$

or

$$\nu_p = 8.97 \times 10^3 \sqrt{n} \text{ Hz}$$

This is called the **plasma frequency**

Index of refraction $n_r^2 = 1 - \frac{\omega_p^2}{\omega^2}$

…for which the wave number

$$k = 2\pi/\lambda = 2\pi/(v/\nu) = 2\pi T/v = \omega/(c/n_r)$$

$$k = \frac{\omega}{c}(1 - \frac{\omega_p^2}{\omega^2})^{1/2}$$

If $\omega > \omega_p$, k is real ⟶ Electrons cannot follow

If $\omega < \omega_p$, k is imaginary ⟶ Electrons follow the motion → absorption → no propagation

- **Metal** --- $\nu_p \sim 10^{15}$ Hz, i.e., in UV region; so metal absorbs EM waves until UV, i.e., transparent to UV radiation
- **Earth's ionosphere** --- n_e depends on height; at H=300 km, $n_e \sim 10^5$-10^6, $\nu_p \sim 10^7$ Hz, $\lambda \sim 30$ m
  - cf. shortwave radio signals get reflected by ionosphere
- **ISM** --- $n_e \sim$ a few, $\nu_p \sim 10^4$ Hz (very low), so usually $\nu >> \nu_p$ ➔ transmission ok

Pulsar Dispersion

Shape of the same pulse varies with freq.

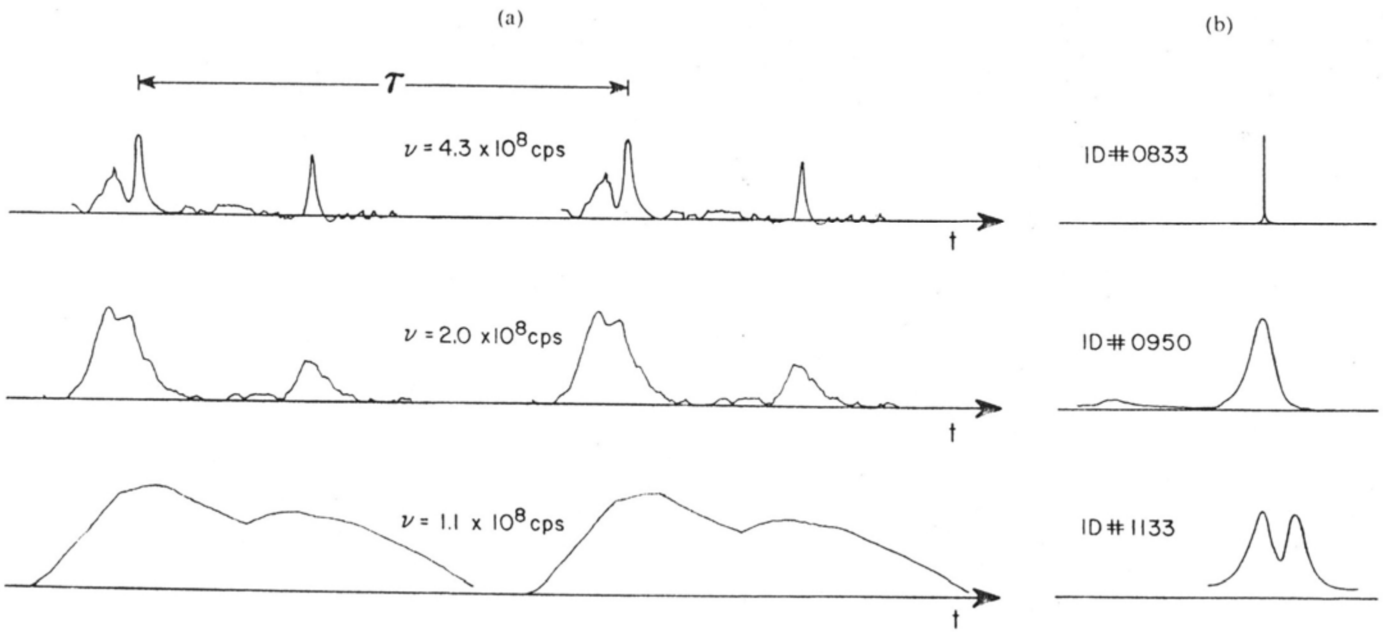
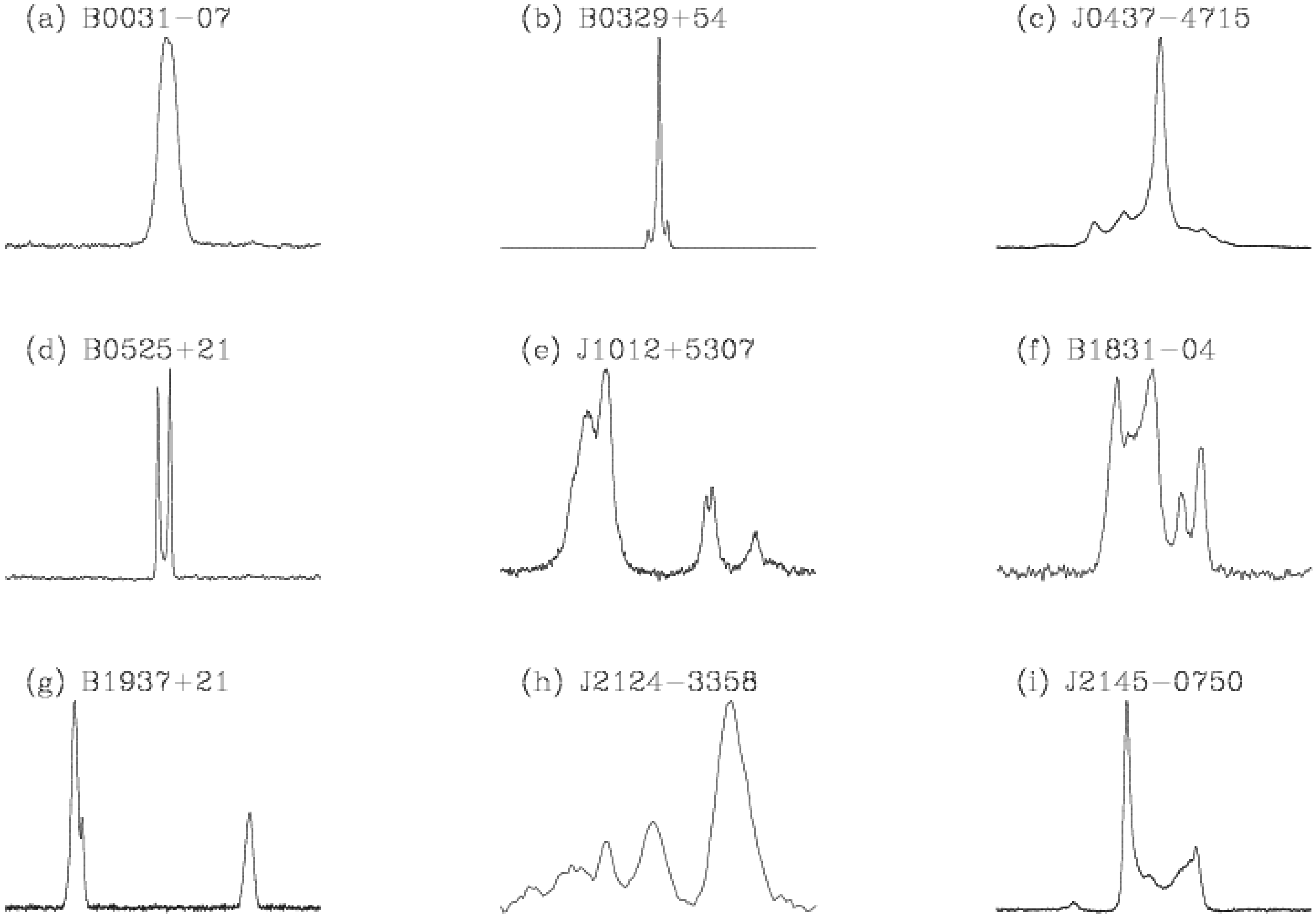


Fig. 5.8. Radio frequency detection of pulsars: (a) periodic pulse shape varying with frequency for a single pulsar; (b) integrated pulse shape for various pulsars.

Every pulsar is different.



$$k = \frac{\omega}{v} = \frac{\omega}{c/n_r}$$

$$n_r^2 = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\omega = \frac{k\,c}{n_r} = \frac{k\,c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}}$$

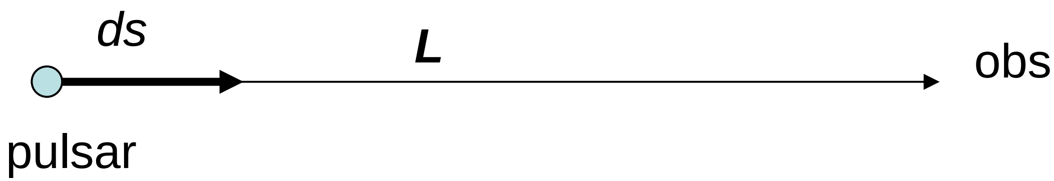
$$\omega^2 - \omega_p^2 = k^2\,c^2$$

$$2\omega\,d\omega = 2k\,dk\ c^2$$

$$\frac{d\omega}{dk} = \frac{k\,c}{\omega} = c\,n_r = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

Pulses propagate at the group velocity, which is frequency dependent.

$$v_{\text{group}} = \frac{d\omega}{dk} = c(1 - \frac{\omega_p^2}{\omega^2})^{1/2}$$



Pulse traveling time = $\tau = \int_0^L \frac{ds}{v_g} = \int \frac{ds}{c(1-\omega_p^2/\omega^2)^{1/2}}$

In ISM, $\omega^2 \gg \omega_p^2$, so $(1-\omega_p^2/\omega^2)^{-1/2} \approx (1+\omega_p^2/2\omega^2)$

$$\tau \approx \int_0^L \frac{ds}{c}(1+\omega_p^2/2\,\omega^2)$$

Since $\omega_p = \sqrt{4\pi\, n\, e^2/m}$, ⟶ $\tau \approx \frac{L}{c} + \frac{4\pi\, e^2}{2\, m\, \omega^2} \int_0^L n_e\, ds$

Traveling time ←→ frequency

Signal arrives earlier at a higher frequency.

Dispersion Measure (DM) [cm^{-3} pc]; typically 10-200

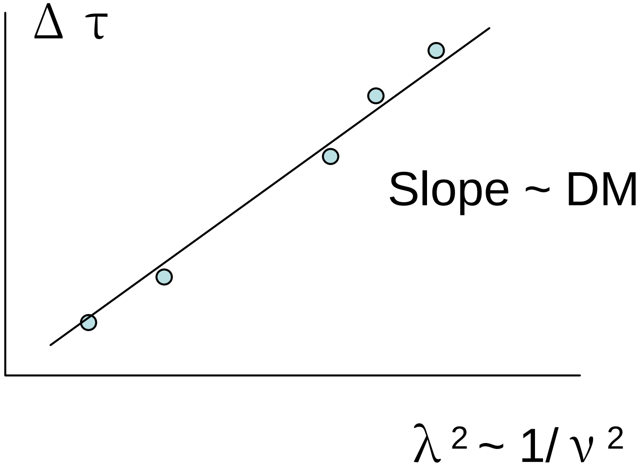
For ω_1 and ω_2,

$$\Delta\tau = \frac{4\pi\, e^2}{2\, m}\left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right) = 4.1 \times 10^3\ DM\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right)$$

In MHz

This gives DM → n_e along the line of sight

Observed <*ne*> ~ 0.03 to 0.08 cm^{-3}



Alternatively, one can assume n_e and estimate the distance.

In fact, n_e varies along the line of sight → scintillation (terrestrial, ISM, IGM)

Atmosphere: 1”

ISM: 1 mas

|b|	< 2	2-5	5-10	10-30	30-90
DM	142	60	59	37	13

Dispersion measure of the ISM from observations of 60 pulsars for various intervals of Galactic latitude b (from Scheffler & Elsässer 1987 based on Pottasch 1974)

Faraday Rotation

What if there is magnetic field?

The effect in which the plane of polarization of an EM wave is rotated under the influence of a magnetic field parallel to the direction of propagation

$n_r^2 = 1 - \omega_p^2/\omega^2$ is modified, $\longrightarrow$ $n_r^2 = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_B})$

where $\omega_B = \frac{e\,B}{m\,c} = \frac{4.8\times10^{-10}\times10^{-6}}{10^{-27}\times3\times10^{10}} \sim 10$ [Hz]

B → different n_r → different phase velocity for 2 opposite circular polarizations (linear polarization with a specific position angle) → PA rotates

In ISM, ω (~10^8 Hz) >> ω_p (~10^4 Hz) >> ω_B (~10 Hz)

$$\frac{\omega_p^2}{\omega^2(1 \pm \omega_B/\omega}) \approx \frac{\omega_p^2}{\omega^2}(1 \mp \omega_B/\omega)$$

original

change

$$n_r^2 = 1 - \frac{\omega_p^2}{\omega^2} \pm \frac{\omega_p^2\,\omega_B}{\omega^3}$$

$$n_r = (1 - \frac{\omega_p^2}{\omega^2} \pm \frac{\omega_p^2\,\omega_B}{\omega^3})^{1/2} \approx (1 - \frac{\omega_p^2}{\omega^2} \pm \frac{\omega_p^2\,\omega_B}{2\omega^3}) \equiv n_{r,0} \pm \Delta n_r$$

$$\begin{aligned}
\text{Phase} &= \varphi = k\, n_r\, \mathfrak{z} = \frac{\omega}{c}\ \frac{\omega_p^2\,\omega_B}{2\omega^3}\mathfrak{z} \\
&= \frac{\lambda^2}{8\pi^2 c^3}\ \frac{4\pi\, e^2}{m}\ \frac{e}{mc} \int B\, n\, ds \\
&\equiv \lambda^2\ RM
\end{aligned}$$

Rotation Measure

$$\textbf{\textit{RM}} = \frac{e^3}{2\pi m^2 c^4} \int n_e B_{\|} \, ds = 8.12\times 10^5 \int_0^L n_e B_{\|} \, ds$$

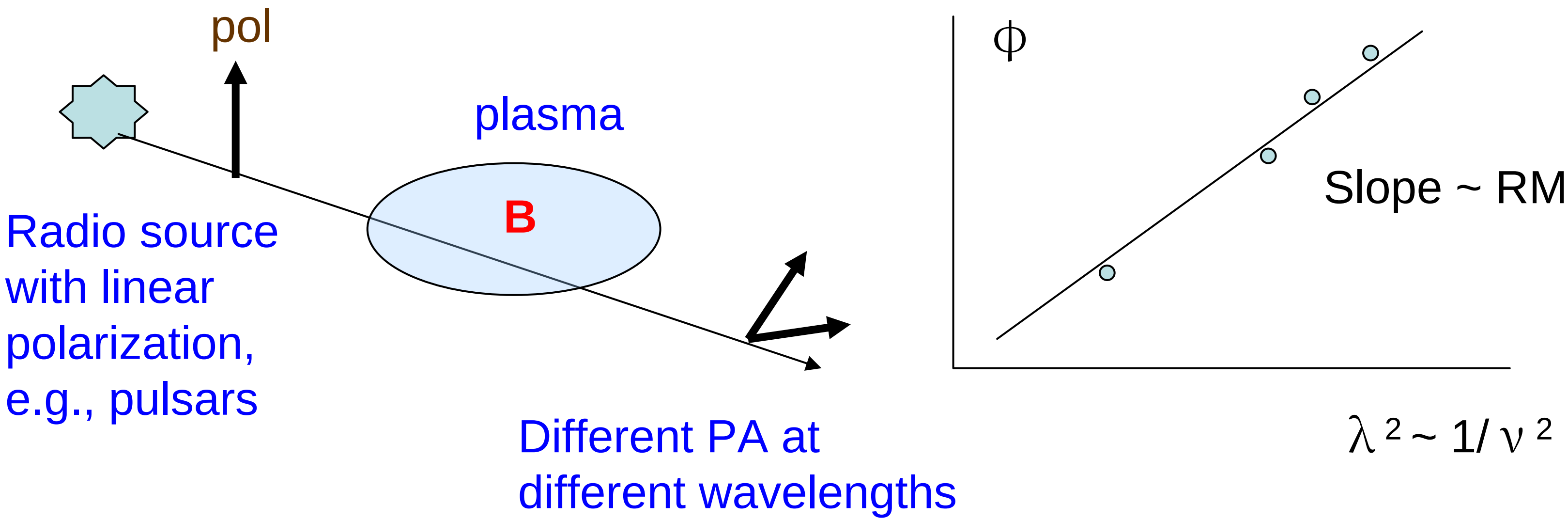
ds [pc]; n_e [cm-3]; *B* [Gauss]; λ [m]; ϕ [radian]

Note:

$$EM = \int n_e^2 \, ds$$

$$DM = \int n_e \, ds$$

$$RM = \int n_e B_{\|} \, ds$$



For polarized pulsars for which DMs are known

$$
\begin{aligned}
< B_{\|} > &= \frac{\int n_e B_{\|} ds}{\int n_e ds} \\
&= \frac{1}{8.1 \times 10^5} \frac{RM}{DM} = 1.23 \frac{RM}{DM} \; [\mu\mathrm{G}]
\end{aligned}
$$

e.g., B(Vela) ~ 0.8 μ G

For galaxies, guess n_e and get B

Local field cancellation

Zeeman splitting, which also measures $B_{//}$, favors high HI column density and narrow line width → cold H I clouds

Faraday rotation samples ionized regions