<u>STELLAR EVOLUTION</u> — (con)sequences of nuclear reactions

$T_*^{center} \sim$ keV, $E_{\substack{Coulomb\\barrier}} \sim$ MeV at nuclear distances

Q — So how can nuclear reactions (strong nuclear force in play!) take place?

A — Quantum mechanics "tunnelling effect"

- Discovered by George Gamow in 1928
- Applied in 1929 to energy source in stars by Robert Atkinson & Fritz Houterma

Cross section for nuclear reactions (penetrating probability)

$$\propto \quad e^{-\pi Z_1 Z_2 e^2/\varepsilon_0 h v} \qquad \uparrow \text{ as } v \uparrow$$

Velocity probability distribution (Maxwellian)

$$\propto \quad e^{-mv^2/2kT} \qquad \downarrow \text{ as } v \uparrow$$

$\therefore$ Product of these 2 factors $\Rightarrow$ $\boxed{\text{Gamow peak}}$

Resonance reactions

Energy of interacting particles $\approx$ Energy level of compound nucleus I+

$$I + J \rightarrow (I+J) \rightarrow K + L$$

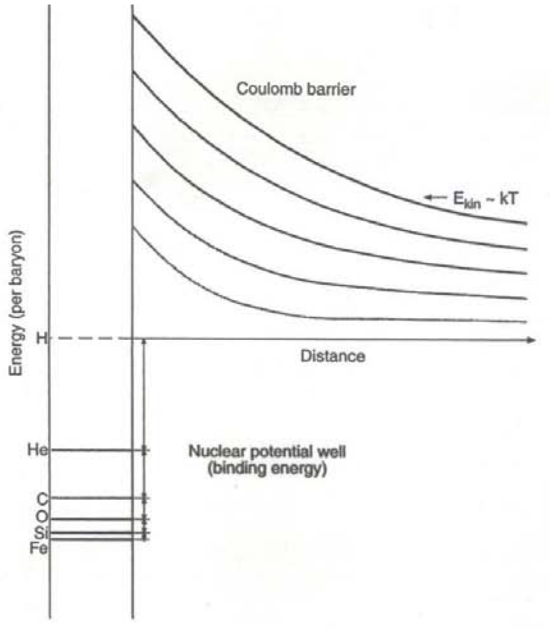


Figure 4.2 Schematic representation of the Coulomb barrier – the repulsive potential encountered by a nucleus in motion relative to another – and the short-range negative potential well that is due to the nuclear force. The height of the barrier and the depth of the well depend on the nuclear charge (atomic number).

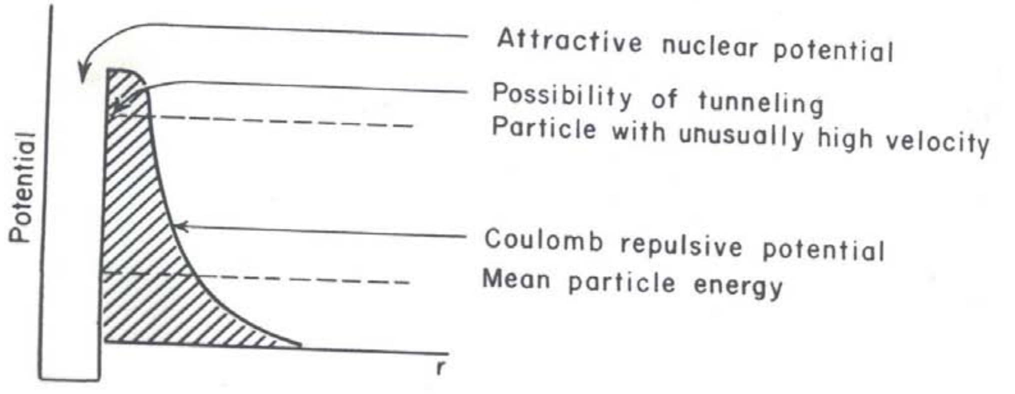


FIGURE 8.6. Energies involved in nuclear reactions.

Quantum mechanics
tunneling effect

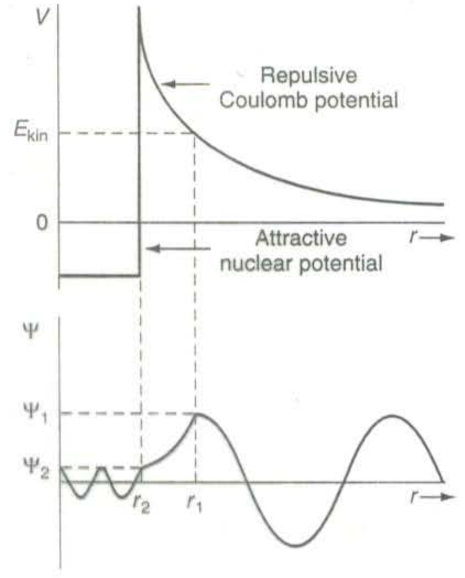


Figure 3.4 Illustration of the potential seen by particle b when approaching particle A with a kinetic energy E_{kin}, and the corresponding wavefunction Ψ; classically, particle b would reach only a distance r_1 from particle A before being repelled by the Coulomb force

Salaris & Cassi

D. Clayton "Principles of Stellar Evolution and Nucleosynthesis"

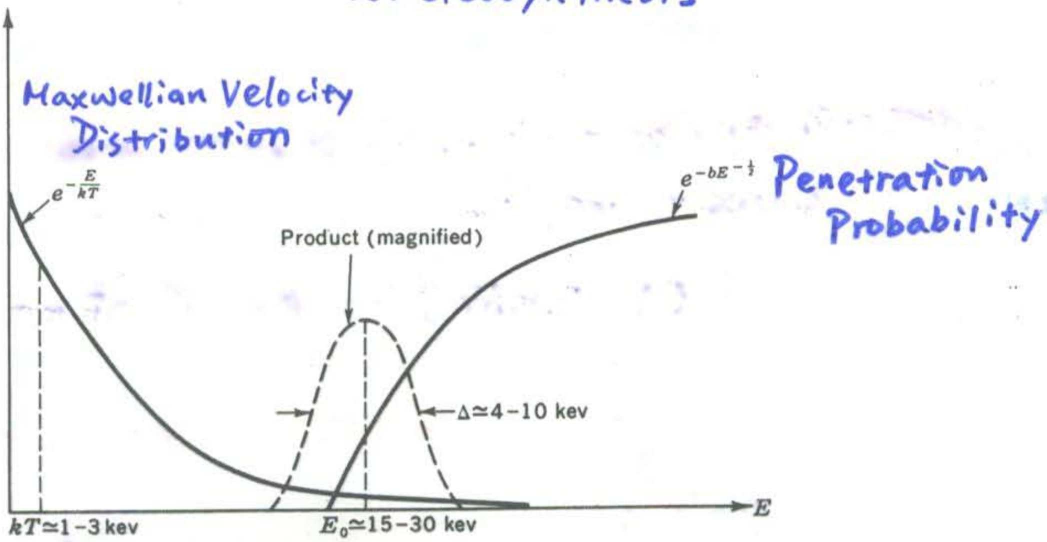


Fig. 4-6 The dominant energy-dependent factors in thermonuclear reactions. Most of the reactions occur in the high-energy tail of the maxwellian energy distribution, which introduces the rapidly falling factor $\exp(-E/kT)$. Penetration through the coulomb barrier introduces the factor $\exp(-bE^{-\frac{1}{2}})$, which vanishes strongly at low energy. Their product is a fairly sharp peak near an energy designated by E_0, which is generally much larger than kT. The peak is pushed out to this energy by the penetration factor, and it is therefore commonly called the *Gamow peak* in honor of the physicist who first studied the penetration through the coulomb barrier.

"ignition" of a nuclear reaction

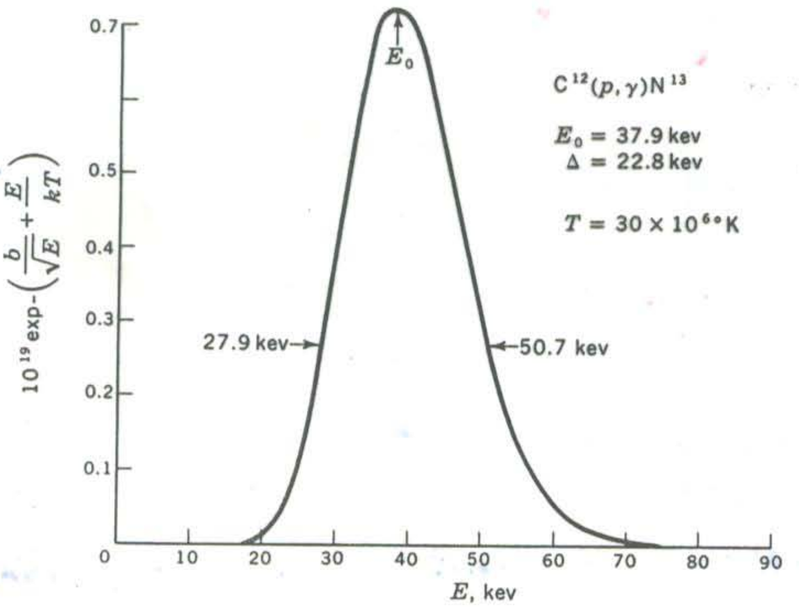


Fig. 4-7 The Gamow peak for the reaction $C^{12}(p,\gamma)N^{13}$ at $T = 30 \times 10^6$ °K. The curve is actually somewhat asymmetric about E_0, but it is nonetheless adequately approximated by a gaussian.

Gamow peak

George Gamow (1904-1968)

Russian-born physicist, stellar and big bang nucleosynthesis, CMB, DNA, Mr. Thompkins series

1929 U Copenhagen

1960s U Colorado

Resonance → v. sharp peak in the reaction rate

⇒ 'ignition' of a nuclear reaction

∃ a narrow range of temperature in which reaction rate ↑ → power law

∃ an ignition temperature (threshold)

Thermonuclear reactions

nucleosynthesis (fusion)

Reaction rate

q [energy released per mass] $\sim \rho^m T^n$

Energy from nuclear reactions (fusion or fission)
= Binding energy of nuclei

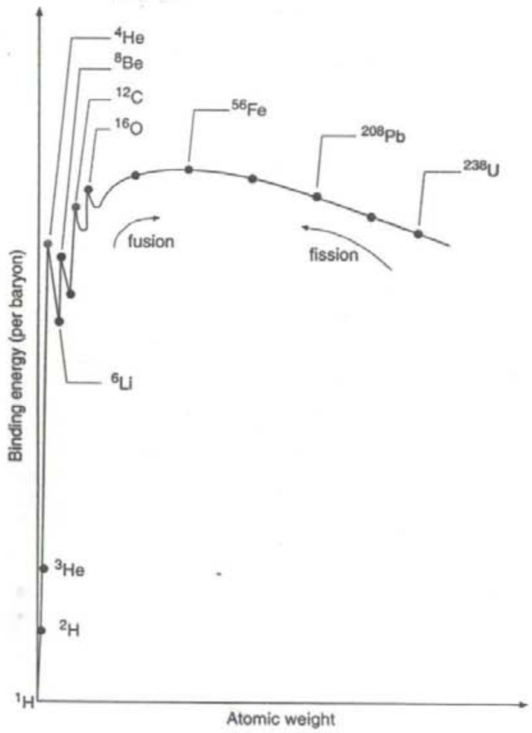


Figure 4.1 Variation of the binding energy per nucleon with baryon number.

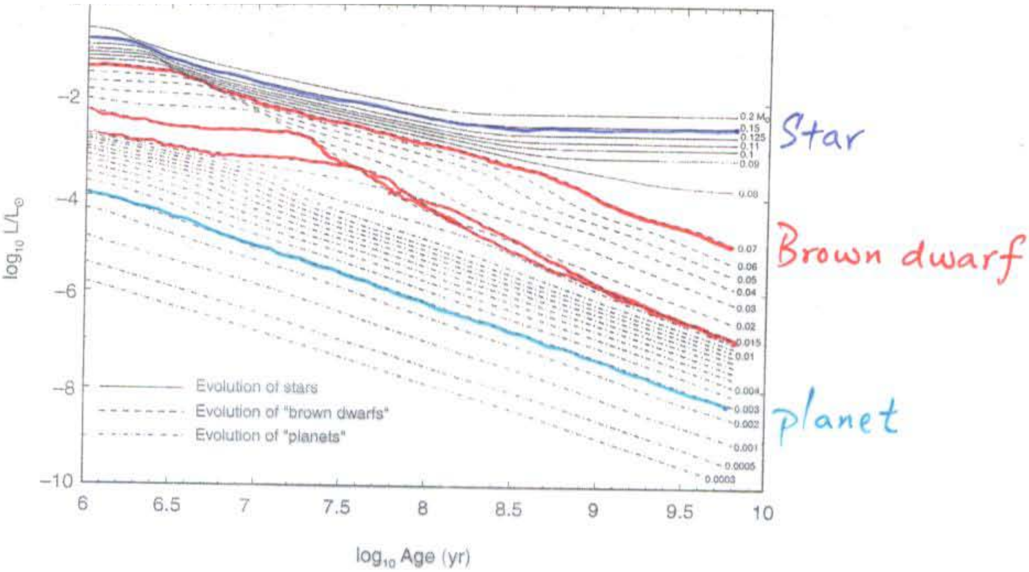


Figure 10.3 Evolution of the luminosity of red dwarf stars (solid curves), brown dwarfs (dashed curves), and planets (dash-dotted curves). Brown dwarfs are here identified as those objects that burn deuterium. Curves are labelled according to mass, the lowest three corresponding to the mass of Jupiter, then half of Jupiter's mass, and finally the mass of Saturn [from A. Burrows et al. (1997), *Astrophys. J.*, 491].

Planets — form in circumstellar disks by aggregation of ever larger dust grains (and gas)

Brown dwarfs — form like stars but evolve like planets

In terms of nuclear reactions

- Stars, $M > 0.08\, M_\odot$, core H burning
- BDs, $M > 0.01\, M_\odot$, short D burning — flat $L(t)$ for $t = 10^6 - 10^8$ yr
  - also for low-mass PMS stars
- Planets, no nuclear burning ever — $L(t) \downarrow$ continuously

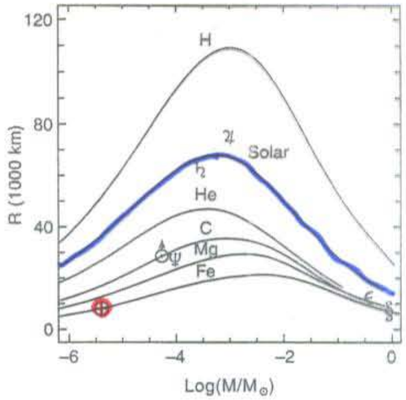


Figure 10.4 Mass-radius relation for low-mass objects [following H. S. Zapolsky & E. E. Salpeter (1969), *Astrophys. J.*, 158]. Different curves correspond to different compositions, as indicated. The locations of several planets – Earth, Jupiter, Saturn, Uranus, and Neptune – are marked by the planets' symbols. Also marked are the locations of two white dwarfs – Sirius B (§) and 40 Eridani B (ϵ) [data from D. Koester (1987), *Astrophys. J.*, 322].

BDs and v. low-mass stars, internal pressure by electron degenerate pressure (but unlike white dwarfs which are nearly completely degenerate)

For WDs, $R \downarrow$ as $M \uparrow$

For terrestrial planets, $R \downarrow$ as $M \downarrow$

complicated equations of states

$\Rightarrow$ Mass-radius max $\approx M_{Jupiter} \approx \frac{1}{1000} M_\odot$

BD mass range $\sim 10\, M_J$ to $80(?)\, M_J$

Deuterium Burning

$${}^2H + {}^1H \longrightarrow {}^3He + \gamma \quad (T > 10^6 K)$$

$${}^2H({}^1H, \gamma)\,{}^3He$$

$$Q_{DP} = 5.5\ \mathrm{MeV}$$

$$q_{DP} = 4.19 \times 10^7 \,[D/H] \left(\frac{\rho}{1\ g\ cm^{-3}}\right)\left(\frac{T}{10^6 K}\right)^{11.8} [erg\ g^{-1} s^{-1}]$$

ISM value, $\langle D/H \rangle \sim 2 \times 10^{-5}$

Recall a star's central temperature

$$T_c \sim \frac{\mu G M}{R} \cdot \alpha$$ mass distr.

Numerically

$$T_c = 7.5 \times 10^6 K \left(\frac{M_*}{M_\odot}\right)\left(\frac{R_*}{R_\odot}\right)^{-1}$$

$$\therefore \quad M_* = 0.4\, M_\odot \longrightarrow T_c \sim 10^6 K$$

Lithium Burning

$$^7Li + {}^1H \longrightarrow {}^4He + {}^4He \quad (T > 3 \times 10^6 K)$$

ISM $[Li/H] \sim 2 \times 10^{-9}$

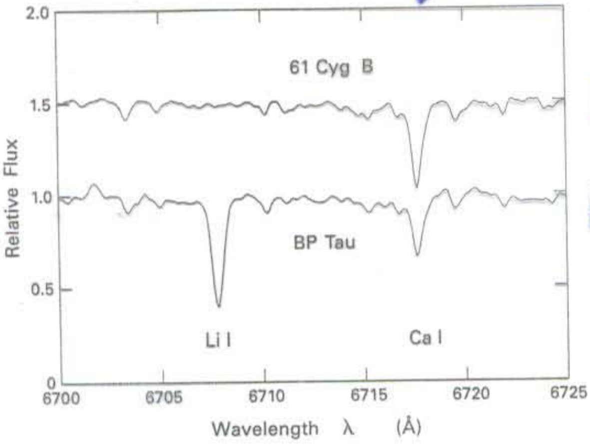
Primordial abundance 10 x lower,
produced by cosmic rays α hitting ^4He
(inverse reaction)

Li measurable in stellar spectra

Li I 6708Å absorption

(actually doublet 6707.78 and 6707.93
but difficult to resolve

6718 Å prominent in late-type stars



MS

PMS

Figure 16.9 Lithium absorption in a pre-main-sequence star. Shown is a portion of the optical spectrum of BP Tau, a T Tauri star of spectral type K7, corresponding to an effective temperature of 4000 K. Also shown, for comparison, is a main-sequence star of the same spectral type, 61 Cyg B. Only in the first star do we see the Li I absorption line at 6708 Å. Both objects also have a strong line due to neutral calcium.

Prensence of Li absorption

→ indicator of youth

M > 1.2 M_sun → shallow convection ➔ surface Li does not deplete during contraction

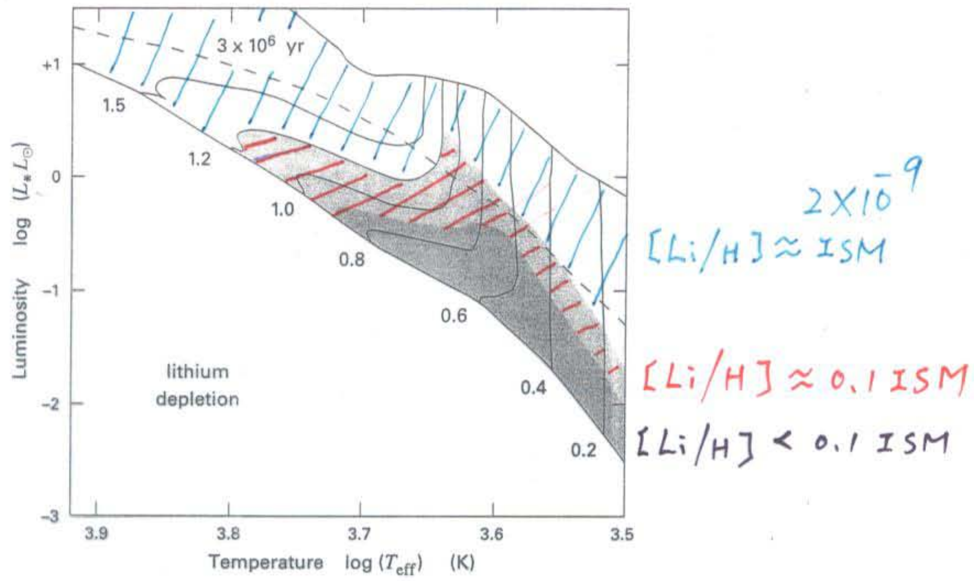
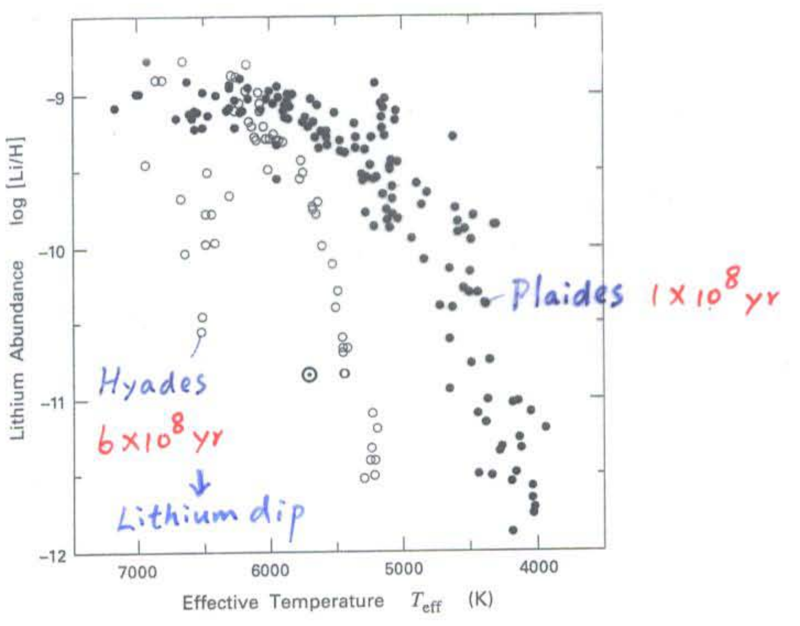


Figure 16.10 Theoretical prediction of pre-main-sequence lithium depletion. Within the white area between the birthline and the ZAMS, the surface [Li/H] is equal to its interstellar value of 2×10^{-9}. Stars in the lightly shaded region have depleted the element down to 0.1 times the interstellar value. The darker shading indicates depletion by at least this amount. Note also the masses on the ZAMS, in solar units, and the indicated isochrone.

Older → depletion at higher T_{eff}



[Li/H] ↓ as T_{eff} ↓

Stahler + Palla book

Q: Why does it take longer for low-mass, young stars to reach the main sequence?

Q: Why do low-mass stars have a higher average material density than higher mass stars?

Q: Why are very low-mass stars fully convective? How does this have anything to do with their surface activities? How does this affect their post-main sequence evolution?

A hydrogen gas — proton-proton chains

$4\,H \rightarrow {}^4He$ unlikely $\Rightarrow$ a chain of reactions

baryon #, lepton #, charges all conserved

$p^+ + p^+ \rightarrow {}^2D^+ + e^+ + \nu$ $\langle 1.4 \times 10^{10}\,yr \rangle$
(1.44 MeV) neutrino $\rightsquigarrow$ 0.26 MeV escaped

${}^2D + p \rightarrow {}^3He + \gamma$ $\langle 6\,s \rangle$
(5.49 MeV)

${}^3He + {}^3He \rightarrow {}^4He + 2p$ $\langle 10^6\,yr \rangle$
(12.85 MeV)

PP I chain

Note: net $6p \rightarrow {}^4He + 2p$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^7Be + e^- \rightarrow {}^7Li + \gamma$

${}^7Li + p \rightarrow 2\,{}^4He$

PP II chain

${}^7Be + p^+ \rightarrow {}^8B + \gamma$

All 3 branches operate simultaneously.

${}^8B \rightarrow {}^8Be + e^+ + \gamma$

${}^8Be \rightarrow 2\,{}^4He$

PP III chain

PP I responsible >90% stellar luminosity

PP I vs PP II

i.e., ^3He to react with ^3He lower temp.

or with ^4He $T > 1.4 \times 10^7 K$

Relative importance of each chain

i.e, branching ratio $\longleftrightarrow T, \rho, \mu$

$T > 3 \times 10^7 K$, PP III dominates

but in reality, at this temperature, CNO reactions take over.

Overall rate of energy generation is determined by the slowest reaction, i.e, the 1st one, $\tau \sim 10^{10} yr$

$\varepsilon_{pp} \sim \rho^1 T^n$, $n \sim 4-6$

$Q_{pp} \sim 26.73$ MeV

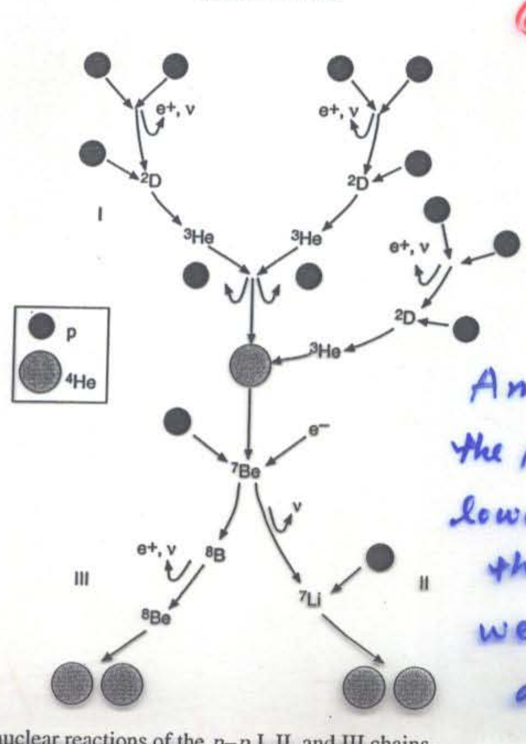


Figure 4.3 The nuclear reactions of the p–p I, II, and III chains.

Among all fusion processes, the p-p chain has the lowest temperature threshold, and the weakest temperature dependence.

CNO cycle

(bi-cycle)

C, N, O as catalysts

$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$	10^6 y	$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$
$^{13}N \rightarrow {}^{13}C + e^+ + \nu$	14 min	$^{15}O \rightarrow {}^{15}N + e^+ + \nu$
$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$	3×10^5 y	$^{15}N + {}^{1}H \rightarrow {}^{16}O + \gamma$
$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$	3×10^8 y	$^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$
$^{15}O \rightarrow {}^{15}N + e^+ + \nu$	82 s	$^{17}F \rightarrow {}^{17}O + e^+ + \nu$
$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$	10^4 y	$^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$

CN cycle more significant

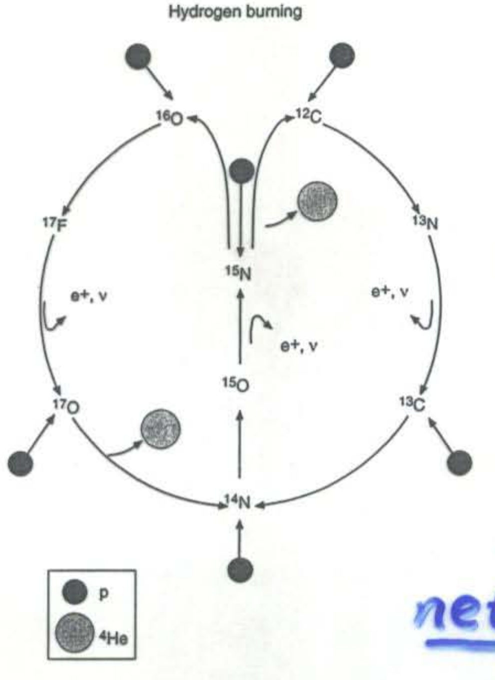


Figure 4.4 The nuclear reactions of the CNO bi-cycle.

net

$4H \rightarrow {}^{4}He$

1938 by Hans Bethe and independently by von Weizsäcker

$Q_{CNO} \sim 25$ MeV (after energy carried away by ν)

$\varepsilon_{CNO} \sim \rho T^{16}$

CNO bi-cycle dominates nuclear energy source in stars > 1.2 M_sun, i.e., of spectral type F7 and earlier.

For comparison, $\epsilon = \epsilon_0 \rho^{\alpha} T^{\beta}$

	α	β
P-P chain	1	~ 4
CNO cycle	1	$\sim 15-18$
3α	2	~ 40

>

In other words

$M \lesssim M_{\odot}$, P-P chain

$M \gtrsim M_{\odot}$, CNO $\Longrightarrow$ Strong temp. dependence

$\Longrightarrow$ higher temp. gradient at center of star

$\Longrightarrow$ convective instability

A He gas — triple-α process

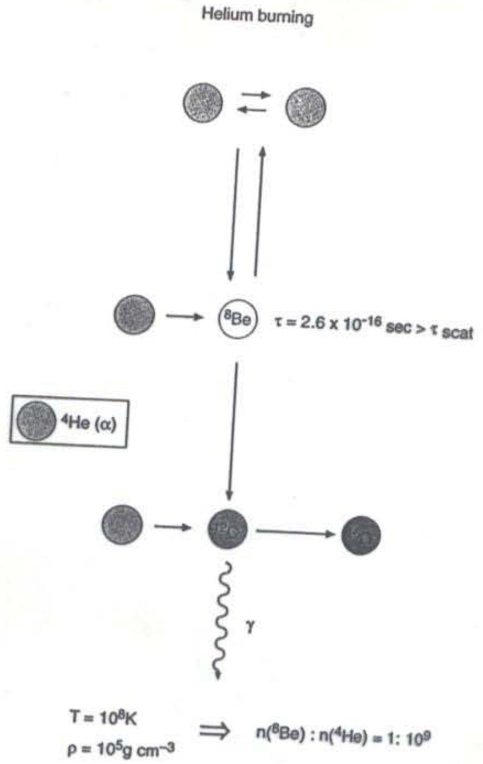


Figure 4.5 The triple-α process.

$^4He + {}^4He \rightarrow {}^8Be$

$(-95\ keV)$

endergonic

lifetime

$\tau \sim 2.6 \times 10^{-16}$ s !

but ~~shorter~~ longer than mean-free time between α particles at T_8 (1952 by Edwin Salpeter)

$^8Be + {}^4He \rightarrow {}^{12}C + \gamma$

$(+7.4\ MeV)$

$Q_{3\alpha} = 7.275$ MeV

net $3\,{}^4He \rightarrow {}^{12}C$

$\hookrightarrow 5.8 \times 10^{17}$ erg g^{-1} ~ 0.1 of $H \rightarrow He$

$q_{3\alpha} \sim \rho^2 T^{40}$

$\because$ bottleneck = 2nd reaction

$\leftrightarrow {}^8Be$

Nucleosynthesis during helium burning

$C^{12}(\alpha, \gamma)O^{16}$ $\quad Q = 7.162$ MeV

$O^{16}(\alpha, \gamma)Ne^{20}$

A carbon/oxygen Gas

$^{12}C + ^{12}C$

$T \gtrsim 5 \times 10^8 K$

$Q_{cc} \sim 13$ MeV

$$
\begin{aligned}
^{12}\mathrm{C} + {}^{12}\mathrm{C} &\to {}^{24}\mathrm{Mg} + \gamma \\
&\to {}^{23}\mathrm{Mg} + n \\
&\to {}^{23}\mathrm{Na} + p \\
&\to {}^{20}\mathrm{Ne} + \alpha \\
&\to {}^{16}\mathrm{O} + 2\alpha
\end{aligned}
\qquad
\begin{aligned}
^{16}\mathrm{O} + {}^{16}\mathrm{O} &\to {}^{32}\mathrm{S} + \gamma \\
&\to {}^{31}\mathrm{S} + n \\
&\to {}^{31}\mathrm{P} + p \\
&\to \boxed{^{28}\mathrm{Si}} + \alpha \\
&\to {}^{24}\mathrm{Mg} + 2\alpha
\end{aligned}
$$

$^{16}O + ^{16}O$

$T \gtrsim 10^9 K$

$Q_{OO} \sim 16$ MeV

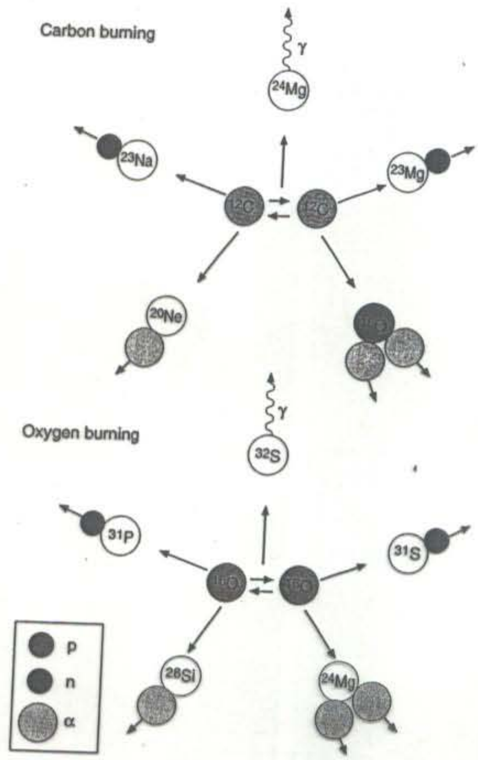


Figure 4.6 The nuclear reactions involved in carbon and in oxygen burning.

(∵ low $E_{coul.}$)

These produce p^+, α, which are captured imm. by heavy elements $\Rightarrow$ isotopes

field)

Does ^{28}Si follow the same scenario?

$$^{28}Si + {}^{28}Si \xrightarrow{?} {}^{56}Fe \; ?$$

C+C 6×10^8K
O+O 10^9K

No!

Coulomb barrier becomes extremly high; another nuclear reaction takes place

e.g. $h\nu \rightsquigarrow$ atom $<$ ion, e^- photoionization

EM binding force

Likewise

$h\nu \rightsquigarrow$ nucleus $<$ photodisintegration

nuclear binding force

e.g. $^{16}O + \alpha \longleftrightarrow {}^{20}Ne + \gamma$

$< 10^9$K $\rightarrow$ but if $\gtrsim 1.5\times10^9 \leftarrow$ (in radiation field)

$\therefore$ ^{28}Si disinteg. $\sim 3\times10^9$K $\rightarrow$ lighter elements

$\rightarrow$ recaptured ....

nuclear statistical equilibrium

not exact $\Rightarrow$ pileup of iron group nuclei (Fe, Co, Ni)

which resist photodisint. until $\lesssim 7\times10^9$K

$^{56}Fe \rightarrow 13\ ^{4}He + 4n$

$\wedge$

$+ 100$ MeV

If $T \uparrow\uparrow\uparrow$, $^{4}He \rightarrow p^{+} + n^{0}$

$\therefore$ Stellar interior has to be between a few T_6 and a few T_9 !

Nuclear reactions that <u>absorb</u> energy from ambient radiation field (in stellar interior) can lead to catastrophic consequences.

Accretion Energy

$$L = \frac{GM}{R}\dot{M}$$

in terms of the Scharzschild radius $R_S \equiv \frac{2GM}{c^2}$

$$\Rightarrow L = \underbrace{\left[\frac{R_S}{2R}\right]}\dot{M}c^2$$

'efficiency; $\uparrow$ as $R \downarrow$

Accretion is highly efficient onto a compact object.

For chemical reactions typically ~ a few eV

e.g., H_2 dissociation, $E \sim 4.48\,eV$

$\therefore \frac{4.48\,eV}{2m_p} \sim 10^{12}\,erg\,g^{-1} \rightarrow 10^{-9}$ eff.

For nuclear reactions typically ~ a few MeV

e.g., $4H \rightarrow He$, $E \sim 7\,MeV$

$\therefore \frac{7\,MeV}{m_p} \sim 10^{19}\,erg\,g^{-1} \rightarrow 10^{-2}$ eff.

For accretion process $E \sim 10^{21}\,erg\,g^{-1}$

Ex. a neutron star $R \sim 15\,km$, $\frac{R_S}{2R} \sim 0.1$

Longair "High-Energy Astrophysic"

When P_{rad} dominates

$$\mathcal{F} = \frac{-dP_{rad}/dr}{\kappa\rho} = \frac{4ac}{3} T^3 \frac{dT}{dr} = \frac{L}{4\pi r^2}$$

$$\frac{dP_{rad}}{dr} \sim \frac{\kappa\rho}{c} \frac{L}{4\pi r^2}$$

By definition

$$\frac{dP_{total}}{dr} = -\rho \frac{Gm}{r^2}$$

$$\Rightarrow \quad \frac{dP_{rad}}{dP_{total}} = \frac{\kappa L}{4\pi c G m}$$

Toward outer layers

$$P_{total} = P_{rad} + P_{gas}$$

$$\downarrow\downarrow P_{total} \Leftarrow \downarrow \qquad \downarrow \quad \Rightarrow dP_{rad} < dP_{total}$$

$$\Rightarrow \boxed{\kappa L \leq 4\pi c G m}$$

At the surface, $m = M$, $P = 0$, always radiative

$$\therefore \boxed{L < \frac{4\pi c G M}{\kappa}}$$

Eddington luminosity limit

Maximum luminosity of a celestial object in balance between the radiation and gravitational force

Eddington Limit

isotropic radiation

Radiation flux $\dfrac{L}{4\pi r^2}$

Momentum flux $\dfrac{L}{4\pi r^2 c}$

$$F_{rad} = \text{Force} = \frac{L}{4\pi r^2 c} \cdot \sigma_T$$

e.g., Thomson scattering

In general, it can be a mixture of processes.

$$F_{grav} = \frac{G M m_p}{r^2}$$

To be stable, $F_{grav} \gtrsim F_{rad}$

or $L \lesssim \left(\dfrac{4\pi G m_p}{\sigma_T} \right) M \equiv \boxed{L_{Edd} \simeq 10^{38} \dfrac{M}{M_\odot} \text{ erg s}^{-1}}$

This is the upper limit on the luminosity of an object of mass M.

For $1 M_\odot$, $L_{Edd} \sim 5 \times 10^4 L_\odot$, $M_{bol} = -7.0$

" 40 " $\qquad M_{bol} = -11.0$

η Carina, $L \sim 5 \times 10^6 L_\odot$, $M_{bol} = -11.6$, $\overset{?}{M} \sim 120 M_\odot$

In general,

$$L_{Edd} \simeq 3.2\times 10^{4} \frac{M}{M_\odot} \frac{\kappa_{e^-}}{\kappa} \, [L_\odot]$$

inequality is violated

L_{Edd} can be exceeded if

① $L \uparrow\uparrow$, e.g., intense thermonuclear burning

② $\kappa \uparrow\uparrow$, e.g. H or He ionization

$\Rightarrow$ Hydrostatic equilibrium can no longer maintained

$\therefore$ need a different heat transfer mechanism

Eddington limit (cont.)

In late 1970s (Humphreys and Davidson, 1979 ApJ 232, 409)

From observations of stars on upper MS

$\rightarrow$ L_{limit} goes nonlinear with T_{eff}

Eddington limit + steady radiation

$T_{eff} \downarrow \Rightarrow L_{limit} \downarrow$ until 10^4 K then

$L_{limit} \approx$ const

They thought $\rightarrow$ sporadic mass loss.

when $T_{eff} \downarrow$, opacity $\uparrow$ due to low-level ions e.g. HI, Fe II begin to appear

$\Rightarrow$ modified limit, $\downarrow$ with $T_{eff} \downarrow$, as observed

\> Langer (1997) : rotation $\uparrow \rightarrow$ disintegrate the star

$$\Omega = \frac{v_{rot}}{v_{crit}} > 1$$ "Omega" limit

$$P_{total} = P_{rad} + P_{gas}$$

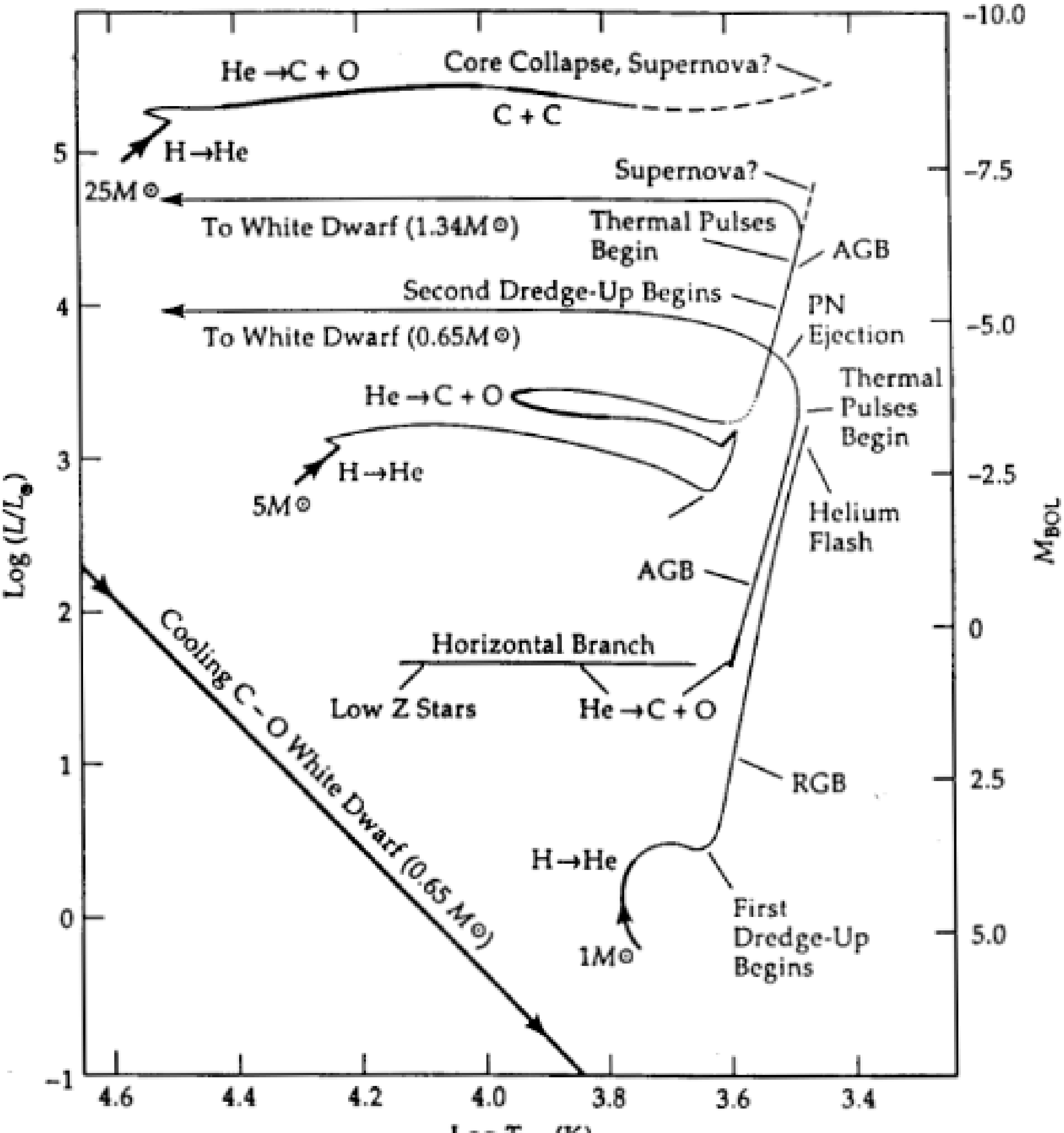
Since $P_{rad} \sim T^4 \sim \dfrac{M^4}{R^4}$

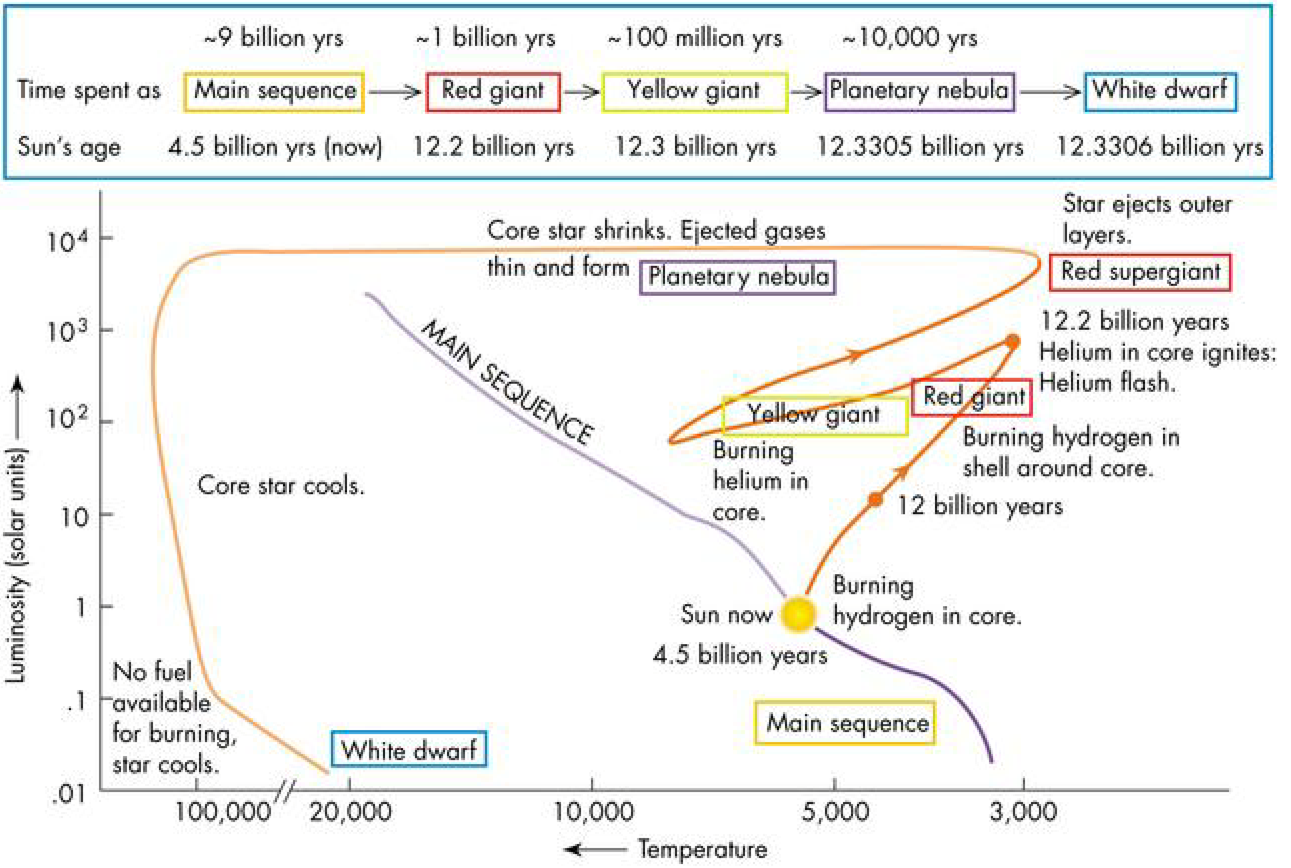
but $P_{total} \sim \dfrac{M^2}{R^4}$

$$\Rightarrow \frac{P_{rad}}{P_{total}} \sim M^2$$

$\therefore$ As stellar mass $\uparrow$, P_{rad} (fraction) $\uparrow$

and γ decreases to $4/3$


He →C + O
Core Collapse, Supernova?
C + C
H→He
25M⊙
Supernova?
To White Dwarf (1.34M⊙)
Thermal Pulses Begin
AGB
Second Dredge-Up Begins
PN Ejection
To White Dwarf (0.65M⊙)
He→C + O
Thermal Pulses Begin
H→He
5M⊙
Helium Flash
AGB
Horizontal Branch
Low Z Stars
He→C + O
Cooling C–O White Dwarf (0.65 M⊙)
RGB
H→He
First Dredge-Up Begins
1M⊙
Log (L/L⊙)
Log Teff (K)
MBOL
5
4
3
2
1
0
−1
4.6
4.4
4.2
4.0
3.8
3.6
3.4
−10.0
−7.5
−5.0
−2.5
0
2.5
5.0



~9 billion yrs
~1 billion yrs
~100 million yrs
~10,000 yrs
Time spent as
Main sequence
Red giant
Yellow giant
Planetary nebula
White dwarf
Sun's age
4.5 billion yrs (now)
12.2 billion yrs
12.3 billion yrs
12.3305 billion yrs
12.3306 billion yrs
Star ejects outer layers.
Core star shrinks. Ejected gases thin and form
Planetary nebula
Red supergiant
12.2 billion years
Helium in core ignites: Helium flash.
MAIN SEQUENCE
Yellow giant
Red giant
Burning helium in core.
Burning hydrogen in shell around core.
Core star cools.
12 billion years
Burning hydrogen in core.
Sun now
4.5 billion years
No fuel available for burning, star cools.
White dwarf
Main sequence
Luminosity (solar units)
10^4
10^3
10^2
10
1
.1
.01
100,000
20,000
10,000
5,000
3,000
Temperature