## Stellar Formation and Evolution

煋尾形成白演化

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$\checkmark$ What is a "star"?
$\checkmark$ How hot is the surface of the Sun? How is this known? The Sun is gaseous, so how come it has a "surface"?
$\checkmark$ How hot is the center of the Sun? How is this known?
$\checkmark$ How the sun derive its energy to shine? How long can the Sun remain as a shining body? How are these known?
$\checkmark$ Describe the radial structure of the Sun. How is this know?

## Stellar Formation and Evolution --- Syllabus

Instructor: Professor Wen-Ping Chen
Office: 906
Class Time: Tuesday 3 to 6 pm
Classroom: Room 914
This course deals with the time variations of the structures of a star's interior (and atmosphere). We will discuss the important physical processes governing the life of a star --- from its birth out of a dense, cold molecular cloud core, to shining with the star's own thermonuclear fuels, to rapid changes in structures when these fuels are no longer available, to the end of a star's life, with matter in extremely compact states.

What it may take for a star billions of years, will take us one semester to cover the following subjects:

- Observational Properties of Stars
- Molecular Clouds and the Interstellar Medium
- Cloud Collapse and Fragmentation
- Stars and Statistical Physics
- Protostars and Jets
- Circumstellar Disks and Planet Formation
- Evolution onto the Main Sequence
- Binaries and Star Clusters
- On the Main Sequence --- Nuclear Reactions
- Effects of Rotation
- Instabilities --- Thermally, Dynamically and Convectively
- Post-MS Evolution of Low-Mass Stars --- RG, AGB, HB, PNe
- Post-MS Evolution of Massive Stars --- SN and SNR
- Mass Loss, Stellar Pulsation and Cepheid Variables
- Compact Objects --- White Dwarfs, Neutron Stars, and Black holes
- Violent End Products --- Supernovae and others


Text: "An Introduction to the Theory of Stellar Structure and Evolution", by Dina Prialnik, Cambridge, $2^{\text {nd }}$ Ed. 2009
In addition to written midterm ( $30 \%$ grade) and final (30\%) exams, there will be homework assignments, plus in-class exercises or projects ( $35 \%$ ).

## References

All the references you have found useful for the course Stellar Atmosphere and Structure will be also of use in this course．The following are the ones I refer to often．
$\checkmark \quad$ Physics of Stellar Evolution and Cosmology，by H．Goldberg \＆Michael Scadron，1982，Gordon and Breach
$\checkmark$ Black Holes，White Dwarfs，and Neutron Stars，by Stuart L．Shapiro \＆Saul A．Teukolsky，1983，Wiley
$\checkmark$ Stellar Structure and Evolution，by R．Kippenhahn \＆W．Weigert，1990，Springer－Verlag
$\checkmark$ Stellar Structure and Evolution，by Huang，R．Q．黃潤乾，Guoshin，1990，originally published in Chinese（恆星物理）
$\checkmark \quad$ Introduction to Stellar Astrophysics，Vol 3 －－－Stellar Structure and Evolution，by Erika Bohm－Vitense，1992，Cambridge
$\checkmark \quad$ The Physics of Stars，by A．C．Phillips，1994，John Wiley \＆Sons
$\checkmark$ Stellar Evolution，by Amos Harpaz，A K Peters， 1994
$\checkmark$ The Stars－－－Their Structure and Evolution，R．J．Tayler，1994，Cambridge
$\checkmark \quad$ Supernovae and Nucleosynthesis，by David Arnett，1996，Princeton
$\checkmark \quad$ Theoretical Astrophysics，Vol II：Stars and Stellar Systems by Padmanabhan，T．，a hefty，mathematical 3 volume set；comprehensive coverage of basic astrophysical processes in vol．1，stars in vol．2，and galaxies and cosmology in vol．3，2001，Cambridge
$\checkmark \quad$ The Formation of Stars，by Steven Stahler \＆Francesco Palla，2004，Wiley－VCH
$\checkmark$ Evolution of Stars and Stellar Populations，by Maurizio Salaris \＆Santi，Cassisi，2005，Wiley
$\checkmark$ The Formation of Stars，by Steven W．Stahler \＆Francesco Palla，2004，Wiley
$\checkmark$ From Dust to Stars，by Norbert S．Schulz，2005，Springer
$\checkmark \quad$ Stellar Physics，2：Stellar Evolution and Stability，by Bisnovatyi－Kogan，2 ${ }^{\text {nd }}$ Ed．，2010，Springer（translated from Russian）
$\checkmark$ Astrophysics of Planet Formation，by Philip J．Armitage，2010，Cambridge
$\checkmark$ Principles of Star Formation，by Peter Bodenheimer，2011，Springer
$\checkmark$ An Introduction to Star Formation，by Derek Ward－Thompson \＆Anthony P．Whitworth，2011，Cambridge
$\checkmark$ Stellar Evolution Physics，by Icko Iben， 2013 （two volumes），Cambridge
$\checkmark \quad$ Star Formation，by Mark R．Krumholz，2017，World Scientific

For star formation，the book＂Molecular Clouds and Star Formation＂，edited by Chi Yuan（袁旅）\＆Junhan You（尤峻漢）and published by World Scientific in 1993，should be a good reference．Unfortunately this book is currently out of print，but Prof Yuan kindly donated his editor copy．

## Class Schedule

| $\#$ | Date | Comments | \# | Date | Comments |
| :--- | :--- | :--- | ---: | :--- | :--- |
| 01 | $02 / 19$ |  | 10 | $04 / 23$ |  |
| 02 | $02 / 26$ |  | 11 | $04 / 30$ |  |
| 03 | $03 / 05$ |  | 12 | $05 / 07$ |  |
| 04 | $03 / 12$ |  | 13 | $05 / 14$ |  |
| 05 | $03 / 19$ |  | 14 | $(05 / 21)$ | To be made up |
| 06 | $03 / 26$ |  | 15 | $05 / 28$ |  |
| 07 | $04 / 02$ |  | 16 | $06 / 04$ |  |
| 08 | $04 / 09$ |  | 17 | $06 / 11$ |  |
| 09 | $04 / 16$ | Midterm | 18 | $06 / 18$ | Final |

## Course Goals

- To know the properties of various phases of the interstellar matter;
- To understand how stars form out of molecular clouds; under what conditions;
- To understand the physical properties of stars, and to know how these properties change with time as a star evolves;
- To understand the basic physics underlying complex stellar evolution models;
- To know how to interpret observational parameters of stars;
- To understand how stars of different masses evolve and what the end products of their evolution are.


## Stellar structure:

balance of forces

## Stellar evolution:

(con)sequence of thermonuclear reactions in different parts of a star, and at different epochs as the star ages

## STRDMAR

 EVOLUTIONInterstellar Matter

Planetary Nebula

## Often used fundamental constants

## Physical

$a$ radiation density constant $7.55 \times 10^{-16} \mathrm{~J} \mathrm{~m}^{-3} \mathrm{~K}^{-4}$
$c$ velocity of light $\quad 3.00 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$
$G$ gravitational constant $\quad 6.67 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}$
$h$ Planck's constant $\quad 6.62 \times 10^{-34} \mathrm{~J} \mathrm{~s}$
$k \quad$ Boltzmann's constant $\quad 1.38 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$
$m_{e}$ mass of electron $\quad 9.11 \times 10^{-31} \mathrm{~kg}$
$m_{H}$ mass of hydrogen atom $\quad 1.67 \times 10^{-27} \mathrm{~kg}$
$N_{A}$ Avogardo's number $\quad 6.02 \times 10^{23} \mathrm{~mol}^{-1}$
$\sigma \quad$ Stefan Boltzmann constant $5.67 \times 10^{-8} \mathrm{~W} \mathrm{~m}^{-2} \mathrm{~K}^{-4}(=\mathrm{ac} / 4)$
$R \quad$ gas constant $\left(k / m_{H}\right) \quad 8.26 \times 10^{3} \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~kg}^{-1}$
$e \quad$ charge of electron $\quad 1.60 \times 10^{-19} \mathrm{C}$

## Astronomical

| $L_{\odot}$ | Solar luminosity | $3.86 \times 10^{26} \mathrm{~W}$ |
| :--- | :--- | :--- |
| $M_{\odot}$ | Solar mass | $1.99 \times 10^{30} \mathrm{~kg}$ |
| $T_{\text {eff } \odot}$ | Solar effective temperature | 5780 K |
| $\mathrm{~T}_{\mathrm{c} \odot}$ | Solar Central temperature | $1.6 \times 10^{7} \mathrm{~K}($ theoretical $)$ |
| $R_{\odot}$ | Solar radius | $6.96 \times 10^{8} \mathrm{~m}$ |
| $\mathrm{~m}_{\odot}$ | apparent mag of Sun | $-26.7 \mathrm{mag}(\mathrm{V})$ |
| $\mathrm{M}_{\odot}$ | absolute mag of Sun | $+4.8 \mathrm{mag}(\mathrm{V})$ |
| $\theta$ | apparent size of Sun | $32^{\prime}$ |
| $\langle\rho\rangle$ | mean density of Sun | $1.4 \mathrm{~g} \mathrm{~cm}^{-3}$ |
| $(B-V)_{\odot}$ | Color of the Sun | $0.6 \mathrm{mag}^{2}$ |
| Parsec | unit of distance | $3.09 \times 10^{16} \mathrm{~m}$ |



## Galactic Ecology

Properties of Stars

## Vocabulary

- Luminosity $\left[\mathrm{erg} \mathrm{s}^{-1}\right] L=$ bolometric luminosity $=$ power
- Spectral luminosity $\left[\mathrm{erg} \mathrm{s}^{-1} \mu \mathrm{~m}^{-1}\right] \boldsymbol{L}_{\lambda} \quad d \lambda=-\left(c / v^{2}\right) d v$
- flux $\left[\mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2}\right] f$
- flux density $\left[\mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2} \mu \mathrm{~m}^{-1}\right.$ ] $f_{\lambda}$ or $f_{v}$

$$
1 \text { Jansky }(\mathrm{Jy})=10^{-23}\left[\mathrm{erg} \mathrm{~s}^{-1} \mathrm{~cm}^{-2} \mathrm{~Hz}^{-1}\right] \quad f\left(m_{V}=0\right)=3640 \mathrm{Jy}
$$

- Brightness/intensity $\left[\mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2} \mathrm{sr}^{-1}\right] \boldsymbol{B}$
- Specific intensity $\left[\mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2} \mathrm{sr}^{-1} \mathrm{~Hz}^{-1}\right] \boldsymbol{I}_{v}$
- Energy density $\left[\mathrm{erg} \mathrm{cm}^{-3}\right] \boldsymbol{u}=(4 \pi / \mathrm{c}) \mathrm{J}$
- $\mathbf{J}=$ mean intensity $=(1 / 4 \pi) \int I d \Omega$

$$
m_{\mathrm{AB}}=-2.5 \log _{10}\left(\frac{f_{v}}{3631 \mathrm{Jy}}\right)
$$

- Magnitude ... apparent, absolute, bolometric, AB

$$
S_{v}[\mu \mathrm{Jy}]=10^{(23.9-\mathrm{AB}) / 2.5}
$$



## Observable properties of stars

Basic parameters to compare between theories and observations

- Mass (M)
- Luminosity ( $L$ )
- Radius (R)
- Effective temperature ( $T_{\mathrm{e}}$ ) $\quad L=4 \pi R^{2} \sigma T_{e}^{4}$
- Distance $\rightarrow$ measured flux $\quad F=L / 4 \pi d^{2}$
$M, R, L$ and $T_{\mathrm{e}}$ not independent
- $L$ and $\mathrm{T}_{\text {eff }} \quad$ Hertzsprung-Russell (HR) diagram or color-magnitude diagram (CMD)
- $L$ and $M$ mass-luminosity relation


$$
\mathcal{M}_{\text {Jupiter }} \sim 0.001 \mathcal{M}_{\odot}
$$

## Stars:

$$
\mathrm{M}>0.08 \mathcal{M}_{\odot}
$$

## Brown Dwarfs:

$0.08 \mathcal{M}_{\odot}>\mathrm{M}>13 \mathcal{M}_{\mathrm{J}}$

## Planet-mass Objects:

$$
\mathrm{M}<13 \mathcal{M}_{\mathrm{J}}
$$

## To measure the stellar distance

- Nearest stars $d>1 \mathrm{pc} \rightarrow p<1$ "
- For a star at $d=100 \mathrm{pc}, p=0.01$ "
- Ground-based observations angular resolution $\sim 1$ "; HST has 0.05 "
- Hipparcos measured the parallaxes of $10^{5}$ bright stars with $p \sim 0.001$ " $\rightarrow$ reliable distance determinations up to $d=100 \mathrm{pc}$
$\rightarrow$ ~100 stars with good parallax distances
Gaia is measuring $10^{9}$ stars!



## Exercise

1. What is Gaia as a space telescope mission?
2. What is the size of the telescope?
3. When was it launched? What kind of an orbit does it have? How long is it expect to last? How faint does it go?
4. What its main mission (what does it measure)?

## Otherwise, the distance is estimated

- Stars with the same spectra are assumed to have identical set of physical parameters (spectroscopic parallax). For example, a G2V star should have the same absolute magnitude as the Sun.
- By comparison of the apparent brightness of an object with known brightness of that particular kind of objects

$$
m_{\lambda}-M_{\lambda}=5 \log d-5+A_{\lambda}(d)
$$

$A_{\lambda}$ is usually unknown; it depends on the intervening dust grains that scatter and absorb the star light, and also depends on the distance to the object.

- Main-sequence fitting; moving-cluster method; Cepheid variables
- Other methods for Galactic molecular clouds, galaxies, etc.


FIg. 1.-Normalized interstellar extinction curves from the far-IR through the UV. Several general features of the curves are noted. The solid and dotted curves are estimates for the case $R \equiv A(V) / E(B-V)=3.1$ derived in the Appendix of this paper and by Cardelli et al. (1989), respectively. The dashed curve shows the average Galactic UV extinction curve from Seaton (1979).

## To measure the stellar size

- Angular diameter of sun at 10 pc $=2 R_{\odot} / 10 \mathrm{pc}=5 \times 10^{-9}$ radians $=10^{-3} \operatorname{arcsec}$
- Even the $\operatorname{HST}(0.05$ ") barely capable of measuring directly the sizes of stars, except for the nearest supergiants
- Radii of $\sim 600$ stars measured with techniques such as interferometry, (lunar) occultation or for eclipsing binaries



## Lunar occultation <br> Beaver \& Eitter (1979)

$\underset{\theta=0 \prime \prime}{\text { Fig. 1.-A }}$ A comparison of the (crosses) observed points and the (line) theoretical pattern for the Aldebaran $\lambda=7460 \AA$ record with $\theta=0$ " 020 .

Optical interferometery CHARA
White et al (2013)


## To measure the stellar temperature

- What is $T_{\text {eff }}$ ? What is the "surface" of a star?
- What is Tanyway? Temperature is often defined by other physical quantities through an equation ("law") (by radiation or by particles) blackbody, radiation, color, excitation, ionization, kinetic, electron, conductive ...
- Only in thermal equilibrium are all these temperatures the same.
- Photometry (spectral energy distribution) gives a rough estimate of $T$, e.g., fluxes/magnitudes measured at different wavelengths, such as the "standard" Johnson system UBVRI
- There are many photometric systems,

| Band | U | B | V | R | I |
| :--- | ---: | ---: | ---: | :--- | :--- |
| $\lambda / \mathrm{nm}$ | 365 | 445 | 551 | 658 | 806 |
| $\Delta \lambda / n m$ | 66 | 94 | 88 | 138 | 149 | using broad bands, intermediate bands, special bands, at optical or infrared wavelengths, etc.





Running (slope) between $B$ and $V$ bands, i.e., the ( $B-V$ ) color (index) $\rightarrow$ photospheric temperature

The larger the value of $(B-V)$, the redder (cooler) the star.


Figure 1.8 Theoretical monochromatic flux emerging form an A type star with $T_{\text {eff }}=8000 \mathrm{~K}$. The first four Balmer absorption lines, as well as the Balmer jump, are identified in this figure. Thousands of other absorption atomic lines can also be seen. This theoretical flux was obtained with the Phoenix stellar atmosphere code (Hauschildt, P.H., Allard, F. and Baron, E., The Astrophysical Journal, 512, 377 (1999)) while using the elemental abundances found in the Sun. The flux at the surface of a blackbody with $T=8000 \mathrm{~K}$ (dotted curve) is also shown.

- Calibration for $B-V=f\left(T_{e}\right)$
- The observed $(B-V)$ must be corrected for interstellar extinction in order to derive the stellar intrinsic $(B-V)_{0}$
- Need more accurate determination of $T$ by spectroscopy and stellar atmosphere models, e.g., the Kurucz's model



## Color Excess

$$
E_{B-V}=(B-V)_{\mathrm{obs}}-(B-V) 0
$$

$$
(B-V)_{\odot}=0.656 \pm 0.005
$$




## Different temperature, elements (at different excitation and ionization levels) $\rightarrow$ different set of spectral lines



## Line ratios $\rightarrow$ Temperature



I --- neutral atoms; II --- ionized once; III --- ionized twice; ...
e.g., $\mathrm{HI}=\mathrm{H}^{0} \ldots \mathrm{H}$ II $=\mathrm{H}^{+} .$. He III $=\mathrm{He}^{+2} \ldots \mathrm{Fe}$ XXVI $=\mathrm{Fe}^{+25}$

Hot stars --- peaked at short wavelengths (UV); mainly He lines, some H lines

Warm stars --- peaked in the visible wavelengths; $H$ lines prominent


Cool stars --- peaked at long wavelengths (IR); molecular lines/bands


## Brown dwarfs and Planetary Objects




Using imaging photometry (time saving) to trace spectral features


Figure 16.15 Near-infrared color-color plot of M dwarfs (filled circles), L dwarfs (open circles), and T dwarfs (filled triangles). The objects are from a variety of regions. Note that the typical measurement errors for the L- and T-dwarfs are quite large, about 0.13 mag.

## To measure the stellar luminosity

- Absolute Magnitude $M$ defined as apparent magnitude of a star if it were placed at a distance of 10 pc

$$
m_{\lambda}-M_{\lambda}=5 \log \left(d_{\mathrm{pc}}\right)-5
$$

But there is extinction $\ldots m_{\lambda}-M_{\lambda}=5 \log \left(d_{\mathrm{pc}}\right)-5+A_{\lambda}$
Bolometric magnitude - the absolute magnitude integrated over all wavelengths. We define the bolometric correction
Bolometric Correction

$$
B C=M_{b o l}-M_{v}
$$

$$
M_{\text {bol }}^{\odot}=+4.74
$$

is a function of the spectral type (min at the F type, why?) and luminosity of a star.
That is, we can apply BC (always negative, why?) to a star to estimate its luminosity (from the photosphere).

Apparent Magnitude $m=-2.5 \log$ (Flux) + ZeroPoint

- The Vega system: 0.0 mag (latest $\sim 0.3 \mathrm{mag}$ ) at every Johnson band
- Gunn system: no Vega; use of F subdwarfs as standards (metal poor so smooth spectra), e.g., BD + 174708
- The AB system: $\mathrm{AB}_{v}=-2.5 \log _{10} f_{v}-48.60$
- STMAG system: used for HST photometry

$$
\text { STMAG }_{\lambda}=-2.5 \log _{10} f_{\lambda}-21.1
$$

Table 7.5. Filter wavelengths, bandwidths, and flux densities for Vega. ${ }^{a}$

| Filter <br> name | $\lambda_{\text {iso }} b$ <br> $(\mu \mathrm{~m})$ | $\Delta \lambda^{c}$ <br> $(\mu \mathrm{~m})$ | $F_{\lambda}$ <br> $\left(\mathrm{W} \mathrm{m}^{-2} \mu_{\mathrm{m}}{ }^{-1}\right)$ | $F_{\nu}$ <br> $(\mathrm{Jy})$ | $N_{\phi}$ <br> $($ photons s <br> $\left.\mathrm{m}^{-1} \mathrm{~m}^{-2} \mu \mathrm{~m}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $V$ | $0.5556^{d}$ | $\ldots$ | $3.44 \times 10^{-8}$ | 3540 | $9.60 \times 10^{10}$ |
| $J$ | 1.215 | 0.26 | $3.31 \times 10^{-9}$ | 1630 | $2.02 \times 10^{10}$ |
| $H$ | 1.654 | 0.29 | $1.15 \times 10^{-9}$ | 1050 | $9.56 \times 10^{9}$ |
| $K_{s}$ | 2.157 | 0.32 | $4.30 \times 10^{-10}$ | 667 | $4.66 \times 10^{9}$ |
| $K$ | 2.179 | 0.41 | $4.14 \times 10^{-10}$ | 655 | $4.53 \times 10^{9}$ |
| $L$ | 3.547 | 0.57 | $6.59 \times 10^{-11}$ | 276 | $1.17 \times 10^{9}$ |
| $L^{\prime}$ | 3.761 | 0.65 | $5.26 \times 10^{-11}$ | 248 | $9.94 \times 10^{8}$ |
| $M$ | 4.769 | 0.45 | $2.11 \times 10^{-11}$ | 160 | $5.06 \times 10^{8}$ |
| 8.7 | 8.756 | 1.2 | $1.96 \times 10^{-12}$ | 50.0 | $8.62 \times 10^{7}$ |
| $N$ | 10.472 | 5.19 | $9.63 \times 10^{-13}$ | 35.2 | $5.07 \times 10^{7}$ |
| 11.7 | 11.653 | 1.2 | $6.31 \times 10^{-13}$ | 28.6 | $3.69 \times 10^{7}$ |
| $Q$ | 20.130 | 7.8 | $7.18 \times 10^{-14}$ | 9.70 | $7.26 \times 10^{6}$ |

$$
\begin{aligned}
1 \text { Jansky } & =10^{-23} \mathrm{erg} \mathrm{~s}^{-1} \mathrm{~cm}^{-2} \mathrm{~Hz}^{-1} \\
& =1.51 \times 10^{7} \text { photons s}
\end{aligned}
$$

| Band | $\lambda_{0}$ | $d \lambda / \lambda$ | $f_{v}(m=0)$ | Reference |
| :--- | ---: | :---: | :--- | :--- |
|  | $\mu \mathrm{m}$ | Jy |  |  |
| U | 0.36 | 0.15 | 1810 | Bessel (1979) |
| B | 0.44 | 0.22 | 4260 | Bessel (1979) |
| V | 0.55 | 0.16 | 3640 | Bessel (1979) |
| R | 0.64 | 0.23 | 3080 | Bessel (1979) |
| I | 0.79 | 0.19 | 2550 | Bessel (1979) |
| J | 1.26 | 0.16 | 1600 | Campins, Reike, \& Lebovsky (1985) |
| H | 1.60 | 0.23 | 1080 | Campins, Reike, \& Lebovsky (1985) |
| K | 2.22 | 0.23 | 670 | Campins, Reike, \& Lebovsky (1985) |
| g | 0.52 | 0.14 | 3730 | Schneider, Gunn, \& Hoessel (1983) |
| r | 0.67 | 0.14 | 4490 | Schneider, Gunn, \& Hoessel (1983) |
| i | 0.79 | 0.16 | 4760 | Schneider, Gunn, \& Hoessel (1983) |
| z | 0.91 | 0.13 | 4810 | Schneider, Gunn, \& Hoessel (1983) |

## Notes

${ }^{a}$ Cohen et al. [1] recommend the use of Sirius rather than Vega as the photometric standard for $\lambda>20 \mu \mathrm{~m}$ because of the infrared excess of Vega at these wavelengths. The magnitude of Vega depends on the photometric system used, and it is either assumed to be 0.0 mag or assumed to be 0.02 or 0.03 mag for consistency with the visual magnitude.
${ }^{b}$ The infrared isophotal wavelengths and flux densities (except for $K_{s}$ ) are taken from Table 1 of [1], and they are based on the UKIRT filter set and the atmospheric absorption at Mauna Kea. See Table 2 of [1] for the case of the atmospheric absorption at Kitt Peak. The isophotal wavelength is defined by $F\left(\lambda_{\text {iso }}\right)=\int F(\lambda) S(\lambda) d \lambda / \int S(\lambda) d \lambda$, where $F(\lambda)$ is the flux density of Vega and $S(\lambda)$ is the (detector quantum efficiency) $\times$ (filter transmission) $\times$ (optical efficiency) $\times$ (atmospheric transmission) [2]. $\lambda_{\text {iso }}$ depends on the spectral shape of the source and a correction must be applied for broadband photometry of sources that deviate from the spectral shape of the standard star [3]. The flux density and $\lambda_{\text {iso }}$ for $K_{s}$ were calculated here. For another filter, $K^{\prime}$, at $2.11 \mu \mathrm{~m}$, see [4].
${ }^{c}$ The filter full width at half maximum.
${ }^{d}$ The wavelength at $V$ is a monochromatic wavelength; see [5].

## References

1. Cohen, M. et al. 1992, AJ, 104, 1650
2. Golay, M. 1974, Introduction to Astronomical Photometry (Reidel, Dordrecht), p. 40
3. Hanner, M.S., et al. 1984, AJ, 89, 162
4. Wainscoat, R.J., \& Cowie, L.L. 1992, AJ, 103, 332
5. Hayes, D.S. 1985, in Calibration of Fundamental Stellar Quantities, edited by D.S. Hayes, et al., Proc. IAU Symp. No. 111 (Reidel, Dordrecht), p. 225

## Exercise

Sirius, the brightest star in the night sky, has been measured $m_{B}=-1.47, m_{V}=-1.47$. The star has an annual parallax of $0.379^{\prime \prime} / \mathrm{yr}$.

1. What is its distance in parsec?
2. What is its absolute V-band magnitude?
3. From the absolute magnitude, what spectral type can be inferred for Sirius?
4. From the observed (B-V) color, what spectral type can be inferred?
5. What kinds of uncertainties/assumptions are associated with the above estimations?

Table 15.7. Calibration of $M K$ spectral types.

| Sp | $M(V)$ | $B-V$ | $U-B$ | $V-R$ | $R-I$ | $T_{\text {eff }}$ | BC |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| MAIN SEQUENCE, V |  |  |  |  |  |  |  |
| O5 | -5.7 | -0.33 | -1.19 | -0.15 | -0.32 | 42000 | -4.40 |
| O9 | -4.5 | -0.31 | -1.12 | -0.15 | -0.32 | 34000 | -3.33 |
| B0 | -4.0 | -0.30 | -1.08 | -0.13 | -0.29 | 30000 | -3.16 |
| B2 | -2.45 | -0.24 | -0.84 | -0.10 | -0.22 | 20900 | -2.35 |
| B5 | -1.2 | -0.17 | -0.58 | -0.06 | -0.16 | 15200 | -1.46 |
| B8 | -0.25 | -0.11 | -0.34 | -0.02 | -0.10 | 11400 | -0.80 |
| A0 | +0.65 | -0.02 | -0.02 | 0.02 | -0.02 | 9790 | -0.30 |
| A2 | +1.3 | +0.05 | +0.05 | 0.08 | 0.01 | 9000 | -0.20 |
| A5 | +1.95 | +0.15 | +0.10 | 0.16 | 0.06 | 8180 | -0.15 |
| F0 | +2.7 | +0.30 | +0.03 | 0.30 | 0.17 | 7300 | -0.09 |
| F2 | +3.6 | +0.35 | 0.00 | 0.35 | 0.20 | 7000 | -0.11 |
| F5 | +3.5 | +0.44 | -0.02 | 0.40 | 0.24 | 6650 | -0.14 |
| F8 | +4.0 | +0.52 | +0.02 | 0.47 | 0.29 | 6250 | -0.16 |
| G0 | +4.4 | +0.58 | +0.06 | 0.50 | 0.31 | 5940 | -0.18 |
| G2 | +4.7 | +0.63 | +0.12 | 0.53 | 0.33 | 5790 | -0.20 |
| G5 | +5.1 | +0.68 | +0.20 | 0.54 | 0.35 | 5560 | -0.21 |
| G8 | +5.5 | +0.74 | +0.30 | 0.58 | 0.38 | 5310 | -0.40 |
| K0 | +5.9 | +0.81 | +0.45 | 0.64 | 0.42 | 5150 | -0.31 |
| K2 | +6.4 | +0.91 | +0.64 | 0.74 | 0.48 | 4830 | -0.42 |
| K5 | +7.35 | +1.15 | +1.08 | 0.99 | 0.63 | 4410 | -0.72 |
| M0 | +8.8 | +1.40 | +1.22 | 1.28 | 0.91 | 3840 | -1.38 |
| M2 | +9.9 | +1.49 | +1.18 | 1.50 | 1.19 | 3520 | -1.89 |
| M5 | +12.3 | +1.64 | +1.24 | 1.80 | 1.67 | 3170 | -2.73 |
| GIANTS |  |  |  |  |  |  |  |
| G5 | +0.9 | +0.86 | +0.56 | 0.69 | 0.48 | 5050 | -0.34 |
| G8 | +0.8 | +0.94 | +0.70 | 0.70 | 0.48 | 4800 | -0.42 |
| K0 | +0.7 | +1.00 | +0.84 | 0.77 | 0.53 | 4660 | -0.50 |
| K2 | +0.5 | +1.16 | +1.16 | 0.84 | 0.58 | 4390 | -0.61 |
| K5 | -0.2 | +1.50 | +1.81 | 1.20 | 0.90 | 4050 | -1.02 |
| M0 | -0.4 | +1.56 | +1.87 | 1.23 | 0.94 | 3690 | -1.25 |
| M2 | -0.6 | +1.60 | +1.89 | 1.34 | 1.10 | 3540 | -1.62 |
| M5 | -0.3 | +1.63 | +1.58 | 2.18 | 1.96 | 3380 | -2.48 |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

Table 15.7. (Continued.)

| $S p$ | $M(V)$ | $B-V$ | $U-B$ | $V-R$ | $R-I$ | $T_{\text {eff }}$ | $B C$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| SUPERGIANTS, I |  |  |  |  |  |  |  |
| O9 | -6.5 | -0.27 | -1.13 | -0.15 | -0.32 | 32000 | -3.18 |
| B2 | -6.4 | -0.17 | -0.93 | -0.05 | -0.15 | 17600 | -1.58 |
| B5 | -6.2 | -0.10 | -0.72 | 0.02 | -0.07 | 13600 | -0.95 |
| B8 | -6.2 | -0.03 | -0.55 | 0.02 | 0.00 | 11100 | -0.66 |
| A0 | -6.3 | -0.01 | -0.38 | 0.03 | 0.05 | 9980 | -0.41 |
| A2 | -6.5 | +0.03 | -0.25 | 0.07 | 0.07 | 9380 | -0.28 |
| A5 | -6.6 | +0.09 | -0.08 | 0.12 | 0.13 | 8610 | -0.13 |
| F0 | -6.6 | +0.17 | +0.15 | 0.21 | 0.20 | 7460 | -0.01 |
| F2 | -6.6 | +0.23 | +0.18 | 0.26 | 0.21 | 7030 | -0.00 |
| F5 | -6.6 | +0.32 | +0.27 | 0.35 | 0.23 | 6370 | -0.03 |
| F8 | -6.5 | +0.56 | +0.41 | 0.45 | 0.27 | 5750 | -0.09 |
| G0 | -6.4 | +0.76 | +0.52 | 0.51 | 0.33 | 5370 | -0.15 |
| G2 | -6.3 | +0.87 | +0.63 | 0.58 | 0.40 | 5190 | -0.21 |
| G5 | -6.2 | +1.02 | +0.83 | 0.67 | 0.44 | 4930 | -0.33 |
| G8 | -6.1 | +1.14 | +1.07 | 0.69 | 0.46 | 4700 | -0.42 |
| K0 | -6.0 | +1.25 | +1.17 | 0.76 | 0.48 | 4550 | -0.50 |
| K2 | -5.9 | +1.36 | +1.32 | 0.85 | 0.55 | 4310 | -0.61 |
| K5 | -5.8 | +1.60 | +1.80 | 1.20 | 0.90 | 3990 | -1.01 |
| M0 | -5.6 | +1.67 | +1.90 | 1.23 | 0.94 | 3620 | -1.29 |
| M2 | -5.6 | +1.71 | +1.95 | 1.34 | 1.10 | 3370 | -1.62 |
| M5 | -5.6 | +1.80 | $+1.60:$ | 2.18 | 1.96 | 2880 | -3.47 |

Table 15.8. Calibration of MK spectral types. ${ }^{a}$

| $S p$ | $\mathcal{M} / \mathcal{M}_{\odot}$ | $R / R_{\odot}$ | $\log (g / g \odot)$ | $\log \left(\bar{\rho} / \bar{\rho}_{\odot}\right)$ | $v_{\text {rot }}\left(\mathrm{km} \mathrm{s}^{-1}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| MAIN SEQUENCE, V |  |  |  |  |  |
| O3 | 120 | 15 | -0.3 | -1.5 |  |
| O5 | 60 | 12 | -0.4 | -1.5 |  |
| O6 | 37 | 10 | -0.45 | -1.45 |  |
| O8 | 23 | 8.5 | -0.5 | -1.4 | 200 |
| B0 | 17.5 | 7.4 | -0.5 | -1.4 | 170 |
| B3 | 7.6 | 4.8 | -0.5 | -1.15 | 190 |
| B5 | 5.9 | 3.9 | -0.4 | -1.00 | 240 |
| B8 | 3.8 | 3.0 | -0.4 | -0.85 | 220 |
| A0 | 2.9 | 2.4 | -0.3 | -0.7 | 180 |
| A5 | 2.0 | 1.7 | -0.15 | -0.4 | 170 |
| F0 | 1.6 | 1.5 | -0.1 | -0.3 | 100 |
| F5 | 1.4 | 1.3 | -0.1 | -0.2 | 30 |
| G0 | 1.05 | 1.1 | -0.05 | -0.1 | 10 |
| G5 | 0.92 | 0.92 | +0.05 | -0.1 | $<10$ |
| K0 | 0.79 | 0.85 | +0.05 | +0.1 | $<10$ |
| K5 | 0.67 | 0.72 | +0.1 | +0.25 | $<10$ |
| M0 | 0.51 | 0.60 | +0.15 | +0.35 |  |
| M2 | 0.40 | 0.50 | +0.2 | +0.8 |  |
| M5 | 0.21 | 0.27 | +0.5 | +1.0 |  |
| M8 | 0.06 | 0.10 | +0.5 | +1.2 |  |

Table 15.8. (Continued.)

| $S p$ | $\mathcal{M} / \mathcal{M}_{\odot}$ | $R / R_{\odot}$ | $\log \left(g / g_{\odot}\right)$ | $\log \left(\bar{\rho} / \bar{\rho}_{\odot}\right)$ | $v_{\text {rot }}\left(\mathrm{km} \mathrm{s}^{-1}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| GIANTS, III |  |  |  |  |  |
| B0 | 20 | 15 | -1.1 | -2.2 | 120 |
| B5 | 7 | 8 | -0.95 | -1.8 | 130 |
| A0 | 4 | 5 |  | -1.5 | 100 |
| G0 | 1.0 | 6 | -1.5 | -2.4 | 30 |
| G5 | 1.1 | 10 | -1.9 | -3.0 | $<20$ |
| K0 | 1.1 | 15 | -2.3 | -3.5 | $<20$ |
| K5 | 1.2 | 25 | -2.7 | -4.1 | $<20$ |
| M0 | 1.2 | 40 | -3.1 | -4.7 |  |
| SUPERGIANTS, I |  |  |  |  |  |
| O5 | 70 | $30:$ | -1.1 | -2.6 |  |
| O6 | 40 | $25:$ | -1.2 | -2.6 |  |
| O8 | 28 | 20 | -1.2 | -2.5 | 125 |
| B0 | 25 | 30 | -1.6 | -3.0 | 102 |
| B5 | 20 | 50 | -2.0 | -3.8 | 40 |
| A0 | 16 | 60 | -2.3 | -4.1 | 40 |
| A5 | 13 | 60 | -2.4 | -4.2 | 38 |
| F0 | 12 | 80 | -2.7 | -4.6 | 30 |
| F5 | 10 | 100 | -3.0 | -5.0 | $<25$ |
| G0 | 10 | 120 | -3.1 | -5.2 | $<25$ |
| G5 | 12 | 150 | -3.3 | -5.3 | $<25$ |
| K0 | 13 | 200 | -3.5 | -5.8 | $<25$ |
| K5 | 13 | 400 | -4.1 | -6.7 | $<25$ |
| M0 | 13 | 500 | -4.3 | -7.0 |  |
| M2 | 19 | 800 | -4.5 | -7.4 |  |

Note
${ }^{a}$ A colon indicates an uncertain value.

Table 15.9. Zero-age main sequence.

| $(B-V)_{0}$ | $(U-B)_{0}$ | $M_{v}$ | $(B-V)_{0}$ | $(U-B)_{0}$ | $M_{v}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $-0 \mathrm{~m}_{33}$ | $-1 \mathrm{~m}_{20}$ | $-5 \mathrm{~m}_{2}$ | +0.40 | -0.01 | +3.4 |
| -0.305 | -1.10 | -3.6 | +0.50 | 0.00 | +4.1 |
| -0.30 | -1.08 | -3.25 | +0.60 | +0.08 | +4.7 |
| -0.28 | -1.00 | -2.6 | +0.70 | +0.23 | +5.2 |
| -0.25 | -0.90 | -2.1 | +0.80 | +0.42 | +5.8 |
| -0.22 | -0.80 | -1.5 | +0.90 | +0.63 | +6.3 |
| -0.20 | -0.69 | -1.1 | +1.00 | +0.86 | +6.7 |
| -0.15 | -0.50 | -0.2 | +1.10 | +1.03 | +7.1 |
| -0.10 | -0.30 | +0.6 | +1.20 | +1.13 | +7.5 |
| -0.05 | -0.10 | +1.1 | +1.30 | +1.20 | +8.0 |
| 0.00 | +0.01 | +1.5 | +1.40 | +1.22 | +8.8 |
| +0.05 | +0.05 | +1.7 | +1.50 | +1.17 | +10.3 |
| +0.10 | +0.08 | +1.9 | +1.60 | +1.20 | +12.0 |
|  |  |  |  |  |  |
| $(B-V)_{0}$ | $(U-B)_{0}$ | $M_{v}$ | $(B-V)_{0}$ | $(U-B)_{0}$ | $M_{v}$ |
| +0.15 | +0.09 | +2.1 | +1.70 | +1.32 | +13.2 |
| +0.20 | +0.10 | +2.4 | +1.80 | +1.43 | +14.2 |
| +0.25 | +0.07 | +2.55 | +1.90 | +1.53 | +15.5 |
| +0.30 | +0.03 | +2.8 | +2.00 | +1.64 | +16.7 |
| +0.35 | 0.00 | +3.1 |  |  |  |

Allen's Astrophysical Quantities (4 $4^{\text {th }}$ edition)

## Main-Sequence Stars (Luminosity Class V)

| Sp. <br> Type | $T_{e}$ <br> $(K)$ | $L / L_{\odot}$ | $R / R_{\odot}$ | $M / M_{\odot}$ | $M_{\text {bol }}$ | $B C$ | $M_{V}$ | $U-B$ | $B-V$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O5 | 42000 | 499000 | 13.4 | 60 | -9.51 | -4.40 | -5.1 | -1.19 | -0.33 |
| O6 | 39500 | 324000 | 12.2 | 37 | -9.04 | -3.93 | -5.1 | -1.17 | -0.33 |
| O7 | 37500 | 216000 | 11.0 | - | -8.60 | -3.68 | -4.9 | -1.15 | -0.32 |
| O8 | 35800 | 147000 | 10.0 | 23 | -8.18 | -3.54 | -4.6 | -1.14 | -0.32 |
|  |  |  |  |  |  |  |  |  |  |
| B0 | 30000 | 32500 | 6.7 | 17.5 | -6.54 | -3.16 | -3.4 | -1.08 | -0.30 |
| B1 | 25400 | 9950 | 5.2 | - | -5.26 | -2.70 | -2.6 | -0.95 | -0.26 |
| B2 | 20900 | 2920 | 4.1 | - | -3.92 | -2.35 | -1.6 | -0.84 | -0.24 |
| B3 | 18800 | 1580 | 3.8 | 7.6 | -3.26 | -1.94 | -1.3 | -0.71 | -0.20 |
| B5 | 15200 | 480 | 3.2 | 5.9 | -1.96 | -1.46 | -0.5 | -0.58 | -0.17 |
| B6 | 13700 | 272 | 2.9 | - | -1.35 | -1.21 | -0.1 | -0.50 | -0.15 |
| B7 | 12500 | 160 | 2.7 | - | -0.77 | -1.02 | +0.3 | -0.43 | -0.13 |
| B8 | 11400 | 96.7 | 2.5 | 3.8 | -0.22 | -0.80 | +0.6 | -0.34 | -0.11 |
| B9 | 10500 | 60.7 | 2.3 | - | +0.28 | -0.51 | +0.8 | -0.20 | -0.07 |
|  |  |  |  |  |  |  |  |  |  |
| A0 | 9800 | 39.4 | 2.2 | 2.9 | +0.75 | -0.30 | +1.1 | -0.02 | -0.02 |
| A1 | 9400 | 30.3 | 2.1 | - | +1.04 | -0.23 | +1.3 | +0.02 | +0.01 |
| A2 | 9020 | 23.6 | 2.0 | - | +1.31 | -0.20 | +1.5 | +0.05 | +0.05 |
| A5 | 8190 | 12.3 | 1.8 | 2.0 | +2.02 | -0.15 | +2.2 | +0.10 | +0.15 |
| A8 | 7600 | 7.13 | 1.5 | - | +2.61 | -0.10 | +2.7 | +0.09 | +0.25 |
|  |  |  |  |  |  |  |  |  |  |
| F0 | 7300 | 5.21 | 1.4 | 1.6 | +2.95 | -0.09 | +3.0 | +0.03 | +0.30 |
| F2 | 7050 | 3.89 | 1.3 | - | +3.27 | -0.11 | +3.4 | +0.00 | +0.35 |
| F5 | 6650 | 2.56 | 1.2 | 1.4 | +3.72 | -0.14 | +3.9 | -0.02 | +0.44 |
| F8 | 6250 | 1.68 | 1.1 | - | +4.18 | -0.16 | +4.3 | +0.02 | +0.52 |

## Main-Sequence Stars (Luminosity Class V)

| Sp. | $T_{e}$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Type | $(K)$ | $L / L_{\odot}$ | $R / R_{\odot}$ | $M / M_{\odot}$ | $M_{\text {bol }}$ | $B C$ | $M_{V}$ | $U-B$ | $B-V$ |
| G0 | 5940 | 1.25 | 1.06 | 1.05 | +4.50 | -0.18 | +4.7 | +0.06 | +0.58 |
| G2 | 5790 | 1.07 | 1.03 | - | +4.66 | -0.20 | +4.9 | +0.12 | +0.63 |
| Sun $^{a}$ | 5777 | 1.00 | 1.00 | 1.00 | +4.74 | -0.08 | +4.82 | +0.195 | +0.650 |
| G8 | 5310 | 0.656 | 0.96 | - | +5.20 | -0.40 | +5.6 | +0.30 | +0.74 |
|  |  |  |  |  |  |  |  |  |  |
| K0 | 5150 | 0.552 | 0.93 | 0.79 | +5.39 | -0.31 | +5.7 | +0.45 | +0.81 |
| K1 | 4990 | 0.461 | 0.91 | - | +5.58 | -0.37 | +6.0 | +0.54 | +0.86 |
| K3 | 4690 | 0.318 | 0.86 | - | +5.98 | -0.50 | +6.5 | +0.80 | +0.96 |
| K4 | 4540 | 0.263 | 0.83 | - | +6.19 | -0.55 | +6.7 | - | +1.05 |
| K5 | 4410 | 0.216 | 0.80 | 0.67 | +6.40 | -0.72 | +7.1 | +0.98 | +1.15 |
| K7 | 4150 | 0.145 | 0.74 | - | +6.84 | -1.01 | +7.8 | +1.21 | +1.33 |
|  |  |  |  |  |  |  |  |  |  |
| M0 | 3840 | 0.077 | 0.63 | 0.51 | +7.52 | -1.38 | +8.9 | +1.22 | +1.40 |
| M1 | 3660 | 0.050 | 0.56 | - | +7.99 | -1.62 | +9.6 | +1.21 | +1.46 |
| M2 | 3520 | 0.032 | 0.48 | 0.40 | +8.47 | -1.89 | +10.4 | +1.18 | +1.49 |
| M3 | 3400 | 0.020 | 0.41 | - | +8.97 | -2.15 | +11.1 | +1.16 | +1.51 |
| M4 | 3290 | 0.013 | 0.35 | - | +9.49 | -2.38 | +11.9 | +1.15 | +1.54 |
| M5 | 3170 | 0.0076 | 0.29 | 0.21 | +10.1 | -2.73 | +12.8 | +1.24 | +1.64 |
| M6 | 3030 | 0.0044 | 0.24 | - | +10.6 | -3.21 | +13.8 | +1.32 | +1.73 |
| M7 | 2860 | 0.0025 | 0.20 | - | +11.3 | -3.46 | +14.7 | +1.40 | +1.80 |

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## Giant Stars (Luminosity Class III)

| Sp. | $T_{e}$ <br> Type | $(K)$ | $L / L_{\odot}$ | $R / R_{\odot}$ | $M / M_{\odot}$ | $M_{\text {bol }}$ | $B C$ | $M_{V}$ | $U-B$ |
| :--- | :---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O5 | 39400 | 741000 | 18.5 | - | -9.94 | -4.05 | -5.9 | -1.18 | -0.32 |
| O6 | 37800 | 519000 | 16.8 | - | -9.55 | -3.80 | -5.7 | -1.17 | -0.32 |
| O7 | 36500 | 375000 | 15.4 | - | -9.20 | -3.58 | -5.6 | -1.14 | -0.32 |
| O8 | 35000 | 277000 | 14.3 | - | -8.87 | -3.39 | -5.5 | -1.13 | -0.31 |
|  |  |  |  |  |  |  |  |  |  |
| B0 | 29200 | 84700 | 11.4 | 20 | -7.58 | -2.88 | -4.7 | -1.08 | -0.29 |
| B1 | 24500 | 32200 | 10.0 | - | -6.53 | -2.43 | -4.1 | -0.97 | -0.26 |
| B2 | 20200 | 11100 | 8.6 | - | -5.38 | -2.02 | -3.4 | -0.91 | -0.24 |
| B3 | 18300 | 6400 | 8.0 | - | -4.78 | -1.60 | -3.2 | -0.74 | -0.20 |
| B5 | 15100 | 2080 | 6.7 | 7 | -3.56 | -1.30 | -2.3 | -0.58 | -0.17 |
| B6 | 13800 | 1200 | 6.1 | - | -2.96 | -1.13 | -1.8 | -0.51 | -0.15 |
| B7 | 12700 | 710 | 5.5 | - | -2.38 | -0.97 | -1.4 | -0.44 | -0.13 |
| B8 | 11700 | 425 | 5.0 | - | -1.83 | -0.82 | -1.0 | -0.37 | -0.11 |
| B9 | 10900 | 263 | 4.5 | - | -1.31 | -0.71 | -0.6 | -0.20 | -0.07 |
|  |  |  |  |  |  |  |  |  |  |
| A0 | 10200 | 169 | 4.1 | 4 | -0.83 | -0.42 | -0.4 | -0.07 | -0.03 |
| A1 | 9820 | 129 | 3.9 | - | -0.53 | -0.29 | -0.2 | +0.07 | +0.01 |
| A2 | 9460 | 100 | 3.7 | - | -0.26 | -0.20 | -0.1 | +0.06 | +0.05 |
| A5 | 8550 | 52 | 3.3 | - | +0.44 | -0.14 | +0.6 | +0.11 | +0.15 |
| A8 | 7830 | 33 | 3.1 | - | +0.95 | -0.10 | +1.0 | +0.10 | +0.25 |


| F0 | 7400 | 27 | 3.2 | - | +1.17 | -0.11 | +1.3 | +0.08 | +0.30 |  |
| :--- | :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F2 | 7000 | 24 | 3.3 | - | +1.31 | -0.11 | +1.4 | +0.08 | +0.35 |  |
| F5 | 6410 | 22 | 3.8 | - | +1.37 | -0.14 | +1.5 | +0.09 | +0.43 |  |
|  |  |  |  |  |  |  |  |  |  |  |
| G0 | 5470 | 29 | 6.0 | 1.0 | +1.10 | -0.20 | +1.3 | +0.21 | +0.65 |  |
| G2 | 5300 | 31 | 6.7 | - | +1.00 | -0.27 | +1.3 | +0.39 | +0.77 |  |
| G8 | 4800 | 44 | 9.6 | - | +0.63 | -0.42 | +1.0 | +0.70 | +0.94 |  |
|  |  |  |  |  |  |  |  |  |  |  |
| K0 | 4660 | 50 | 10.9 | 1.1 | +0.48 | -0.50 | +1.0 | +0.84 | +1.00 |  |
| K1 | 4510 | 58 | 12.5 | - | +0.32 | -0.55 | +0.9 | +1.01 | +1.07 |  |
| K3 | 4260 | 79 | 16.4 | - | -0.01 | -0.76 | +0.8 | +1.39 | +1.27 |  |
| K4 | 4150 | 93 | 18.7 | - | -0.18 | -0.94 | +0.8 | - | +1.38 |  |
| K5 | 4050 | 110 | 21.4 | 1.2 | -0.36 | -1.02 | +0.7 | +1.81 | +1.50 |  |
| K7 | 3870 | 154 | 27.6 | - | -0.73 | -1.17 | +0.4 | +1.83 | +1.53 |  |
|  |  |  |  |  |  |  |  |  |  |  |
| M0 | 3690 | 256 | 39.3 | 1.2 | -1.28 | -1.25 | +0.0 | +1.87 | +1.56 |  |
| M1 | 3600 | 355 | 48.6 | - | -1.64 | -1.44 | -0.2 | +1.88 | +1.58 |  |
| M2 | 3540 | 483 | 58.5 | 1.3 | -1.97 | -1.62 | -0.4 | +1.89 | +1.60 |  |
| M3 | 3480 | 643 | 69.7 | - | -2.28 | -1.87 | -0.4 | +1.88 | +1.61 |  |
| M4 | 3440 | 841 | 82.0 | - | -2.57 | -2.22 | -0.4 | +1.73 | +1.62 |  |
| M5 | 3380 | 1100 | 96.7 | - | -2.86 | -2.48 | -0.4 | +1.58 | +1.63 |  |
| M6 | 3330 | 1470 | 116 | - | -3.18 | -2.73 | -0.4 | +1.16 | +1.52 |  |

## Supergiant Stars (Luminosity Class Approximately Iab)

| Sp. <br> Type | $T_{e}$ <br> $(K)$ | $L / L_{\odot}$ | $R / R_{\odot}$ | $M / M_{\odot}$ | $M_{\text {bol }}$ | $B C$ | $M_{V}$ | $U-B$ | $B-V$ |
| :---: | :---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O5 | 40900 | 1140000 | 21.2 | 70 | -10.40 | -3.87 | -6.5 | -1.17 | -0.31 |
| O6 | 38500 | 998000 | 22.4 | 40 | -10.26 | -3.74 | -6.5 | -1.16 | -0.31 |
| O7 | 36200 | 877000 | 23.8 | - | -10.12 | -3.48 | -6.6 | -1.14 | -0.31 |
| O8 | 34000 | 769000 | 25.3 | 28 | -9.98 | -3.35 | -6.6 | -1.13 | -0.29 |
|  |  |  |  |  |  |  |  |  |  |
| B0 | 26200 | 429000 | 31.7 | 25 | -9.34 | -2.49 | -6.9 | -1.06 | -0.23 |
| B1 | 21400 | 261000 | 37.3 | - | -8.80 | -1.87 | -6.9 | -1.00 | -0.19 |
| B2 | 17600 | 157000 | 42.8 | - | -8.25 | -1.58 | -6.7 | -0.94 | -0.17 |
| B3 | 16000 | 123000 | 45.8 | - | -7.99 | -1.26 | -6.7 | -0.83 | -0.13 |
| B5 | 13600 | 79100 | 51.1 | 20 | -7.51 | -0.95 | -6.6 | -0.72 | -0.10 |
| B6 | 12600 | 65200 | 53.8 | - | -7.30 | -0.88 | -6.4 | -0.69 | -0.08 |
| B7 | 11800 | 54800 | 56.4 | - | -7.11 | -0.78 | -6.3 | -0.64 | -0.05 |
| B8 | 11100 | 47200 | 58.9 | - | -6.95 | -0.66 | -6.3 | -0.56 | -0.03 |
| B9 | 10500 | 41600 | 61.8 | - | -6.81 | -0.52 | -6.3 | -0.50 | -0.02 |
|  |  |  |  |  |  |  |  |  |  |
| A0 | 9980 | 37500 | 64.9 | 16 | -6.70 | -0.41 | -6.3 | -0.38 | -0.01 |
| A1 | 9660 | 35400 | 67.3 | - | -6.63 | -0.32 | -6.3 | -0.29 | +0.02 |
| A2 | 9380 | 33700 | 69.7 | - | -6.58 | -0.28 | -6.3 | -0.25 | +0.03 |
| A5 | 8610 | 30500 | 78.6 | 13 | -6.47 | -0.13 | -6.3 | -0.07 | +0.09 |
| A8 | 7910 | 29100 | 91.1 | - | -6.42 | -0.03 | -6.4 | +0.11 | +0.14 |


| F0 | 7460 | 28800 | 102 | 12 | -6.41 | -0.01 | -6.4 | +0.15 | +0.17 |
| :--- | ---: | ---: | ---: | :--- | ---: | ---: | ---: | ---: | ---: |
| F2 | 7030 | 28700 | 114 | - | -6.41 | 0.00 | -6.4 | +0.18 | +0.23 |
| F5 | 6370 | 29100 | 140 | 10 | -6.42 | -0.03 | -6.4 | +0.27 | +0.32 |
| F8 | 5750 | 29700 | 174 | - | -6.44 | -0.09 | -6.4 | +0.41 | +0.56 |
|  |  |  |  |  |  |  |  |  |  |
| G0 | 5370 | 30300 | 202 | 10 | -6.47 | -0.15 | -6.3 | +0.52 | +0.76 |
| G2 | 5190 | 30800 | 218 | - | -6.48 | -0.21 | -6.3 | +0.63 | +0.87 |
| G8 | 4700 | 32400 | 272 | - | -6.54 | -0.42 | -6.1 | +1.07 | +1.15 |
|  |  |  |  |  |  |  |  |  |  |
| K0 | 4550 | 33100 | 293 | 13 | -6.56 | -0.50 | -6.1 | +1.17 | +1.24 |
| K1 | 4430 | 34000 | 314 | - | -6.59 | -0.56 | -6.0 | +1.28 | +1.30 |
| K3 | 4190 | 36100 | 362 | - | -6.66 | -0.75 | -5.9 | +1.60 | +1.46 |
| K4 | 4090 | 37500 | 386 | - | -6.70 | -0.90 | -5.8 | - | +1.53 |
| K5 | 3990 | 39200 | 415 | 13 | -6.74 | -1.01 | -5.7 | +1.80 | +1.60 |
| K7 | 3830 | 43200 | 473 | - | -6.85 | -1.20 | -5.6 | +1.84 | +1.63 |
|  |  |  |  |  |  |  |  |  |  |
| M0 | 3620 | 51900 | 579 | 13 | -7.05 | -1.29 | -5.8 | +1.90 | +1.67 |
| M1 | 3490 | 60300 | 672 | - | -7.21 | -1.38 | -5.8 | +1.90 | +1.69 |
| M2 | 3370 | 72100 | 791 | 19 | -7.41 | -1.62 | -5.8 | +1.95 | +1.71 |
| M3 | 3210 | 89500 | 967 | - | -7.64 | -2.13 | -5.5 | +1.95 | +1.69 |
| M4 | 3060 | 117000 | 1220 | - | -7.93 | -2.75 | -5.2 | +2.00 | +1.76 |
| M5 | 2880 | 165000 | 1640 | 24 | -8.31 | -3.47 | -4.8 | +1.60 | +1.80 |
| M6 | 2710 | 264000 | 2340 | - | -8.82 | -3.90 | -4.9 | - | - |

Carroll \& Ostelie

Adopted calibration of MK spectral types in absolute magnitudes $M_{V}$

| Sp | ZAMS | V | IV | III | II | Ib | Iab | Ia |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| O5 | -4.6 | -5.6 | -5.8 | -6.0 | -6.3 | -6.6 | -6.9 | -7.2 |
| O6 | -4.0 | -5.4 | -5.7 | -5.9 | -6.3 | -6.6 | -6.9 | -7.2 |
| O7 | -3.9 | -5.2 | -5.5 | -5.8 | -6.2 | -6.5 | -6.8 | -7.2 |
| O8 | -3.7 | -4.9 | -5.2 | -5.6 | -6.1 | -6.4 | -6.7 | -7.2 |
| O9 | -3.5 | -4.5 | -4.9 | -5.3 | -5.9 | -6.3 | -6.6 | -7.2 |
| B0 | -3.1 | -4.0 | -4.4 | -4.9 | -5.6 | -6.1 | -6.5 | -7.2 |
| B1 | -2.3 | -3.3 | -3.9 | -4.5 | -5.2 | -5.9 | -6.4 | -7.2 |
| B2 | -1.6 | -2.5 | -3.1 | -3.7 | -5.0 | -5.9 | -6.4 | -7.2 |
| B3 | -1.0 | -1.7 | -2.3 | -3.0 | -4.8 | -5.9 | -6.4 | -7.2 |
| B5 | -0.1 | -0.8 | -1.2 | -1.7 | -4.6 | -5.9 | -6.4 | -7.2 |
| B6 | 0.3 | -0.5 | -0.9 | -1.3 | -4.4 | -5.8 | -6.4 | -7.2 |
| B7 | 0.6 | -0.2 | -0.6 | -1.0 | -4.2 | -5.8 | -6.4 | -7.2 |
| B8 | 1.0 | 0.1 | -0.3 | -0.7 | -3.9 | -5.8 | -6.4 | -7.2 |
| B9 | 1.4 | 0.5 | 0.1 | -0.4 | -3.6 | -5.7 | -6.4 | -7.2 |
| A0 | 1.6 | 0.8 | 0.4 | -0.1 | -3.4 | -5.5 | -6.4 | -7.2 |
| A1 | 1.7 | 1.1 | 0.7 | 0.2 | -3.2 | -5.3 | -6.4 | -7.2 |
| A2 | 1.8 | 1.3 | 0.9 | 0.4 | -3.1 | -5.2 | -6.4 | -7.3 |
| A3 | 1.9 | 1.5 | 1.0 | 0.5 | -3.0 | -5.1 | -6.4 | -7.3 |
| A5 | 2.3 | 1.9 | 1.4 | 0.8 | -2.9 | -5.0 | -6.5 | -7.5 |
| A7 | 2.6 | 2.3 | 1.7 | 1.1 | -2.8 | -5.0 | -6.7 | -7.7 |
| F0 | 3.0 | 2.8 | 2.2 | 1.5 | -2.7 | -5.0 | -6.9 | -7.9 |
| F2 | 3.2 | 3.1 | 2.4 | 1.8 | -2.6 | -4.9 | -7.0 | -8.0 |
| F5 | 3.7 | 3.6 | 2.6 | 2.0 | -2.6 | -4.8 | -7.1 | -8.0 |
| F8 | 4.2 | 4.1 | 2.8 |  | -2.5 | -4.7 | -7.2 | -8.1 |
| G0 | 4.5 | 4.4 | 2.9 |  | -2.4 | -4.6 | -7.2 | -8.2 |
| G2 |  | 4.7 | 3.0 | $1.1:$ | -2.4 | -4.5 | -7.2 | -8.2 |
| G5 |  | 5.1 | 3.1 | 1.0 | -2.4 | -4.4 | -7.2 | -8.2 |
| G8 |  | 5.6 | 3.2 | 0.9 | -2.5 | -4.3 | -7.0 | -8.1 |
| K0 |  | 6.0 | 3.2 | 0.8 | -2.5 | -4.3 | -6.8 | -7.9 |
| K1 |  | 6.2 | 3.2 | 0.8 | -2.5 | -4.3 | -6.7 | -7.7 |
| K2 |  | 6.5 |  | 0.7 | -2.5 | -4.3 | -6.6 | -7.6 |
| K3 |  | 6.7 |  | 0.6 | -2.5 | -4.3 | -6.5 | -7.5 |
| K4 |  | 7.0 |  | 0.5 | -2.6 | -4.4 | -6.4 | -7.4 |
| K5 |  | 7.3 |  | 0.3 | -2.6 | -4.4 | -6.2 | -7.2 |
| K7 |  | 8.1 |  | 0.0 | -2.7 | -4.5 | -6.0 | -7.0 |
| M0 |  | 8.9 |  | -0.6 | -2.8 | -4.6 | -5.8 | -6.9 |
| M1 |  | 9.4 |  | -0.8 | -2.9 | -4.6 | -5.8 | -6.8 |
| M2 |  | 10.0 |  | -0.9 | -3.0 | -4.7 | -5.8 | -6.7 |
| M3 |  | 10.5 |  | -1.0 | -3.0 | -4.7 | -5.8 | -6.7 |
| M4 |  | 11.5 |  | -0.6 | -3.1 | -4.7 | -5.8 | -6.7 |
| M5 |  | 13.5 |  | -0.1 | -3.1 | -4.7 | -5.8 | -6.7 |


| Sp | $\log T_{\text {eff }}$ |  |  |  | Bol. Correction |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | V |  | III | I-II | V |  | III | I-II |
| O5 |  | 4.626 |  | 4.618 |  | -4.15 |  | -3.80 |
| O6 |  | 4.593 |  | 4.585 |  | -3.90 |  | -3.55 |
| 07 |  | 4.568 |  | 4.556 |  | -3.65 |  | -3.30 |
| O8 |  | 4.550 |  | 4.535 |  | -3.40 |  | -3.15 |
| O9 |  | 4.525 |  | 4.512 |  | -3.15 |  | -2.95 |
| B0 |  | 4.498 |  | 4.431 |  | -2.95 |  | -2.50 |
| B1 |  | 4.423 |  | 4.371 |  | -2.60 |  | -2.15 |
| B2 |  | 4.362 |  | 4.307 |  | -2.20 |  | -1.75 |
| B3 |  | 4.286 |  | 4.243 |  | -1.85 |  | -1.40 |
| B5 |  | 4.188 |  | 4.137 |  | -1.30 |  | -0.90 |
| B6 |  | 4.152 |  | 4.100 |  | -1.05 |  | -0.75 |
| B7 |  | 4.107 |  | 4.068 |  | -0.80 |  | -0.60 |
| B8 |  | 4.061 |  | 4.041 |  | -0.55 |  | -0.45 |
| B9 |  | 4.017 |  | 4.013 |  | -0.35 |  | -0.35 |
| A0 |  | 3.982 |  | 3.991 |  | -0.25 |  | -0.25 |
| A1 |  | 3.973 |  | 3.978 |  | -0.16 |  | -0.16 |
| A2 |  | 3.961 |  | 3.964 |  | -0.10 |  | -0.10 |
| A3 |  | 3.949 |  | 3.949 |  | -0.03 |  | -0.03 |
| A5 |  | 3.924 |  | 3.919 |  | 0.02 |  | 0.05 |
| A7 |  | 3.903 |  | 3.897 |  | 0.02 |  | 0.09 |
| F0 |  | 3.863 |  | 3.869 |  | 0.02 |  | 0.13 |
| F2 |  | 3.845 |  | 3.851 |  | 0.01 |  | 0.11 |
| F5 |  | 3.813 |  | 3.813 |  | -0.02 |  | 0.08 |
| F8 | 3.789 |  | 3.782 | 3.778 |  | -0.03 |  | 0.03 |
| G0 | 3.774 |  | 3.763 | 3.756 |  | -0.05 |  | 0.00 |
| G2 | 3.763 |  | 3.740 | 3.732 |  | -0.07 |  | -0.05 |
| G5 | 3.740 |  | 3.712 | 3.699 | -0.09 |  | -0.22 | -0.13 |
| G8 | 3.720 |  | 3.695 | 3.663 | -0.13 |  | -0.28 | -0.22 |
| K0 | 3.703 |  | 3.681 | 3.643 | -0.19 |  | -0.37 | -0.29 |
| K1 | 3.695 |  | 3.663 | 3.633 |  |  | -0.43 | -0.35 |
| K2 | 3.686 |  | 3.648 | 3.623 | -0.30 |  | -0.49 | -0.42 |
| K3 | 3.672 |  | 3.628 | 3.613 |  |  | -0.66 | -0.57 |
| K4 | 3.663 |  | 3.613 |  |  |  | -0.86 | -0.75 |
| K5 | 3.643 |  | 3.602 | 3.585 | -0.62 |  | -1.15 | -1.17 |
| K7 | 3.602 |  |  |  | -0.89 |  |  |  |
| M0 | 3.591 |  | 3.591 | 3.568 | -1.17 |  | -1.25 | -1.25 |
| M1 | 3.574 |  | 3.580 | 3.556 | -1.45 |  | -1.45 | -1.40 |
| M2 | 3.550 |  | 3.574 | 3.544 | -1.71 |  | -1.65 | -1.60 |
| M3 | 3.531 |  | 3.562 | 3.518 | -1.92 |  | -1.95 | -2.0 |
| M4 | 3.512 |  | 3.550 | 3.491 | -2.24 |  | -2.4 | -2.6 |
| M5 | 3.491 |  | 3.531 | 3.470 | -2.55 |  | -3.1 | -3.3 |
| M6 |  |  | 3.512 |  | -4.4 |  | -4.0 |  |

Straižys \&

Stellar masses $\log \mathfrak{M} / \mathbb{M}_{\odot}$ for different MK spectral types derived from the evolutionary tracks

| Sp | ZAMS | V | IV | III | II | Ib | Iab | Ia |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O5 | 1.60 | 1.81 | 1.85 | 1.89 | 1.90 | 1.92 | 1.99 |  |  |
| 06 | 1.48 | 1.70 | 1.76 | 1.80 | 1.80 | 1.87 | 1.91 | 2.00 |  |
| O7 | 1.40 | 1.59 | 1.65 | 1.68 | 1.71 | 1.76 | 1.83 | 1.92 |  |
| O8 | 1.34 | 1.48 | 1.54 | 1.60 | 1.65 | 1.72 | 1.76 | 1.90 |  |
| O9 | 1.28 | 1.38 | 1.45 | 1.49 | 1.58 | 1.66 | 1.72 | 1.83 |  |
| B0 | 1.20 | 1.30 | 1.34 | 1.40 | 1.40 | 1.48 | 1.56 | 1.70 |  |
| B1 | 1.04 | 1.11 | 1.18 | 1.23 | 1.28 | 1.38 | 1.46 | 1.64 |  |
| B2 | 0.92 | 0.99 | 1.04 | 1.08 | 1.18 | 1.30 | 1.38 | 1.54 |  |
| B3 | 0.78 | 0.84 | 0.88 | 0.94 | 1.11 | 1.23 | 1.32 | 1.45 |  |
| B5 | 0.62 | 0.68 | 0.72 | 0.75 | 1.00 | 1.18 | 1.26 | 1.40 |  |
| B6 | 0.56 | 0.61 | 0.64 | 0.68 | 0.94 | 1.15 | 1.26 | 1.38 |  |
| B7 | 0.49 | 0.53 | 0.57 | 0.60 | 0.91 | 1.11 | 1.23 | 1.36 |  |
| B8 | 0.43 | 0.48 | 0.49 | 0.52 | 0.88 | 1.08 | 1.20 | 1.34 |  |
| B9 | 0.36 | 0.41 | 0.45 | 0.49 | 0.85 | 1.04 | 1.20 | 1.32 |  |
| A0 | 0.32 | 0.35 | 0.39 | 0.43 | 0.81 | 1.04 | 1.18 | 1.30 |  |
| A1 | 0.31 | 0.34 | 0.36 | 0.41 | 0.78 | 1.00 | 1.18 | 1.30 |  |
| A2 | 0.29 | 0.32 | 0.34 | 0.39 | 0.75 | 0.98 | 1.15 | 1.30 |  |
| A3 | 0.27 | 0.30 | 0.32 | 0.36 | 0.75 | 0.97 | 1.11 | 1.30 |  |
| A5 | 0.23 | 0.26 | 0.29 | 0.33 | 0.74 | 0.95 | 1.11 | 1.30 |  |
| A7 | 0.20 | 0.22 | 0.26 | 0.30 | 0.73 | 0.94 | 1.15 | 1.32 |  |
| F0 | 0.16 | 0.16 | 0.20 | 0.23 | 0.72 | 0.93 | 1.20 | 1.38 |  |
| F2 | 0.13 | 0.13 | 0.16 | 0.20 | 0.72 | 0.93 | 1.20 | 1.40 |  |
| F5 | 0.08 | 0.08 | 0.13 | 0.18 | 0.72 | 0.93 | 1.26 | 1.40 |  |
| F8 | 0.04 | 0.04 | 0.11 |  | 0.72 | 0.93 | 1.28 | 1.41 |  |
| G0 | 0.02 | 0.02 | 0.10 |  | 0.72 | 0.93 | 1.30 | 1.43 |  |
| G2 | 0.00 | 0.00 | 0.10 | 0.33 | 0.72 | 0.93 | 1.30 | 1.45 |  |
| G5 |  | -0.02 | 0.08 | 0.39 | 0.73 | 0.94 | 1.32 | 1.46 |  |
| G8 |  | -0.04 | 0.08 | 0.42 | 0.76 | 0.94 | 1.32 | 1.46 |  |
| K0 |  | -0.07 | 0.11 | 0.46 | 0.78 | 0.96 | 1.30 | 1.45 |  |
| K1 |  | -0.10 | 0.13 | 0.46 | 0.78 | 0.96 | 1.30 | 1.45 |  |
| K2 |  | -0.10 |  | 0.45 | 0.79 | 0.98 | 1.28 | 1.43 |  |
| K3 |  | -0.12 |  | 0.38 | 0.80 | 1.00 | 1.30 | 1.43 |  |
| K4 |  | -0.15 |  | 0.36 |  |  |  |  |  |
| K5 |  | -0.19 |  | 0.37 | 0.83 | 1.08 | 1.30 | 1.45 |  |
| K7 |  | -0.22 |  |  |  |  |  |  |  |
| M0 |  | -0.26 |  | 0.48 | 0.83 | 1.15 | 1.32 | 1.46 |  |
| M1 |  | -0.30 |  | 0.54 | 0.83 | 1.18 | 1.34 | 1.48 |  |
| M2 |  | -0.35 |  | 0.54 | 0.81 | 1.18 | 1.36 | 1.50 |  |
| M3 |  | -0.40 |  | 0.52 | 0.84 | 1.20 | 1.38 | 1.56 |  |
| M4 |  | -0.52 |  | 0.51 |  |  |  |  |  |
| M5 |  | (-0.82) |  | (0.41) |  |  |  |  |  |
| M6 |  |  |  | (0.40) |  |  |  |  | Kurliene (1981) |

Calibration of MK spectral types in surface gravities $(\log g)$

| Sp | ZAMS | V | IV | III | II | Ib | Iab | Ia |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| O5 | 4.13 | 3.90 | 3.86 | 3.82 | 3.76 | 3.74 | 3.69 |  |
| O6 | 4.16 | 3.86 | 3.80 | 3.76 | 3.69 | 3.64 | 3.60 | 3.53 |
| O7 | 4.18 | 3.85 | 3.80 | 3.74 | 3.64 | 3.57 | 3.52 | 3.45 |
| O8 | 4.17 | 3.87 | 3.81 | 3.75 | 3.62 | 3.53 | 3.49 | 3.39 |
| O9 | 4.21 | 3.95 | 3.82 | 3.74 | 3.58 | 3.50 | 3.44 | 3.31 |
| B0 | 4.22 | 4.00 | 3.88 | 3.74 | 3.39 | 3.27 | 3.19 | 3.05 |
| B1 | 4.28 | 4.00 | 3.86 | 3.71 | 3.31 | 3.17 | 3.01 | 2.87 |
| B2 | 4.28 | 4.06 | 3.88 | 3.68 | 3.19 | 3.00 | 2.84 | 2.68 |
| B3 | 4.31 | 4.06 | 3.89 | 3.71 | 3.12 | 2.79 | 2.68 | 2.49 |
| B5 | 4.32 | 4.10 | 3.98 | 3.81 | 2.90 | 2.52 | 2.40 | 2.22 |
| B6 | 4.32 | 4.09 | 3.96 | 3.84 | 2.77 | 2.42 | 2.29 | 2.13 |
| B7 | 4.35 | 4.07 | 3.95 | 3.82 | 2.77 | 2.33 | 2.21 | 2.02 |
| B8 | 4.34 | 4.07 | 3.92 | 3.79 | 2.79 | 2.27 | 2.11 | 1.97 |
| B9 | 4.34 | 4.03 | 3.94 | 3.75 | 2.81 | 2.20 | 2.04 | 1.88 |
| A0 | 4.32 | 4.07 | 3.91 | 3.75 | 2.85 | 2.23 | 2.01 | 1.81 |
| A1 | 4.35 | 4.10 | 3.96 | 3.78 | 2.88 | 2.22 | 1.96 | 1.76 |
| A2 | 4.32 | 4.16 | 3.98 | 3.78 | 2.87 | 2.23 | 1.92 | 1.71 |
| A3 | 4.34 | 4.20 | 4.03 | 3.83 | 2.85 | 2.20 | 1.86 | 1.65 |
| A5 | 4.36 | 4.22 | 4.06 | 3.86 | 2.81 | 2.14 | 1.74 | 1.53 |
| A7 | 4.36 | 4.26 | 4.10 | 3.86 | 2.75 | 2.08 | 1.65 | 1.38 |
| F0 | 4.32 | 4.28 | 4.05 | 3.83 | 2.67 | 2.00 | 1.51 | 1.25 |
| F2 | 4.30 | 4.26 | 4.01 | 3.81 | 2.63 | 1.92 | 1.39 | 1.15 |
| F5 | 4.32 | 4.28 | 3.93 | 3.74 | 2.48 | 1.81 | 1.22 | 1.00 |
| F8 | 4.39 | 4.35 | 3.89 |  | 2.38 | 1.71 | 1.06 | 0.83 |
| G0 | 4.39 | 4.39 | 3.84 |  | 2.29 | 1.62 | 0.95 | 0.72 |
| G2 | 4.40 | 4.40 | 3.77 | 3.20 | 2.20 | 1.53 | 0.86 | 0.61 |
| G5 |  | 4.49 | 3.71 | 3.07 | 2.04 | 1.45 | 0.71 | 0.45 |
| G8 |  | 4.55 | 3.64 | 2.95 | 1.84 | 1.30 | 0.60 | 0.30 |
| K0 |  | 4.57 | 3.57 | 2.89 | 1.74 | 1.20 | 0.54 | 0.25 |
| K1 |  | 4.55 | 3.55 | 2.78 | 1.66 | 1.16 | 0.54 | 0.25 |
| K2 |  | 4.55 |  | 2.63 | 1.59 | 1.10 | 0.48 | 0.23 |
| K3 |  | 4.56 |  | 2.36 | 1.52 | 1.00 | 0.46 | 0.19 |
| K4 |  | 4.57 |  | 2.16 |  |  |  |  |
| K5 |  | 4.57 |  | 1.93 | 1.20 | 0.77 | 0.35 | 0.10 |
| K7 |  | 4.62 |  |  |  |  |  |  |
| M0 |  | 4.61 |  | 1.63 | 1.01 | 0.61 | 0.30 | 0.00 |
| M1 |  | 4.67 |  | 1.41 | 0.84 | 0.51 | 0.19 | -0.07 |
| M2 |  | 4.69 |  | 1.31 | 0.70 | 0.39 | 0.09 | -0.13 |
| M3 |  | 4.71 |  | 1.12 | 0.38 | 0.10 | -0.16 | -0.34 |
| M4 |  | 4.77 |  | 0.98 |  |  |  |  |
| M5 |  | 5.06 |  | $0.76)$ |  |  |  |  |
| M6 |  |  |  | $0.52)$ |  |  |  |  |
|  |  |  |  |  |  |  |  |  |


| Sp | ZAMS | V | IV | III | II | Ib | Iab | Ia |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O5 | 0.95 | 1.17 | 1.21 | 1.25 | 1.28 | 1.30 | 1.36 |  |  |
| O6 | 0.87 | 1.13 | 1.19 | 1.23 | 1.27 | 1.33 | 1.37 | 1.45 |  |
| 07 | 0.82 | 1.08 | 1.14 | 1.18 | 1.25 | 1.31 | 1.37 | 1.45 |  |
| O8 | 0.80 | 1.02 | 1.08 | 1.14 | 1.23 | 1.31 | 1.35 | 1.47 |  |
| O9 | 0.75 | 0.93 | 1.03 | 1.09 | 1.22 | 1.30 | 1.36 | 1.48 |  |
| B0 | 0.70 | 0.86 | 0.94 | 1.04 | 1.20 | 1.32 | 1.40 | 1.54 |  |
| B1 | 0.59 | 0.77 | 0.87 | 0.97 | 1.20 | 1.32 | 1.44 | 1.60 |  |
| B2 | 0.54 | 0.68 | 0.80 | 0.92 | 1.21 | 1.37 | 1.49 | 1.65 |  |
| B3 | 0.45 | 0.61 | 0.71 | 0.83 | 1.21 | 1.43 | 1.53 | 1.69 |  |
| B5 | 0.36 | 0.50 | 0.58 | 0.68 | 1.27 | 1.55 | 1.65 | 1.81 |  |
| B6 | 0.34 | 0.48 | 0.56 | 0.64 | 1.30 | 1.58 | 1.70 | 1.84 |  |
| B7 | 0.29 | 0.45 | 0.53 | 0.61 | 1.28 | 1.60 | 1.72 | 1.88 |  |
| B8 | 0.26 | 0.42 | 0.50 | 0.58 | 1.26 | 1.62 | 1.76 | 1.90 |  |
| B9 | 0.23 | 0.41 | 0.47 | 0.59 | 1.23 | 1.63 | 1.79 | 1.93 |  |
| A0 | 0.22 | 0.36 | 0.46 | 0.56 | 1.20 | 1.62 | 1.80 | 1.96 |  |
| A1 | 0.19 | 0.33 | 0.41 | 0.53 | 1.16 | 1.60 | 1.82 | 1.98 |  |
| A2 | 0.20 | 0.30 | 0.40 | 0.52 | 1.15 | 1.59 | 1.83 | 2.01 |  |
| A3 | 0.18 | 0.26 | 0.36 | 0.48 | 1.16 | 1.60 | 1.84 | 2.04 |  |
| A5 | 0.15 | 0.23 | 0.33 | 0.45 | 1.18 | 1.62 | 1.90 | 2.10 |  |
| A7 | 0.13 | 0.19 | 0.29 | 0.43 | 1.21 | 1.65 | 1.97 | 2.19 |  |
| F0 | 0.13 | 0.15 | 0.29 | 0.41 | 1.24 | 1.68 | 2.06 | 2.28 |  |
| F2 | 0.13 | 0.15 | 0.29 | 0.41 | 1.26 | 1.72 | 2.12 | 2.34 |  |
| F5 | 0.09 | 0.11 | 0.31 | 0.43 | 1.30 | 1.77 | 2.23 | 2.41 |  |
| F8 | 0.04 | 0.06 | 0.33 |  | 1.38 | 1.82 | 2.32 | 2.50 |  |
| G0 | 0.03 | 0.03 | 0.34 |  | 1.43 | 1.87 | 2.39 | 2.57 |  |
| G2 | 0.01 | 0.01 | 0.38 | 0.78 | 1.48 | 1.92 | 2.44 | 2.64 |  |
| G5 |  | -0.04 | 0.41 | 0.88 | 1.56 | 1.96 | 2.52 | 2.72 |  |
| G8 |  | -0.08 | 0.43 | 0.95 | 1.67 | 2.03 | 2.57 | 2.79 |  |
| K0 |  | -0.11 | 0.48 | 1.00 | 1.73 | 2.09 | 2.59 | 2.81 |  |
| K1 |  | -0.11 | 0.50 | 1.05 | 1.77 | 2.11 | 2.61 | 2.81 |  |
| K2 |  | -0.11 |  | 1.12 | 1.81 | 2.15 | 2.61 | 2.81 |  |
| K3 |  | -0.12 |  | 1.22 | 1.85 | 2.21 | 2.63 | 2.83 |  |
| K4 |  | -0.15 |  | 1.31 |  |  |  |  |  |
| K5 |  | -0.17 |  | 1.44 | 2.03 | 2.37 | 2.69 | 2.89 |  |
| K7 |  | -0.20 |  |  |  |  |  |  |  |
| M0 |  | -0.22 |  | 1.64 | 2.12 | 2.48 | 2.72 | 2.92 |  |
| M1 |  | -0.27 |  | 1.78 | 2.21 | 2.55 | 2.78 | 2.99 |  |
| M2 |  | -0.30 |  | 1.83 | 2.27 | 2.61 | 2.85 | 3.03 |  |
| M3 |  | -0.36 |  | 1.92 | 2.44 | 2.76 | 2.98 | 3.16 |  |
| M4 |  | -0.42 |  | 1.98 |  |  |  |  |  |
| M5 |  | -0.72 |  | (2.04) |  |  |  |  |  |
| M6 |  |  |  | (2.16) |  |  |  |  | Kurliene (1981) |

## SIMBAD Astronomical Database

| (2) | Portal | Simbad | VizieR | R Aladin | X-Match | Other | Help |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | sirius |
| other query modes : | Identifier query | er Coord |  | Criteria query | Reference query | Basic query | Script submission | $\begin{aligned} & \text { Output Help } \\ & \text { options } \end{aligned}$ |
| Query : sirius |  |  |  |  |  |  |  |  |



```
Basic data :
* alf CMa -- Double or multiple star
Other object types: \(\quad *(*, B D, G C, H D, H I C, H I P, H R, S A O, U B V), I R(A K A R I, I R A S, I R C, 2 M A S S, R A F G L),{ }^{* *}(* *\), WDS \()\)
```

ICRS coord. (ep=J2000) :
FK5 coord. (ep=J2000 eq=2000) :
FK4 coord. ( $e p=$ B1950 eq=1950) :
Gal coord. (ep=J2000) :
Proper motions mas/yr:
Radial velocity / Redshift / cz :
Parallaxes mas:
Spectral type:
Fluxes (8) :

PM* (LHS) , $\mathrm{V}^{*}$ (NSV), UV (TD1)
$064508.91728-164258.0171$ ( Optical ) [ 11.70 10.90 90 ] A 2007A\&A...474..653V
$064508.917-164258.02$ [ 11.7010 .9090 ]
064256.72 -16 3845.4 [ 67.3963 .090 ]
227.2303 -08.8903 [ 11.7010 .9090 ]
-546.01-1223.07 [1.33 1.24 0] A 2007A\&A...474..653V
$\mathrm{V}(\mathrm{km} / \mathrm{s})-5.50$ [0.4] / $\mathrm{z}(\sim)-0.000018$ [0.000001] / cz -5.50 [0.40] (~) A 2006AstL...32..759G
379.21 [1.58] A 2007A\&A...474..653V

A1V+DA C 2013yCat....1.2023s
U -1.51 [~] C 2002yCat.2237....0D
B -1.46 [~] C 2002yCat.2237....0D
V -1.46 [~] C 2002yCat.2237....0D
R-1.46 [~] C 2002yCat.2237....0D
I -1.43 [~] C $2002 y C a t .2237 \ldots .00$
J -1.36 [~] C 2002yCat. 2237....0D
H -1.33 [~] C 2002yCat. 2237....0D
K -1.35 [~] C 2002yCat.2237....0D

## To measure the stellar mass

- Stellar mass difficult to measure, direct measurements, except the Sun, only by binary systems
(but uncertain even for these, why?)
- Then one gets the mass-luminosity relation $L \propto M^{\alpha}$ where the slope $\alpha=3$ to 5 , depending on the mass range
- The main-sequence (MS) is a sequence of stellar mass under hydrostatic equilibrium
- Why are lower mass stars cooler on the surface and fainter in luminosity?


$$
\begin{aligned}
& M_{\max } \sim 120 M_{\odot} \\
& M_{\min } \sim 0.08 M_{\odot} \\
& L_{\max } \sim 10^{+6} L_{\odot} \\
& L_{\min } \sim 10^{-4} L_{\odot}
\end{aligned}
$$



Luminosity versus mass for a selection of stars in binaries

## Luminosity class and surface gravity

$\log g=\log \mathrm{GM} / \mathrm{R}^{2}$

- Betelgeuse ... (M2 I) $\log g \approx-0.6$ [cgs]
- Jupiter ... $\log g=3.4$
- $\operatorname{Sun}(\mathrm{G} 2 \mathrm{~V}) . . . \log g=4.44$
- Gl229B ... (T6.5) $\log g \approx 5$
- Sirius B... (WD) $\log g \approx 8$



Composite Hertzsprung-Russell Diagram. Stars of different absolute luminosity, $L$ - right axis, or bolometric absolute magnitude, $\mathrm{M}_{\mathrm{bol}}$ - left axis, are plotted as a function of surface temperature, $\mathrm{T}_{\mathrm{s}}$ bottom axis, or spectral type - top axis. (Adapted from L. Goldberg and E.R. Dyer, Science in
Space, eds. L.V. Berkner and H. Odishaw (1961).)

## Exercise

1. What is the spectral type of Alpha Scorpii?
2. What is its apparent magnitude? Expected absolute magnitude? Bolometric luminosity?
3. What is its distance estimated from its apparent magnitude? Measured directly by parallax? Why do these differ?
4. What is the expected diameter of the star in km , in $R_{\odot}$ and in AU? What is then the expected angular diameter seen from Earth? Can it be resolved by the HST?
(Always show your work clearly, and cite the references.)

## To measure the stellar abundance

- By spectroscopy
- Stellar composition $X, Y, Z=$ mass fraction of H , He and all other elements ("metals") $Z$ : metallicity $\quad X+Y+Z=1$
-Solar abundance: $X_{\odot}=0.747 ; Y_{\odot}=0.236 ; Z_{\odot}=0.017$
- One often compares the iron abundance of a star to that of the sun. Iron is not the most abundant (only 0.001 ), but easy to measure in spectra. Why?


$$
\begin{aligned}
& {[\mathrm{Fe} / \mathrm{H}]=\log _{10}\left(\frac{\mathrm{~N}_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\text {star }}-\log _{10}\left(\frac{\mathrm{~N}_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\odot}} \\
& \quad \log \left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\odot}=-4.33 \\
& \text { i.e., } 1 \text { iron atom for } 20,000 \mathrm{H} \text { atoms }
\end{aligned}
$$

$$
[M / H] \approx \log \left(Z / Z_{\odot}\right)
$$

Younger stars tend to be more metal-rich. Stars older than 10 Gyr almost all have $[\mathrm{Fe} / \mathrm{H}] \lesssim-0.5$; stars younger than 5 Gyr have $[\mathrm{Fe} / \mathrm{H}] \gtrsim-0.5$.




## Cosmic element factories --- the Big

Bang, stellar nucleosynthesis, supernova explosions, and compact mergers

## To measure the stellar age

- Very tricky. Often one relies on measurements of $M v, T e f f$, [ $\mathrm{Fe} / \mathrm{H}]$, and then uses some kind of theoretically computed isochrones to interpolate the age (and mass)
- Crude diagnostics include
$\checkmark$ Lithium absorption line, e.g., 6707A
$\checkmark$ Chromospheric activities, e.g., X-ray or Ca II emission
$\checkmark$ Evolving off the main sequence
- ... hence subject to large uncertainties


## References:

Edvardsson et al., 1993, A\&A, 275, 101
Nordström et al., 2004, A\&A, 418, 989


## An MS star of the same spectral type

## A PMS (young) star

Figure 16.9 Lithium absorption in a pre-main-sequence star. Shown is a portion of the optical spectrum of BP Tau, a T Tauri star of spectral type K7, corresponding to an effective temperature of 4000 K . Also shown, for comparison, is a main-sequence star of the same spectral type, 61 Cyg B. Only in the first star do we see the Li I absorption line at $6708 \AA$. Both objects also have a strong line due to neutral calcium.




Fig. 1. Kurucz's (1991a) new model for Vega compared with a series of independent ilv_antinal macome ments, specifically those by Hayes \& Latham (1985) and by Tug et al. (1977).

## Check out Aumann+84 for

 discovery of debris materials by IRAS.
## Pre-main sequence evolutionary models (tracks)


http://stev.oapd.inaf.it/

## Stellar populations

- Population I ....... Stars in the Galactic disk; like the Sun; metal rich
- Population II ..... Stars like those in the globular clusters; metal poor
- Population III .... Stars formed in the early universe; perhaps very hot and luminous; metal free


Distribution of Star Populations in Milky Way


Typical properties of Stellar Populations in the Milky Way

|  | Population I |  |  | Population II |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | very young | young |  | old |  | very old |
| Scale height $[\mathrm{kpc}]$ | 60 | 100 |  | 500 | 2000 |  |
| $\Sigma_{\mathrm{w}}\left[\mathrm{km} \mathrm{s}^{-1}\right]$ | 8 | 10 |  | 25 | 75 |  |
| $Z$ | $>0.02$ | 0.01 |  | 0.005 | $<0.002$ |  |
| Age (rel. to the Universe) | $<0.05$ | 0.25 |  | 0.75 | 1 |  |
| Distribution | generally in aggregates |  | spherical |  |  |  |

Radial abundance gradient in the disk


Figure 2 (a) The radial abundance gradient in the galactic disk. Mean metallicities from DDO and UBV photometry from Janes (1979) (triangles) are plotted versus galactocentric distance relative to the Sun. Also shown are results from Washington photometry of classical Cepheids by Harris (1981) (solid circles) and high-dispersion abundance analysis of $G$ to $M$ supergiants by Luck \& Bond (1980) (open circles); (b) The relation between age and metallicity for the open cluster samples of Janes (1979, Table 8). Ages are taken from McClure \& Twarog (1978), Jennens \& Helfer (1975), Cannon (1970), and sources quoted by Janes. (Reliable ages were not found for six clusters.) Open circles distinguish clusters with galactocentric radius larger than the solar value by more than 1 kpc . No correction has been made for any vertical abundance gradient.

## Age and metallicity for open clusters

