Thermonuclear Reactions

- Eddington in 1920s hypothesized that fusion reactions between light elements were the energy source of the stars.
- Stellar evolution = (con) sequence of nuclear reactions
- $E_{\text{kinetic}} \approx kT_c \approx 8.62 \times 10^{-8}\, T \sim$ keV,

  but $E_{\text{Coulomb barrier}} = \frac{Z_1 Z_2 e^2}{r} = \frac{1.44\, Z_1 Z_2}{r[\text{fm}]} \sim$ MeV, 3 orders higher than the kinetic energy of the particles.

- Tunneling effect in QM proposed by Gamow (1928, Z. Physik, **52**, 510);
  applied to energy source in stars by Atkinson
  & Houtermans (1929, Z. Physik, **54**, 656)

George Gamow (1904-1968)

Russian-born physicist, stellar and big bang nucleosynthesis, CMB, DNA, Mr. Thompkins series

1929 U Copenhagen

1960s U Colorado

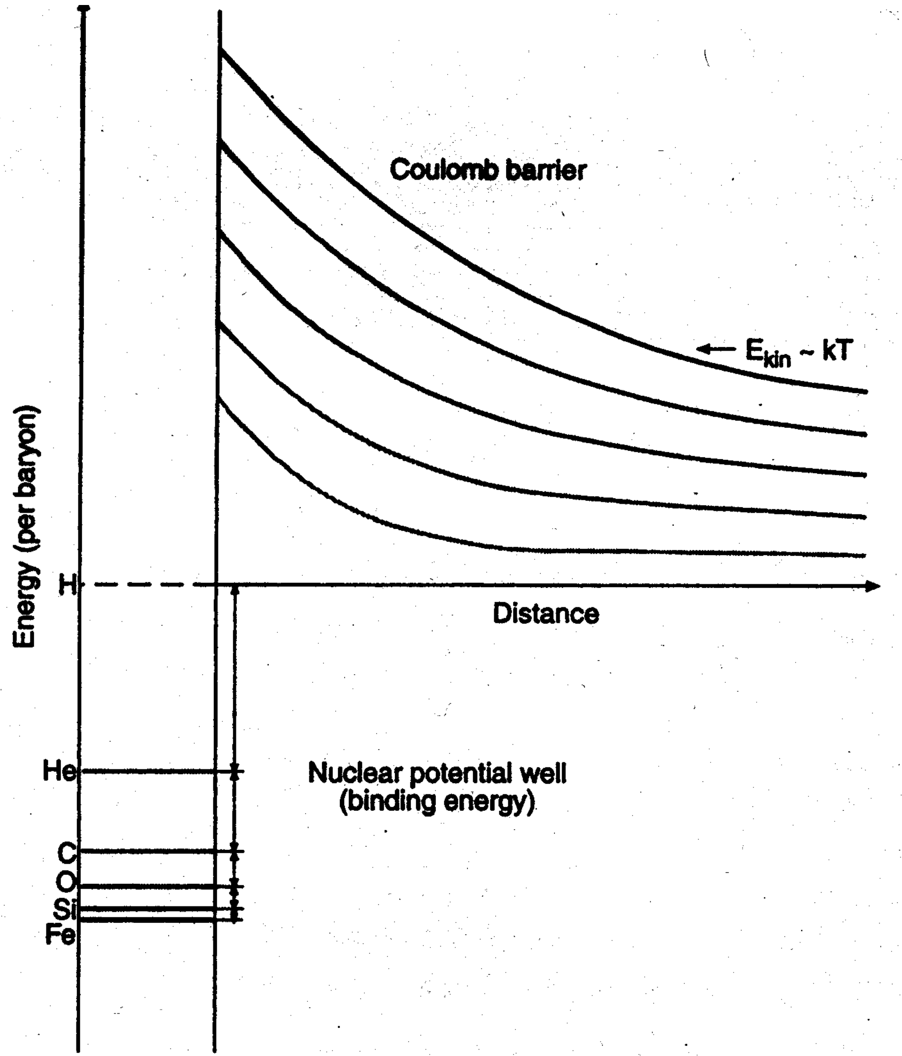


Figure 4.2 Schematic representation of the Coulomb barrier – the repulsive potential encountered by a nucleus in motion relative to another – and the short-range negative potential well that is due to the nuclear force. The height of the barrier and the depth of the well depend on the nuclear charge (atomic number).

Quantum mechanics tunneling effect

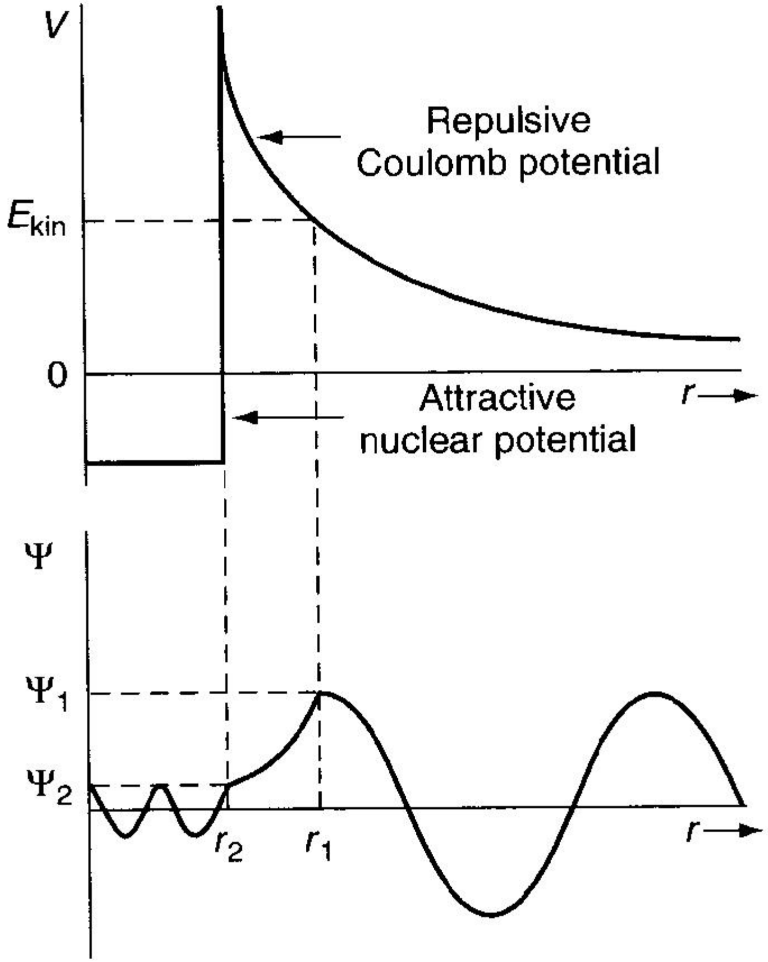


Figure 3.4 Illustration of the potential seen by particle b when approaching particle A with a kinetic energy E_{kin}, and the corresponding wavefunction Ψ; classically, particle b would reach only a distance r_1 from particle A before being repelled by the Coulomb force

Cross section for nuclear reactions (penetrating probability)

$$\propto\ e^{-\pi Z_1 Z_2 e^2/\varepsilon_0 h\nu}$$

This ↗ as v ↗

Velocity probability distribution (Maxwellian)

$$\propto\ e^{-mv^2/2kT}$$

This ↘ as v ↗

∴ **Product of these 2 factors ➔ <u>Gamow peak</u>**

D. Clayton "Principles of Stellar Evolution and Nucleosynthesis"

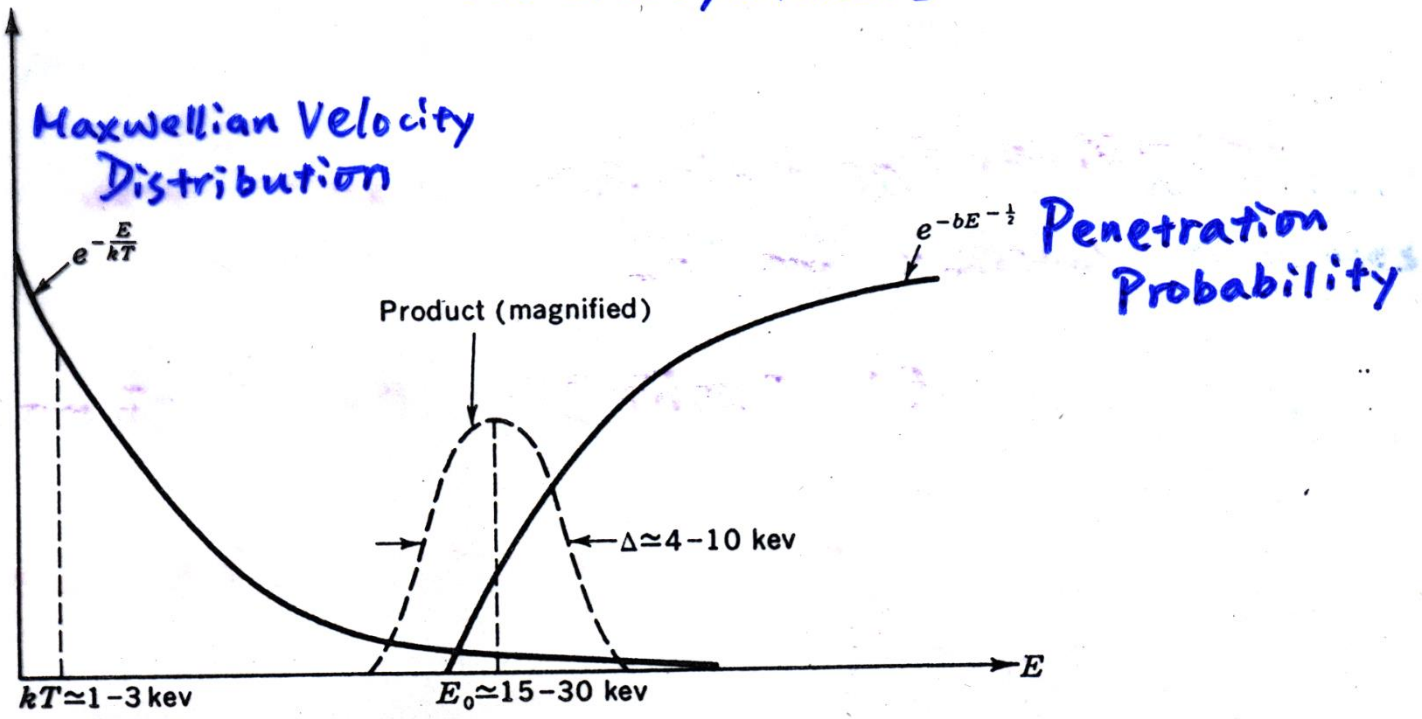


Fig. 4-6 The dominant energy-dependent factors in thermonuclear reactions. Most of the reactions occur in the high-energy tail of the maxwellian energy distribution, which introduces the rapidly falling factor $\exp(-E/kT)$. Penetration through the coulomb barrier introduces the factor $\exp(-bE^{-\frac{1}{2}})$, which vanishes strongly at low energy. Their product is a fairly sharp peak near an energy designated by E_0, which is generally much larger than kT. The peak is pushed out to this energy by the penetration factor, and it is therefore commonly called the *Gamow peak* in honor of the physicist who first studied the penetration through the coulomb barrier.

Clayton

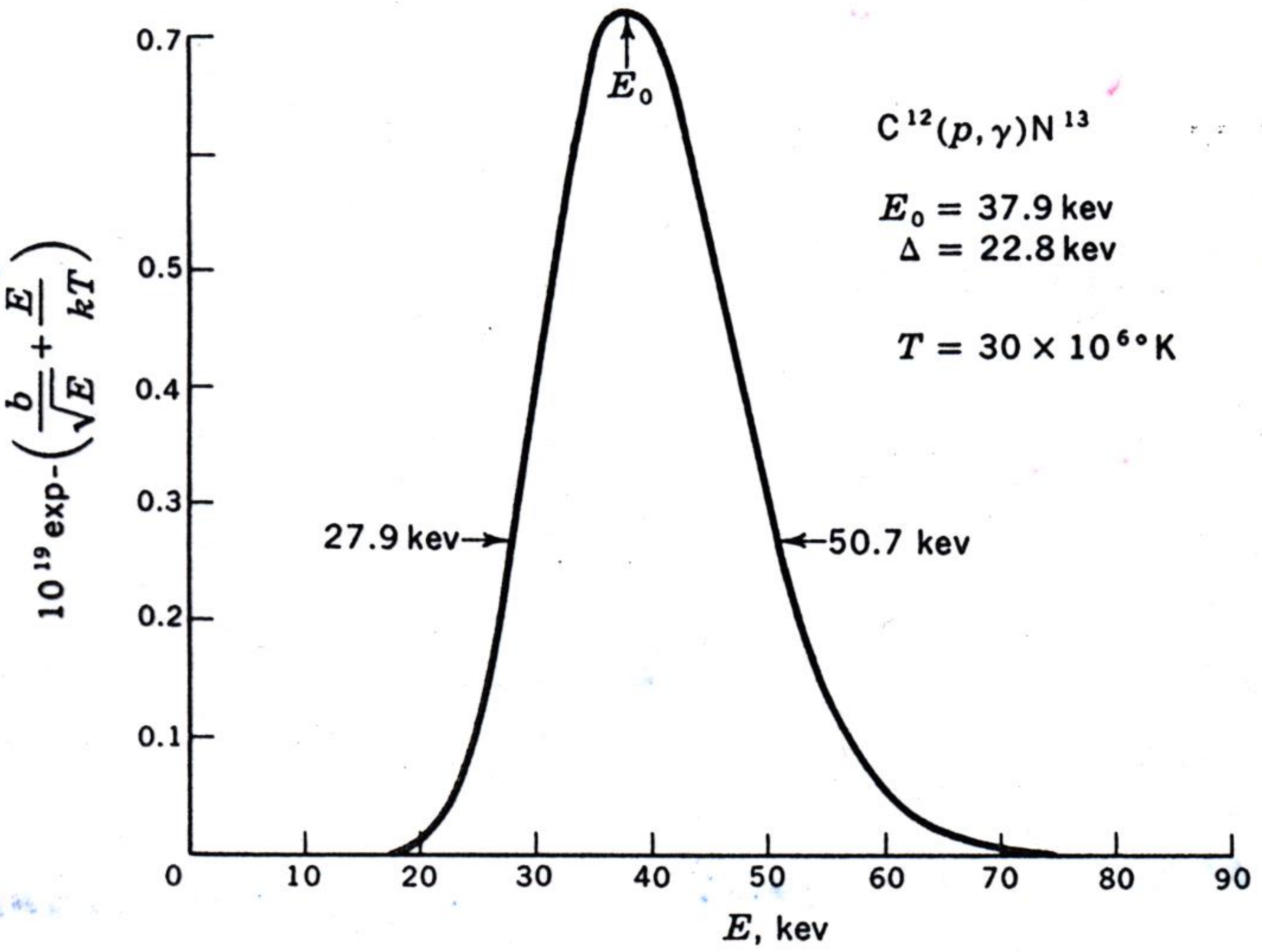


Fig. 4-7 The Gamow peak for the reaction $C^{12}(p,\gamma)N^{13}$ at $T = 30 \times 10^6$ °K. The curve is actually somewhat asymmetric about E_0, but it is nonetheless adequately approximated by a gaussian.

Clayton

Resonance → very sharp peak in the reaction rate
➔ ‘ignition’ of a nuclear reaction

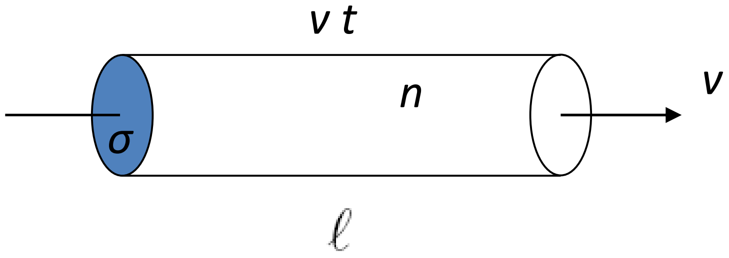
Resonance reactions
Energy of interacting particles ≈ Energy level of compound nucleus

So there exists a narrow range of temperature in which the reaction rate ↑↑
→ a power law
→ an ignition (threshold) temperature

For a thermonuclear reaction or a nucleosynthesis (fusion) process, the reaction rate is expressed as

$$\boxed{q \text{ [energy released per mass]} \propto \rho^m T^n}$$

Collision



A two-body encounter,

[# of collisions] = [total # of particles in the (moving) volume]

so $N = n\,(\sigma v t)$

✓ **# of collisions per unit time** $= {}^{N}/_{t} = n\,\sigma v$

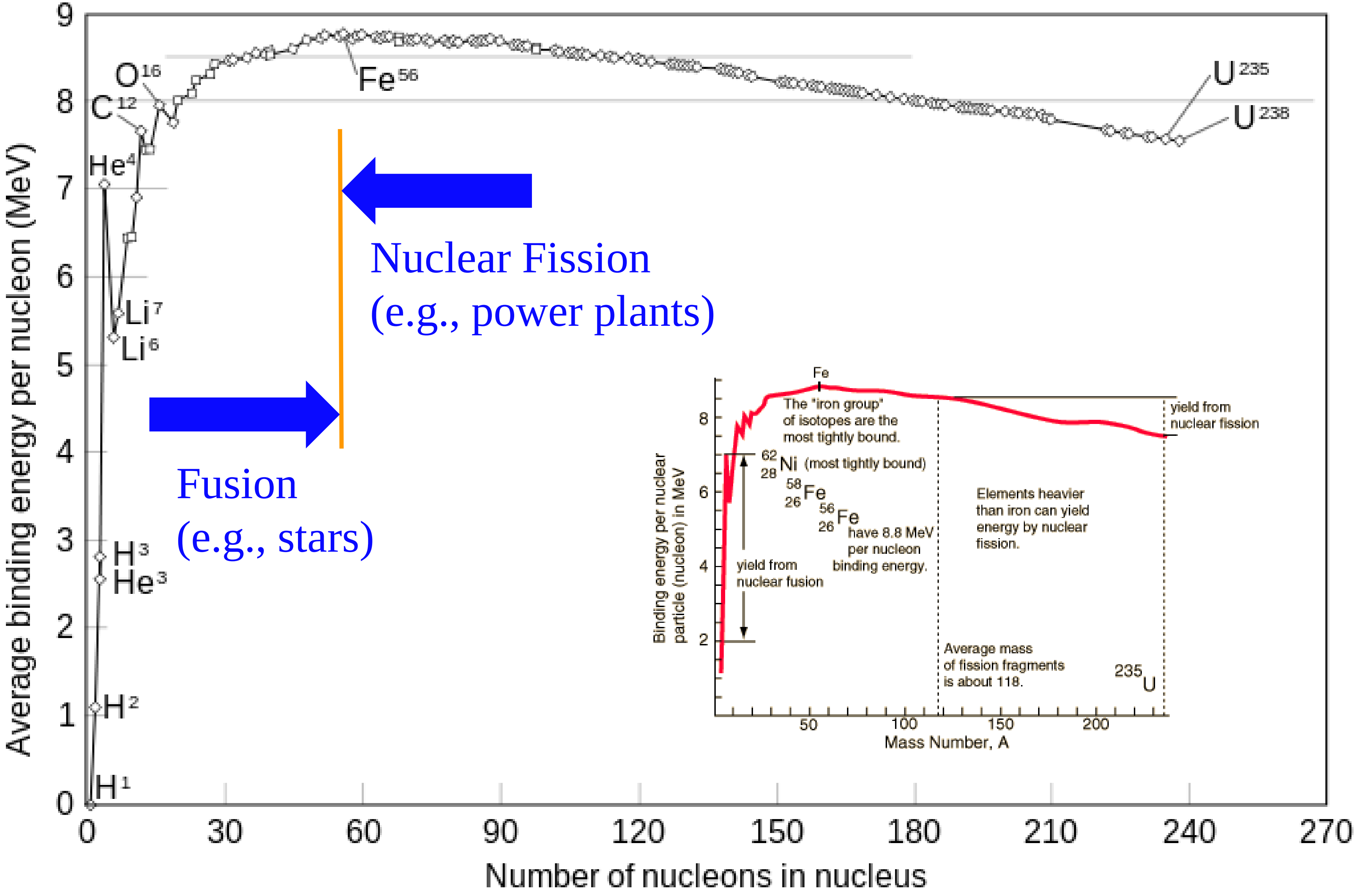
✓ **Time between 2 consecutive collisions,**

<u>mean free time</u> $(N{=}1)$, $t_{\text{col}} = 1/\,n\,\sigma v$

✓ **<u>Mean free path</u>** $\ell = v t_{\text{col}} = 1/n\,\sigma$

Nuclear reaction rate

- ✓ $r_{12} \propto \; n_1 n_2 \langle \sigma v \rangle \; \propto \; n_1 n_2 \exp\left[-C \left(\frac{z_1^2 z_2^2}{T_6} \right)^{1/3} \right] \; [\mathrm{cm}^{-3}\mathrm{s}^{-1}]$
- ✓ **As $\boldsymbol{T} \nearrow$, $\boldsymbol{r_{12}} \nearrow \nearrow$**
- ✓ **Major reactions are those with smallest $\boldsymbol{Z_1 Z_2}$**
- ✓ n_i **is the particle volume number density,** $n_i m_i = \rho X_i$, **where** X_i **is the mass fraction**
- ✓ $q_{12} \propto Q \; \rho \; X_1 \; X_2 / m_1 m_2 \; [\mathrm{erg\,g}^{-1}\mathrm{s}^{-1}]$



Nuclear Fission
(e.g., power plants)

Fusion
(e.g., stars)

$M_{\odot}c^2 = 2 \times 10^{54}$ ergs
1 amu = 931 Mev/c^2

Deuterium Burning

$$^2H + {}^1H \longrightarrow {}^3He + \gamma \quad (\underline{T > 10^6 K})$$

$$^2H({}^1H, \gamma){}^3He$$

$$Q_{DP} = 5.5\ MeV$$

$$q_{DP} = 4.19 \times 10^7 \, [D/H] \left(\frac{\rho}{1\ g\ cm^{-3}} \right) \left(\frac{T}{10^6 K} \right)^{11.8} [erg\ g^{-1} s^{-1}$$

ISM value, $\langle D/H \rangle \sim 2 \times 10^{-5}$

$n + p \rightarrow D + \gamma$ (production of D)

$D + D \rightarrow {}^{4}He + \gamma$ (destruction) → faster

The lower the mass density, the more the D abundant

→ D as a sensitive tracer of the density of the early Universe

Before the Big Bang nucleosynthesis, there were plenty of neutrons, but much less abundant than protons, so all neutrons go into making ${}^{4}He$

$$\rightarrow {}^{4}He \approx \frac{n/2}{(n+p)/4} = \frac{2n}{n+p}$$

Current value $n/p \approx 0.12$, so ${}^{4}He \approx 2/9$, as observed today.

D/H

- 156 ppm … Terrestrial seawater (1.56×10^{-4})
- 22~26 ppm … Jupiter
- 17 ppm … Saturn
- 55 ppm … Uranus
- 200 ppm … Halley’s Comet

Recall a star's central temperature

$$T_c \sim \frac{\mu G M}{R} \cdot \alpha$$

mass distr.

Numerically

$$T_c = 7.5 \times 10^6 \, K \left(\frac{M_*}{M_\odot} \right) \left(\frac{R_*}{R_\odot} \right)^{-1}$$

$$\therefore \quad M_* = 0.4 \, M_\odot \longrightarrow T_c \sim 10^6 \, K$$

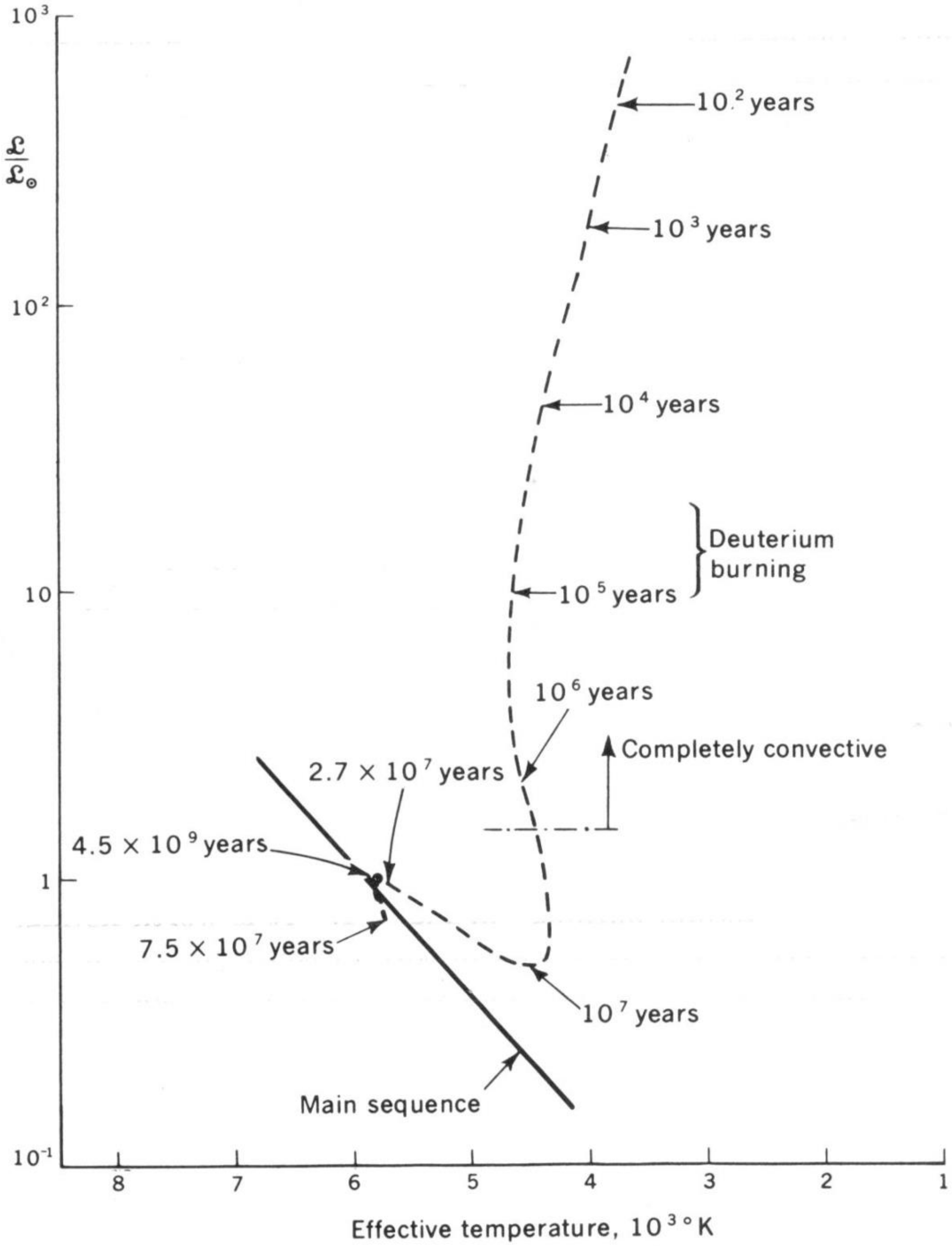
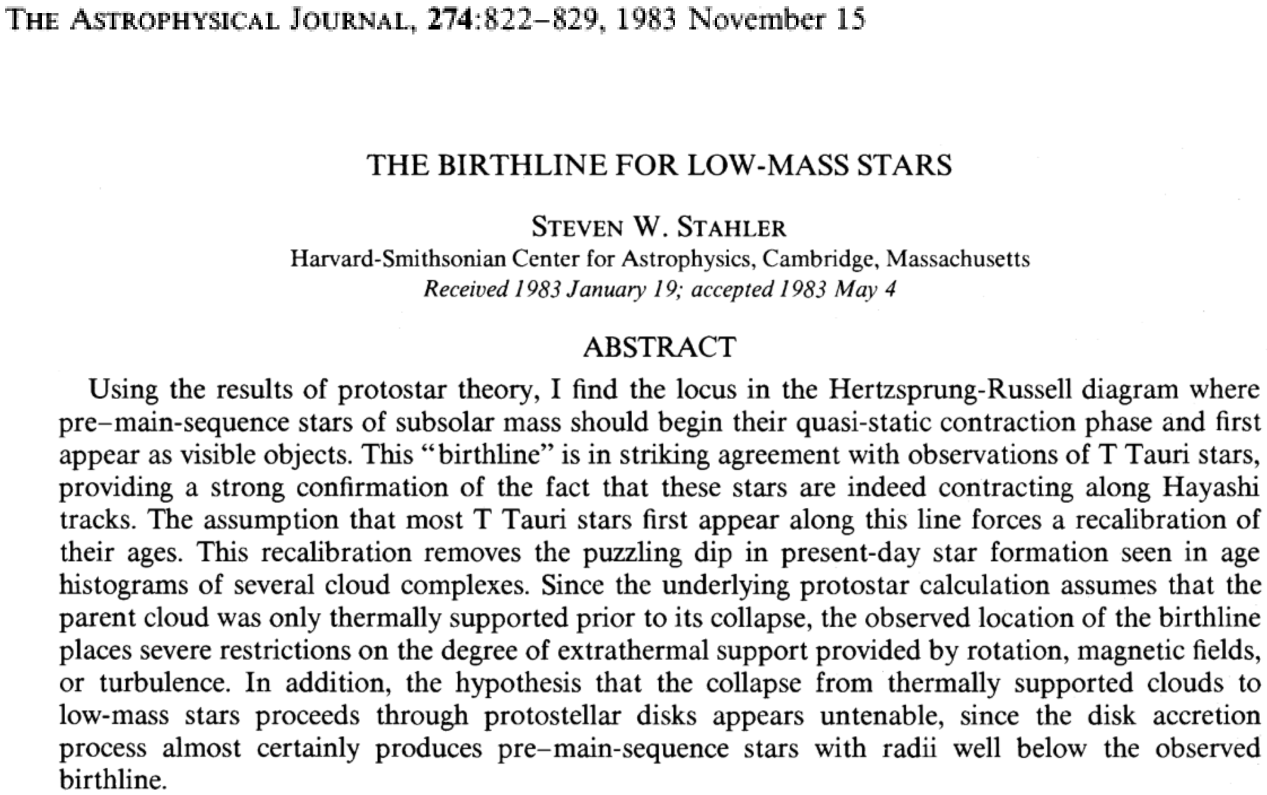


Fig. 5-1 The path on the H-R diagram of the contraction of the sun to the main sequence. The interior has become sufficiently hot to burn deuterium after about 10^5 years. The contraction ceases near the main sequence when the core has become hot enough to replenish the solar luminosity with the thermonuclear power generated by the fusion of hydrogen into helium. [*After D. Ezer and A. G. W. Cameron, The Contraction Phase of Stellar Evolution, in R. F. Stein and A. G. W. Cameron (eds.), "Stellar Evolution," Plenum Press, New York,* 1966.]

Clayton



The Astrophysical Journal, 274:822–829, 1983 November 15

THE BIRTHLINE FOR LOW-MASS STARS

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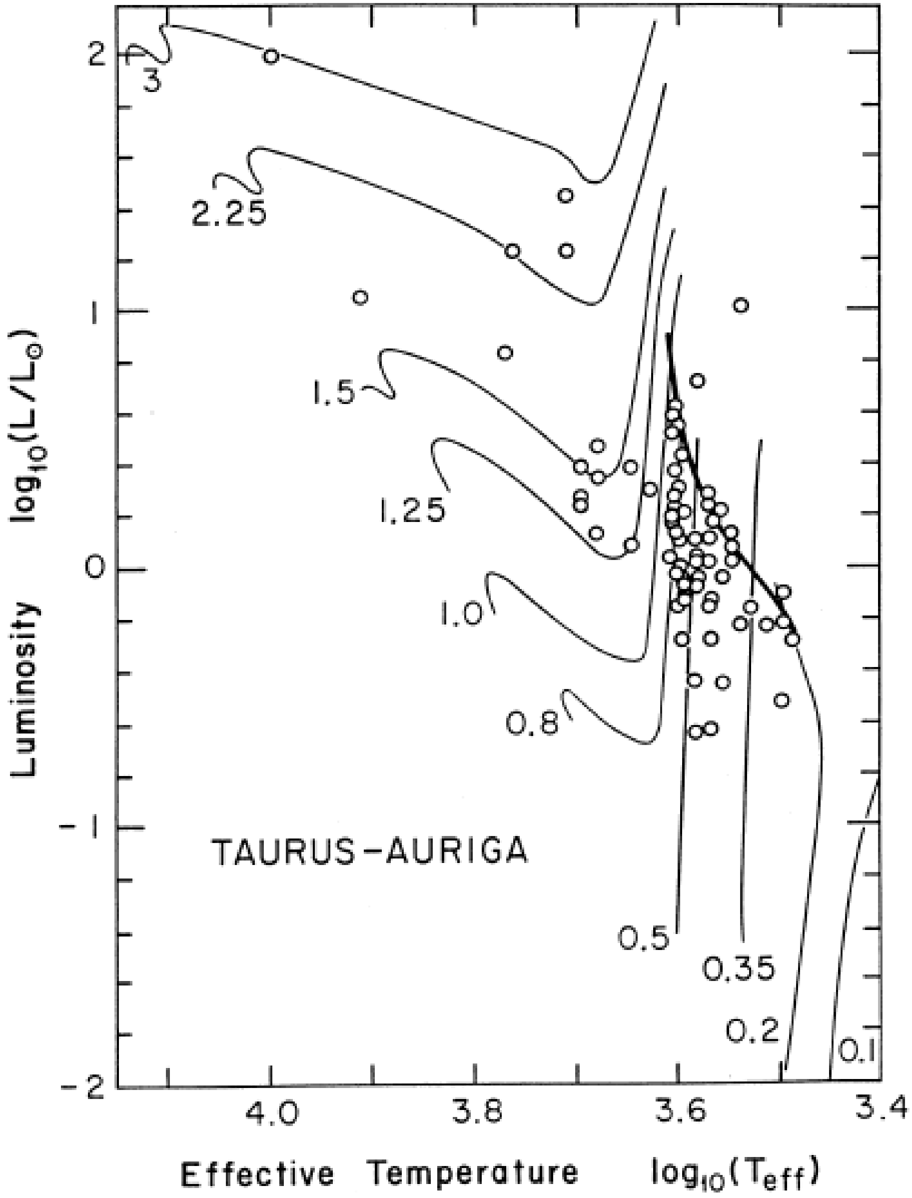
Received 1983 January 19; accepted 1983 May 4

ABSTRACT

Using the results of protostar theory, I find the locus in the Hertzsprung-Russell diagram where pre–main-sequence stars of subsolar mass should begin their quasi-static contraction phase and first appear as visible objects. This “birthline” is in striking agreement with observations of T Tauri stars, providing a strong confirmation of the fact that these stars are indeed contracting along Hayashi tracks. The assumption that most T Tauri stars first appear along this line forces a recalibration of their ages. This recalibration removes the puzzling dip in present-day star formation seen in age histograms of several cloud complexes. Since the underlying protostar calculation assumes that the parent cloud was only thermally supported prior to its collapse, the observed location of the birthline places severe restrictions on the degree of extrathermal support provided by rotation, magnetic fields, or turbulence. In addition, the hypothesis that the collapse from thermally supported clouds to low-mass stars proceeds through protostellar disks appears untenable, since the disk accretion process almost certainly produces pre–main-sequence stars with radii well below the observed birthline.



Protostars are heavily embedded in clouds, so obscured, with no definition of $T_{\rm eff}$

Birthline=beginning of PMS; star becomes optically visible ≈ deuterium main sequence

Stahler (1983, 1988),
Palla & Stahler (1990)

… compared with observations

1987ARA&A..25...23S

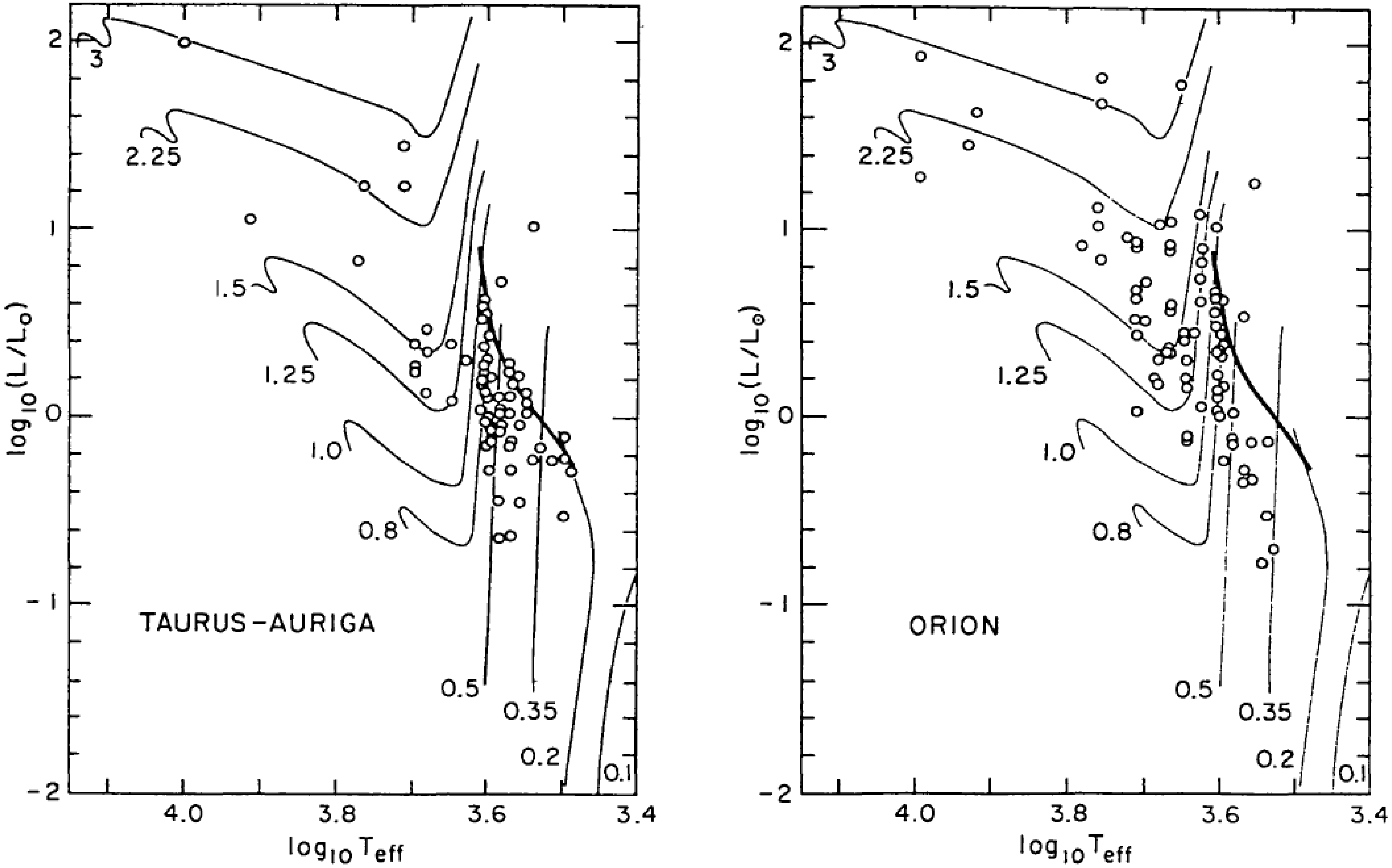


Figure 4 Hertzsprung-Russell diagrams from Cohen & Kuhi (1979) showing theoretical pre-main-sequence contraction tracks and T Tauri stars in the Taurus-Auriga and Orion cloud complexes. The heavy solid curve is the theoretical "birthline" of Stahler (1983).

Lithium Burning

$^{7}Li + {}^{1}H \longrightarrow {}^{4}He + {}^{4}He$ $(\underline{T > 3 \times 10^{6} K})$

ISM $[Li/H] \sim 2 \times 10^{-9}$

Primordial abundance 10 x lower,
produced by cosmic rays α hitting ^4He
(inverse reaction)

Li measurable in stellar spectra

Li I 6708Å absorption

(actually doublet 6707.78 and 6707.93
but difficult to resolve

Presence of Li I λ6707 absorption → stellar youth

Ca I λ6718 prominent in late-type stars

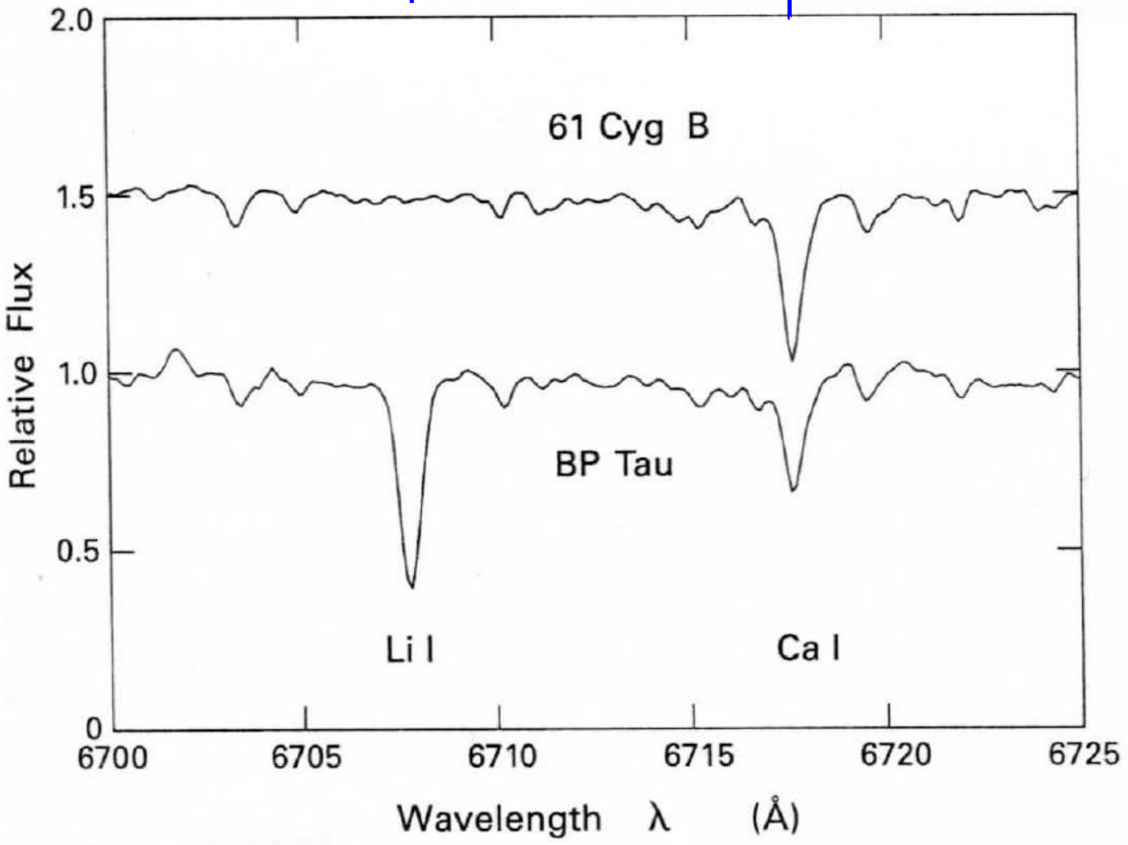


Figure 16.9 Lithium absorption in a pre-main-sequence star. Shown is a portion of the optical spectrum of BP Tau, a T Tauri star of spectral type K7, corresponding to an effective temperature of 4000 K. Also shown, for comparison, is a main-sequence star of the same spectral type, 61 Cyg B. Only in the first star do we see the Li I absorption line at 6708 Å. Both objects also have a strong line due to neutral calcium.

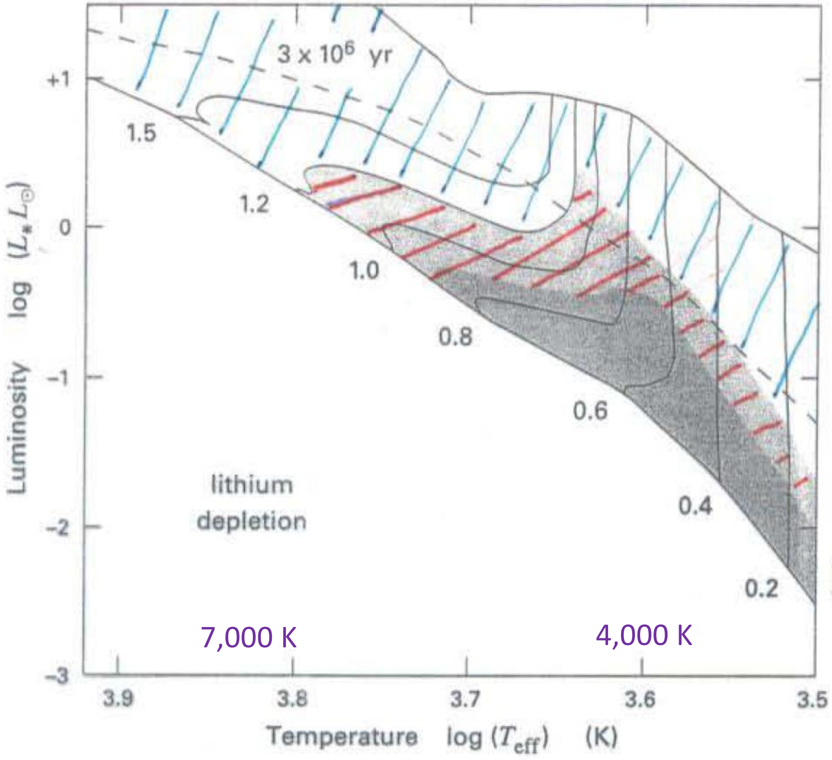
Stahler & Palla

M > 1.2 M_sun → shallow convection → surface Li does not deplete during contraction

For protostars with $T_c \geq 3 \times 10^6$K, the central lithium is readily destroyed.

Stars $\geq 0.9\ M_\odot$ become radiative at the core, so Li not fully depleted.

Li abundance → age clock



2×10^{-9}

[Li/H] ≈ ISM

[Li/H] ≈ 0.1 ISM

[Li/H] < 0.1 ISM

Figure 16.10 Theoretical prediction of pre-main-sequence lithium depletion. Within the white area between the birthline and the ZAMS, the surface [Li/H] is equal to its interstellar value of 2×10^{-9}. Stars in the lightly shaded region have depleted the element down to 0.1 times the interstellar value. The darker shading indicates depletion by at least this amount. Note also the masses on the ZAMS, in solar units, and the indicated isochrone.

Stahler & Palla

Stars	$\mathcal{M}/\mathrm{M}_{\odot} > 0.08$, core H fusion Spectral types O, B, A, F, G, K, M
Brown Dwarfs	$0.065 > \mathcal{M}/\mathrm{M}_{\odot} > 0.013$, core D fusion $0.080 > \mathcal{M}/\mathrm{M}_{\odot} > 0.065$, core Li fusion Spectral types M6.5–9, L, T, Y Electron degenerate core ✓$10\ \mathrm{g\ cm^{-3}} < \rho_c < 10^3\ \mathrm{g\ cm^{-3}}$ ✓$T_c < 3 \times 10^6$ K
Planets	$\mathcal{M}/\mathrm{M}_{\odot} < 0.013$, no fusion ever

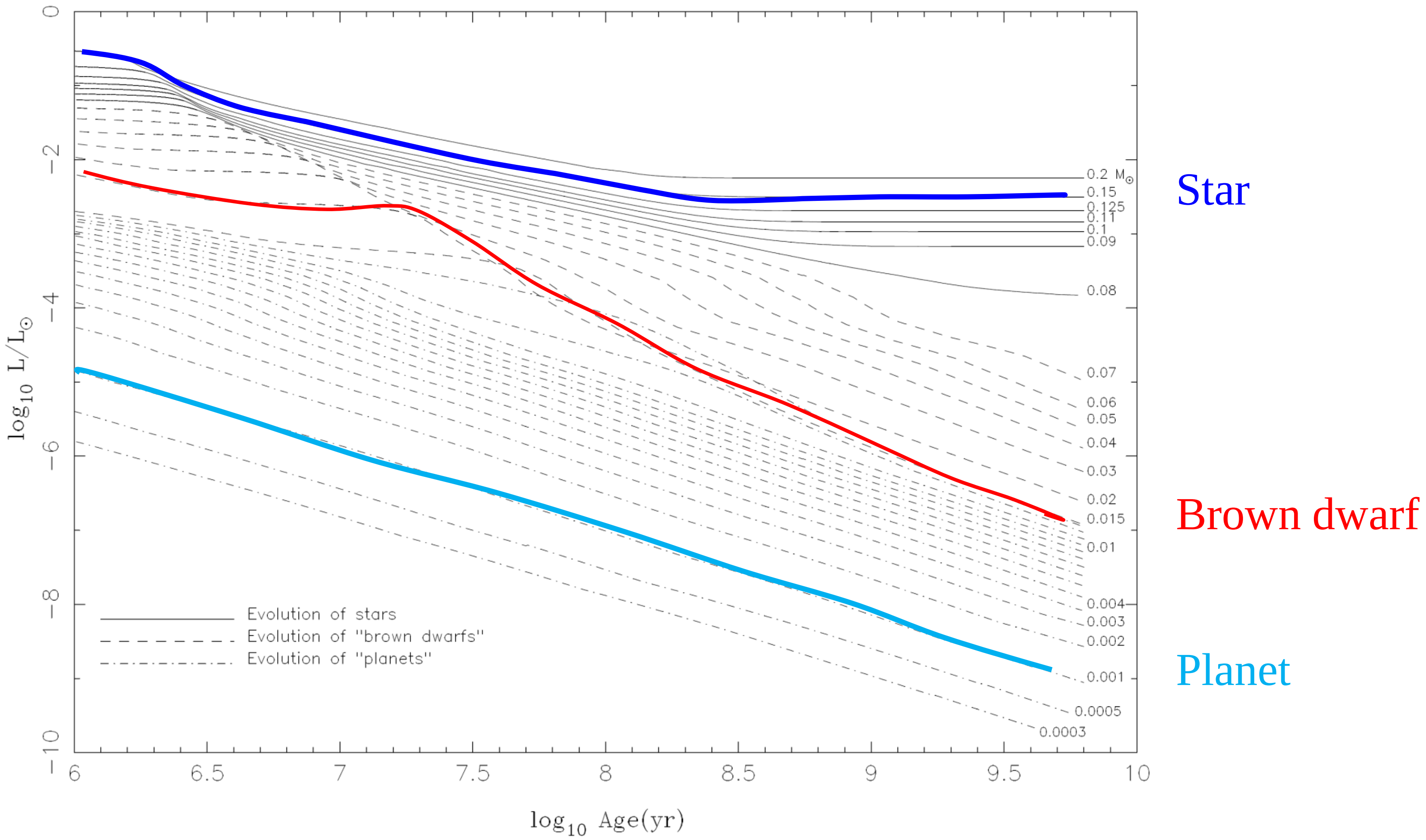
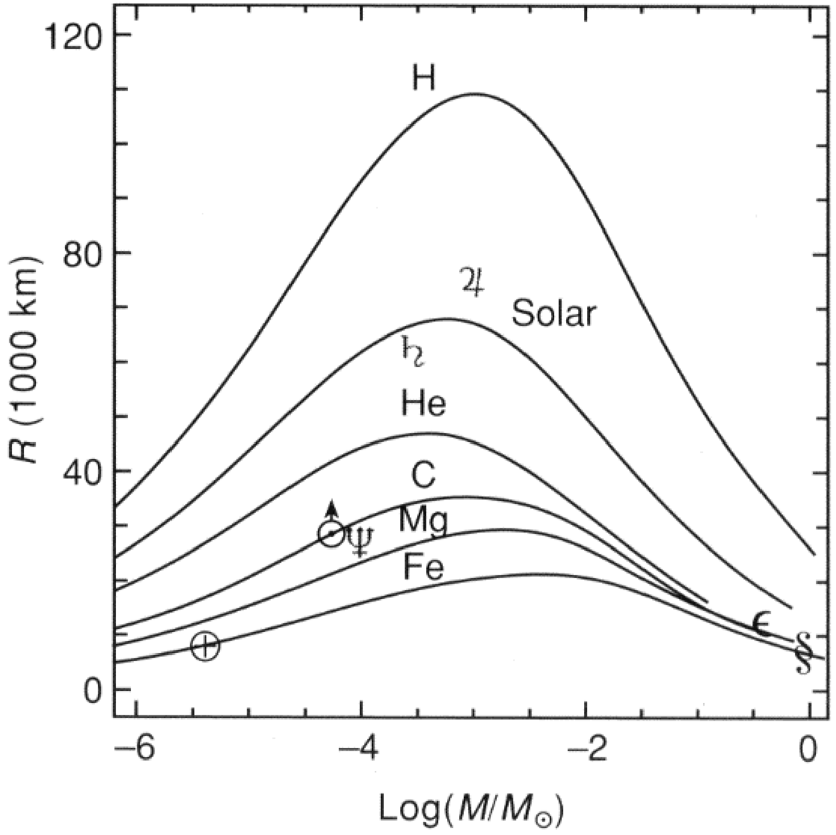


FIG. 7.—Evolution of the luminosity (in $L_\odot$) of solar-metallicity M dwarfs and substellar objects vs. time (in yr) after formation. The stars, "brown dwarfs" and "planets" are shown as solid, dashed, and dot-dashed curves, respectively. In this figure, we arbitrarily designate as "brown dwarfs" those objects that burn deuterium, while we designate those that do not as "planets." The masses (in $M_\odot$) label most of the curves, with the lowest three corresponding to the mass of Saturn, half the mass of Jupiter, and the mass of Jupiter.

Burrows



Brown dwarfs and very low-mass stars … partial P_{deg}^{e-}

White dwarfs

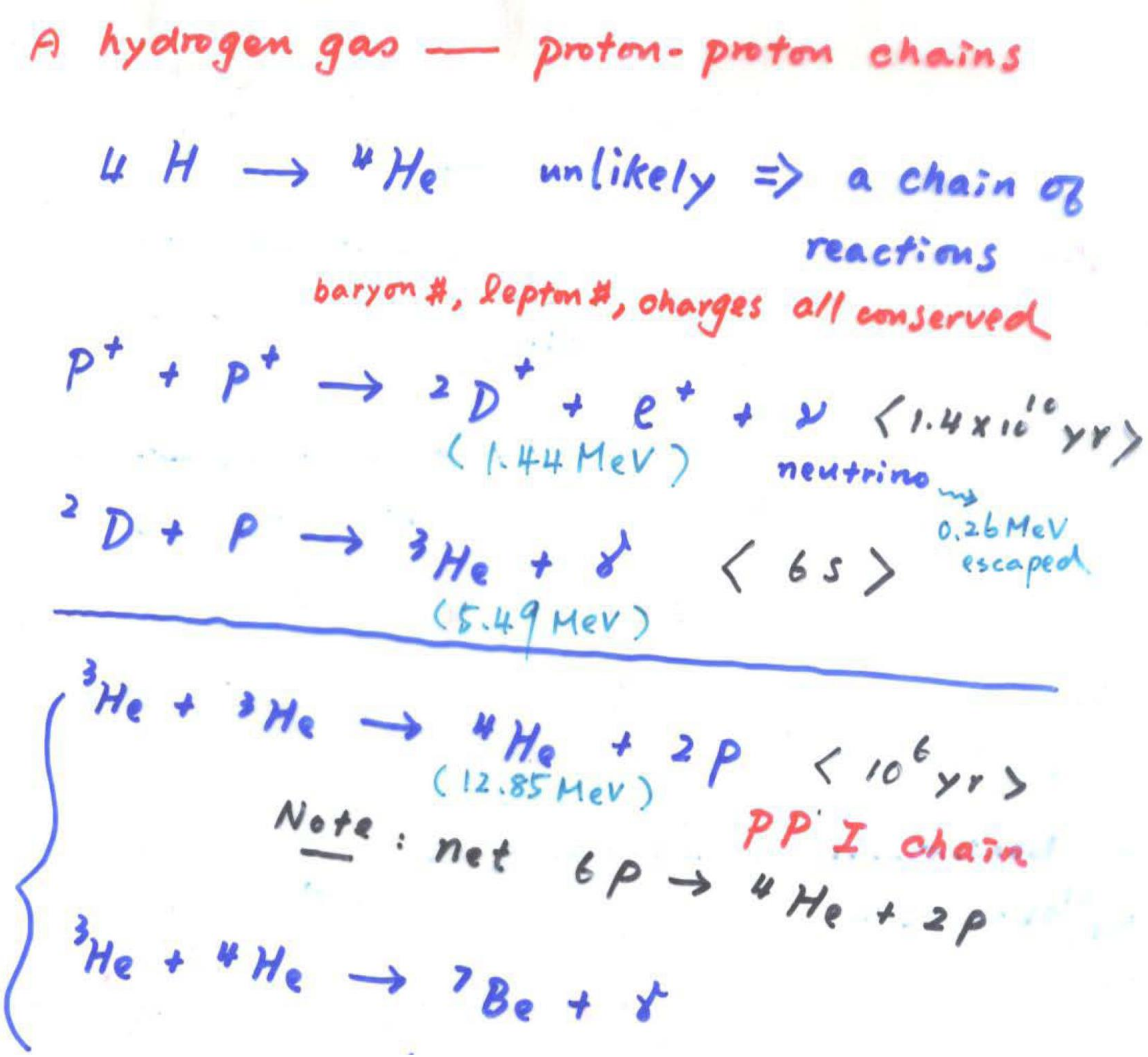
≈ completely degenerate,
R↘ as M↗

Terrestrial planets

R ↗ as M↗ ← complicated EoSs

Figure 12.4 Mass-radius relation for low-mass objects (following H. S. Zapolsky & E. E. Salpeter, *Astrophys. J.* 158). Different curves correspond to different compositions, as indicated. The locations of several planets – Earth, Jupiter, Saturn, Uranus and Neptune – are marked by the planets' symbols. Also marked are the locations of two white dwarfs, Sirius B (§) and 40 Eridani B (ϵ) (data from D. Koester (1987), *Astrophys. J.*, 322).

Mass-radius relation max @ $M_{\text{Jupiter}} \approx$ $(1/1000)\ M_{\odot}$



A hydrogen gas — proton-proton chains

$4H \rightarrow {}^4He$ unlikely ⇒ a chain of reactions

baryon #, lepton #, charges all conserved

$p^+ + p^+ \rightarrow {}^2D^+ + e^+ + \nu$ $\langle 1.4 \times 10^{10}\ yr \rangle$

(1.44 MeV) neutrino ⇝ 0.26 MeV escaped

${}^2D + p \rightarrow {}^3He + \gamma$ $\langle 6\ s \rangle$

(5.49 MeV)

${}^3He + {}^3He \rightarrow {}^4He + 2p$ $\langle 10^6\ yr \rangle$

(12.85 MeV) PP I chain

Note: net $6p \rightarrow {}^4He + 2p$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

0.420 MeV to the positron and neutrino; position and electron (each 0.511 MeV rest energy) annihilate ➔ 1.442 MeV

… but the nucleus of deuterium, a deuteron, consists of a proton and a neutron!

- ✓ $p + p \rightarrow {}^{2}He$ (unstable)$\rightarrow p + p$
- ✓ **Hans Bethe (1939) realized that the weak interaction was capable of converting a proton to a neutron (!) first**
- ✓ **Weak interaction → very small cross section**
- ✓ **The neutron is more massive, so this requires energy, i.e., it is an endothermic process, but neutron + proton → deuteron (releasing binding energy, so exothermic)**

$^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p$ $\langle 10^{6}\,yr \rangle$

(12.85 MeV)

PP I chain

Note: net $6p \rightarrow {}^{4}He + 2p$

$^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$

$^{7}Be + e^{-} \rightarrow {}^{7}Li + \gamma$

$^{7}Li + p \rightarrow 2\,{}^{4}He$

PP II chain

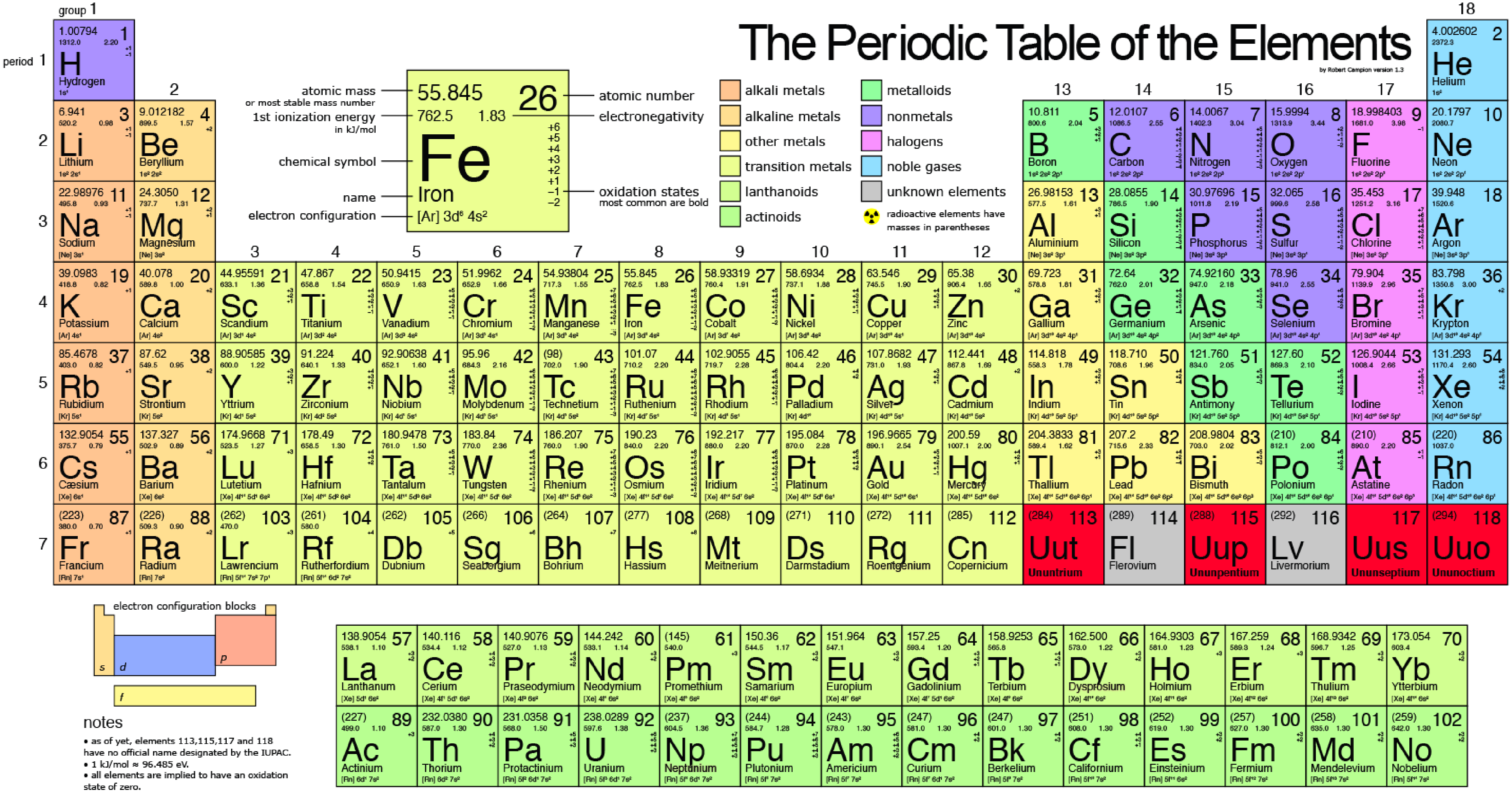
$^{7}Be + p^{+} \rightarrow {}^{8}B + \gamma$

$^{8}B \rightarrow {}^{8}Be + e^{+} + \gamma$

$^{8}Be \rightarrow 2\,{}^{4}He$

PP III chain

All 3 branches operate simultaneously.



The Periodic Table of the Elements

by Robert Campion version 1.3

26
Fe
Iron
55.845
762.5 1.83
[Ar] $3d^6 4s^2$
+6 +5 +4 +3 +2 +1 −1 −2

atomic mass — or most stable mass number
1st ionization energy — in kJ/mol
chemical symbol
name
electron configuration
atomic number
electronegativity
oxidation states — most common are bold

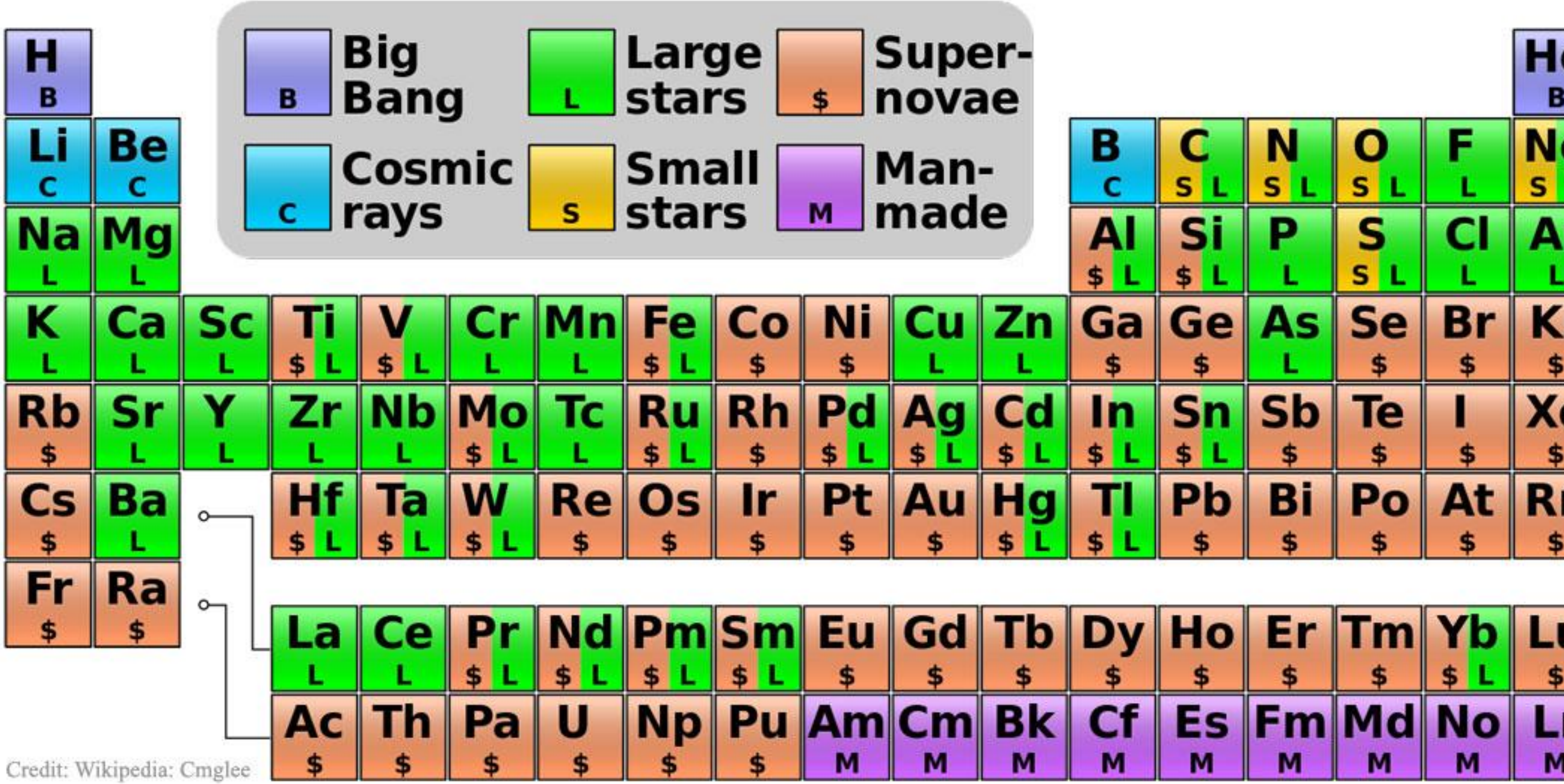
🟧 alkali metals	🟩 metalloids
🟨 alkaline metals	🟪 nonmetals
🟨 other metals	🟪 halogens
🟨 transition metals	🟦 noble gases
🟩 lanthanoids	⬜ unknown elements
🟩 actinoids	☢ radioactive elements have masses in parentheses

	group 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
period 1	1 **H** Hydrogen 1.00794 1312.0 2.20 $1s^1$																	2 **He** Helium 4.002602 2372.3 $1s^2$
2	3 **Li** Lithium 6.941 520.2 0.98 $1s^2 2s^1$	4 **Be** Beryllium 9.012182 899.5 1.57 $1s^2 2s^2$											5 **B** Boron 10.811 800.6 2.04 $1s^2 2s^2 2p^1$	6 **C** Carbon 12.0107 1086.5 2.55 $1s^2 2s^2 2p^2$	7 **N** Nitrogen 14.0067 1402.3 3.04 $1s^2 2s^2 2p^3$	8 **O** Oxygen 15.9994 1313.9 3.44 $1s^2 2s^2 2p^4$	9 **F** Fluorine 18.998403 1681.0 3.98 $1s^2 2s^2 2p^5$	10 **Ne** Neon 20.1797 2080.7 $1s^2 2s^2 2p^6$
3	11 **Na** Sodium 22.98976 495.8 0.93 [Ne] $3s^1$	12 **Mg** Magnesium 24.3050 737.7 1.31 [Ne] $3s^2$											13 **Al** Aluminium 26.98153 577.5 1.61 [Ne] $3s^2 3p^1$	14 **Si** Silicon 28.0855 786.5 1.90 [Ne] $3s^2 3p^2$	15 **P** Phosphorus 30.97696 1011.8 2.19 [Ne] $3s^2 3p^3$	16 **S** Sulfur 32.065 999.6 2.58 [Ne] $3s^2 3p^4$	17 **Cl** Chlorine 35.453 1251.2 3.16 [Ne] $3s^2 3p^5$	18 **Ar** Argon 39.948 1520.6 [Ne] $3s^2 3p^6$
4	19 **K** Potassium 39.0983 418.8 0.82 [Ar] $4s^1$	20 **Ca** Calcium 40.078 589.8 1.00 [Ar] $4s^2$	21 **Sc** Scandium 44.95591 633.1 1.36 [Ar] $3d^1 4s^2$	22 **Ti** Titanium 47.867 658.8 1.54 [Ar] $3d^2 4s^2$	23 **V** Vanadium 50.9415 650.9 1.63 [Ar] $3d^3 4s^2$	24 **Cr** Chromium 51.9962 652.9 1.66 [Ar] $3d^5 4s^1$	25 **Mn** Manganese 54.93804 717.3 1.55 [Ar] $3d^5 4s^2$	26 **Fe** Iron 55.845 762.5 1.83 [Ar] $3d^6 4s^2$	27 **Co** Cobalt 58.93319 760.4 1.91 [Ar] $3d^7 4s^2$	28 **Ni** Nickel 58.6934 737.1 1.88 [Ar] $3d^8 4s^2$	29 **Cu** Copper 63.546 745.5 1.90 [Ar] $3d^{10} 4s^1$	30 **Zn** Zinc 65.38 906.4 1.65 [Ar] $3d^{10} 4s^2$	31 **Ga** Gallium 69.723 578.8 1.81 [Ar] $3d^{10} 4s^2 4p^1$	32 **Ge** Germanium 72.64 762.0 2.01 [Ar] $3d^{10} 4s^2 4p^2$	33 **As** Arsenic 74.92160 947.0 2.18 [Ar] $3d^{10} 4s^2 4p^3$	34 **Se** Selenium 78.96 941.0 2.55 [Ar] $3d^{10} 4s^2 4p^4$	35 **Br** Bromine 79.904 1139.9 2.96 [Ar] $3d^{10} 4s^2 4p^5$	36 **Kr** Krypton 83.798 1350.8 3.00 [Ar] $3d^{10} 4s^2 4p^6$
5	37 **Rb** Rubidium 85.4678 403.0 0.82 [Kr] $5s^1$	38 **Sr** Strontium 87.62 549.5 0.95 [Kr] $5s^2$	39 **Y** Yttrium 88.90585 600.0 1.22 [Kr] $4d^1 5s^2$	40 **Zr** Zirconium 91.224 640.1 1.33 [Kr] $4d^2 5s^2$	41 **Nb** Niobium 92.90638 652.1 1.60 [Kr] $4d^4 5s^1$	42 **Mo** Molybdenum 95.96 684.3 2.16 [Kr] $4d^5 5s^1$	43 **Tc** Technetium (98) 702.0 1.90 [Kr] $4d^5 5s^2$	44 **Ru** Ruthenium 101.07 710.2 2.20 [Kr] $4d^7 5s^1$	45 **Rh** Rhodium 102.9055 719.7 2.28 [Kr] $4d^8 5s^1$	46 **Pd** Palladium 106.42 804.4 2.20 [Kr] $4d^{10}$	47 **Ag** Silver 107.8682 731.0 1.93 [Kr] $4d^{10} 5s^1$	48 **Cd** Cadmium 112.441 867.8 1.69 [Kr] $4d^{10} 5s^2$	49 **In** Indium 114.818 558.3 1.78 [Kr] $4d^{10} 5s^2 5p^1$	50 **Sn** Tin 118.710 708.6 1.96 [Kr] $4d^{10} 5s^2 5p^2$	51 **Sb** Antimony 121.760 834.0 2.05 [Kr] $4d^{10} 5s^2 5p^3$	52 **Te** Tellurium 127.60 869.3 2.10 [Kr] $4d^{10} 5s^2 5p^4$	53 **I** Iodine 126.9044 1008.4 2.66 [Kr] $4d^{10} 5s^2 5p^5$	54 **Xe** Xenon 131.293 1170.4 2.60 [Kr] $4d^{10} 5s^2 5p^6$
6	55 **Cs** Cæsium 132.9054 375.7 0.79 [Xe] $6s^1$	56 **Ba** Barium 137.327 502.9 0.89 [Xe] $6s^2$	71 **Lu** Lutetium 174.9668 523.5 1.27 [Xe] $4f^{14} 5d^1 6s^2$	72 **Hf** Hafnium 178.49 658.5 1.30 [Xe] $4f^{14} 5d^2 6s^2$	73 **Ta** Tantalum 180.9478 761.0 1.50 [Xe] $4f^{14} 5d^3 6s^2$	74 **W** Tungsten 183.84 770.0 2.36 [Xe] $4f^{14} 5d^4 6s^2$	75 **Re** Rhenium 186.207 760.0 1.90 [Xe] $4f^{14} 5d^5 6s^2$	76 **Os** Osmium 190.23 840.0 2.20 [Xe] $4f^{14} 5d^6 6s^2$	77 **Ir** Iridium 192.217 880.0 2.20 [Xe] $4f^{14} 5d^7 6s^2$	78 **Pt** Platinum 195.084 870.0 2.28 [Xe] $4f^{14} 5d^9 6s^1$	79 **Au** Gold 196.9665 890.1 2.54 [Xe] $4f^{14} 5d^{10} 6s^1$	80 **Hg** Mercury 200.59 1007.1 2.00 [Xe] $4f^{14} 5d^{10} 6s^2$	81 **Tl** Thallium 204.3833 589.4 1.62 [Xe] $4f^{14} 5d^{10} 6s^2 6p^1$	82 **Pb** Lead 207.2 715.6 2.33 [Xe] $4f^{14} 5d^{10} 6s^2 6p^2$	83 **Bi** Bismuth 208.9804 703.0 2.02 [Xe] $4f^{14} 5d^{10} 6s^2 6p^3$	84 **Po** Polonium (210) 812.1 2.00 [Xe] $4f^{14} 5d^{10} 6s^2 6p^4$	85 **At** Astatine (210) 890.0 2.20 [Xe] $4f^{14} 5d^{10} 6s^2 6p^5$	86 **Rn** Radon (220) 1037.0 [Xe] $4f^{14} 5d^{10} 6s^2 6p^6$
7	87 **Fr** Francium (223) 380.0 0.70 [Rn] $7s^1$	88 **Ra** Radium (226) 509.3 0.90 [Rn] $7s^2$	103 **Lr** Lawrencium (262) 470.0 [Rn] $5f^{14} 7s^2 7p^1$	104 **Rf** Rutherfordium (261) 580.0 [Rn] $5f^{14} 6d^2 7s^2$	105 **Db** Dubnium (262)	106 **Sg** Seaborgium (266)	107 **Bh** Bohrium (264)	108 **Hs** Hassium (277)	109 **Mt** Meitnerium (268)	110 **Ds** Darmstadium (271)	111 **Rg** Roentgenium (272)	112 **Cn** Copernicium (285)	113 **Uut** **Ununtrium** (284)	114 **Fl** Flerovium (289)	115 **Uup** **Ununpentium** (288)	116 **Lv** Livermorium (292)	117 **Uus** **Ununseptium**	118 **Uuo** **Ununoctium** (294)

57 **La** Lanthanum 138.9054 538.1 1.10 [Xe] $5d^1 6s^2$	58 **Ce** Cerium 140.116 534.4 1.12 [Xe] $4f^1 5d^1 6s^2$	59 **Pr** Praseodymium 140.9076 527.0 1.13 [Xe] $4f^3 6s^2$	60 **Nd** Neodymium 144.242 533.1 1.14 [Xe] $4f^4 6s^2$	61 **Pm** Promethium (145) 540.0 [Xe] $4f^5 6s^2$	62 **Sm** Samarium 150.36 544.5 1.17 [Xe] $4f^6 6s^2$	63 **Eu** Europium 151.964 547.1 [Xe] $4f^7 6s^2$	64 **Gd** Gadolinium 157.25 593.4 1.20 [Xe] $4f^7 5d^1 6s^2$	65 **Tb** Terbium 158.9253 565.8 [Xe] $4f^9 6s^2$	66 **Dy** Dysprosium 162.500 573.0 1.22 [Xe] $4f^{10} 6s^2$	67 **Ho** Holmium 164.9303 581.0 1.23 [Xe] $4f^{11} 6s^2$	68 **Er** Erbium 167.259 589.3 1.24 [Xe] $4f^{12} 6s^2$	69 **Tm** Thulium 168.9342 596.7 1.25 [Xe] $4f^{13} 6s^2$	70 **Yb** Ytterbium 173.054 603.4 [Xe] $4f^{14} 6s^2$
89 **Ac** Actinium (227) 499.0 1.10 [Rn] $6d^1 7s^2$	90 **Th** Thorium 232.0380 587.0 1.30 [Rn] $6d^2 7s^2$	91 **Pa** Protactinium 231.0358 568.0 1.50 [Rn] $5f^2 6d^1 7s^2$	92 **U** Uranium 238.0289 597.6 1.38 [Rn] $5f^3 6d^1 7s^2$	93 **Np** Neptunium (237) 604.5 1.36 [Rn] $5f^4 6d^1 7s^2$	94 **Pu** Plutonium (244) 584.7 1.28 [Rn] $5f^6 7s^2$	95 **Am** Americium (243) 578.0 1.30 [Rn] $5f^7 7s^2$	96 **Cm** Curium (247) 581.0 1.30 [Rn] $5f^7 6d^1 7s^2$	97 **Bk** Berkelium (247) 601.0 1.30 [Rn] $5f^9 7s^2$	98 **Cf** Californium (251) 608.0 1.30 [Rn] $5f^{10} 7s^2$	99 **Es** Einsteinium (252) 619.0 1.30 [Rn] $5f^{11} 6s^2$	100 **Fm** Fermium (257) 627.0 1.30 [Rn] $5f^{12} 7s^2$	101 **Md** Mendelevium (258) 635.0 1.30 [Rn] $5f^{13} 7s^2$	102 **No** Nobelium (259) 642.0 1.30 [Rn] $5f^{14} 7s^2$

notes

- as of yet, elements 113,115,117 and 118 have no official name designated by the IUPAC.
- 1 kJ/mol ≈ 96.485 eV.
- all elements are implied to have an oxidation state of zero.



B Big Bang
L Large stars
$ Super-novae
C Cosmic rays
S Small stars
M Man-made

H B																	H B
Li C	Be C											B C	C S L	N S L	O S L	F L	N S
Na L	Mg L											Al $ L	Si $ L	P L	S S L	Cl L	A L
K L	Ca L	Sc L	Ti $ L	V $ L	Cr L	Mn L	Fe $ L	Co $	Ni $	Cu L	Zn L	Ga $	Ge $	As L	Se $	Br $	K $
Rb $	Sr L	Y L	Zr L	Nb L	Mo $ L	Tc L	Ru $ L	Rh $	Pd $ L	Ag $ L	Cd $ L	In $ L	Sn $ L	Sb $	Te $	I $	X $
Cs $	Ba L		Hf $ L	Ta $ L	W $ L	Re $	Os $	Ir $	Pt $	Au $	Hg $ L	Tl $ L	Pb $	Bi $	Po $	At $	R $
Fr $	Ra $																
			La L	Ce L	Pr $ L	Nd $ L	Pm $ L	Sm $ L	Eu $	Gd $	Tb $	Dy $	Ho $	Er $	Tm $	Yb $ L	L $
			Ac $	Th $	Pa $	U $	Np $	Pu $	Am M	Cm M	Bk M	Cf M	Es M	Fm M	Md M	No M	L M

Credit: Wikipedia: Cmglee

https://apod.nasa.gov/apod/ap160125.html

The proton-proton chain

$^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{D} + \mathrm{e}^+ + \nu_e \quad (1.44\ \mathrm{MeV},\ 1.4 \times 10^{10}\ \mathrm{yr})$
$^2\mathrm{D} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma \quad (5.49\ \mathrm{MeV},\ 6\ \mathrm{s})$

pp I important when
$T_c > 5 \times 10^6\mathrm{K}$

pp I chain

$^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + {}^1\mathrm{H} + {}^1\mathrm{H} \quad (12.85\ \mathrm{MeV},\ 10^6\ \mathrm{yr})$

Note: net $6\ {}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2\ {}^1\mathrm{H}$

$$Q_{total} = 1.44 \times 2 + 5.49 \times 2 + 12.85 = 27.7\ \mathrm{MeV}$$

$$Q_{net} = 27.7 - 0.26 \times 2 = 26.2\ \mathrm{MeV}$$

pp II chain

$^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$
$^7\mathrm{Be} + \mathrm{e}^- \rightarrow {}^7\mathrm{Li} + \nu_e$
$^7\mathrm{Li} + {}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$

pp III chain

$^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$
$^7\mathrm{Be} + {}^1\mathrm{H} \rightarrow {}^8\mathrm{B} + \gamma$
$^8\mathrm{B} + \rightarrow {}^8\mathrm{Be} + \mathrm{e}^+ + \nu_e$
$^8\mathrm{Be} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$

The baryon number, lepton number, and charges are all conserved.

All 3 branches operate simultaneously.

pp I is responsible for > 90% stellar luminosity

Exercise

Assuming that the solar luminosity is provided by $4\ ^1H \rightarrow\ ^4He$, liberating 26.73 MeV, and that the neutrinos carry off about 2% of the total energy. Estimate how many neutrinos are produced each second from the sun? What is the solar neutrino flux at the earth? (How many neutrinos pass through your body per second?)

Solution

2% is carried away by neutrinos, so the actual energy produced for radiation

$$E = (0.98 \times 26.731 \text{ MeV}) \times 1.6 \times 10^{-12} \text{ erg/eV}$$

Each alpha particle produced → 2 neutrinos, so with $L_{\odot} = 3.846 \times 10^{33}$ ergs/s, the neutrino production rate is $2 \times 10^{38}\ \nu/\text{s}$, and the flux at earth is $2 \times 10^{38}/4\pi\,(1\ \text{AU})^2 \approx 6.6 \times 10^{10}\ \nu\ \text{cm}^{-2}\text{s}^{-1}$

The thermonuclear reaction rate,

$$r_{pp} = 3.09 \times 10^{-37}\ n_p^2\ T_6^{-2/3} \exp\left(-33.81\, T_6^{-1/3}\right)$$

$$(1\ +\ 0.0123\, T_6^{1/3} + 0.0109\, T_6^{2/3} + 0.0009\, T_6)\quad [\mathrm{cm^{-3} s^{-1}}],$$

where the factor $3.09 \times 10^{-37} n_p^2\ = 11.05 \times 10^{10}\ \rho^2 X_H^2$

$$q_{pp} = 2.38 \times 10^{6}\ \rho\ X_H^2\ T_6^{-2/3} \exp\left(-33.81\, T_6^{-1/3}\right)$$

$$(1\ +\ 0.0123\, T_6^{1/3} + 0.0109\, T_6^{2/3} + 0.0009\, T_6)\ \ [\mathrm{erg\ g^{-1} s^{-1}}]$$

PP I vs PP II

i.e., ^{3}He to react with ^{3}He   lower temp.

or with ^{4}He $T > 1.4 \times 10^7$ K

Relative importance of each chain

i.e, branching ratio $\longleftrightarrow T, \rho, \mu$

$T > 3 \times 10^7$ K, PP III dominates

but in reality, at this temperature, CNO reactions take over.

Overall rate of energy generation is determined by the slowest reaction, i.e, the 1st one, $\tau \sim 10^{10}$ yr

$\varepsilon_{pp} \sim \rho^1 T^n$, $n \sim 4-6$

$Q_{pp} \sim 26.73$ MeV ≈ 6.54 MeV per proton

$n \sim 6$ for $T \approx 5 \times 10^6$ K
$n \sim 3.8$ for $T \approx 15 \times 10^6$ K (Sun)
$n \sim 3.5$ for $T \approx 20 \times 10^6$ K

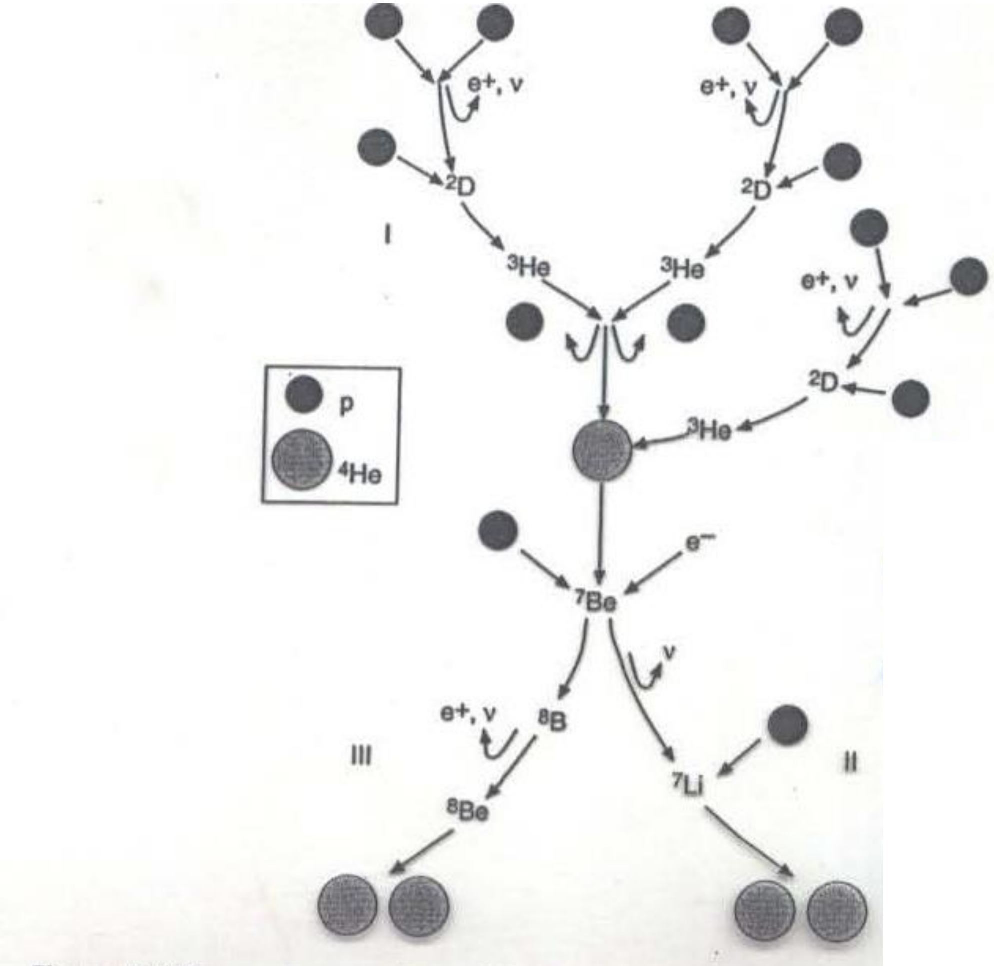


Figure 4.3 The nuclear reactions of the p–p I, II, and III chains.

Among all fusion processes, the p-p chain has the lower temperature threshold, and the weakest temperature dependence.

$$Q_{pp} = (M_{4H} - M_{He})\, c^2$$
$$= 26.73 \text{ MeV}$$

But some energy (up to a few MeV) is carried away by neutrinos.

CNO cycle

(bi-cycle)

C, N, O as catalysts

$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$ 10^6 y

$^{13}N \rightarrow {}^{13}C + e^+ + \nu$ 14 min

$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$ 3×10^5 y

$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$ 3×10^8 y

$^{15}O \rightarrow {}^{15}N + e^+ + \nu$ 82 s

→ $^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$ 10^4 y

$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$

$^{15}O \rightarrow {}^{15}N + e^+ + \nu$

$^{15}N + {}^{1}H \rightarrow {}^{16}O + \gamma$ ←

$^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$

$^{17}F \rightarrow {}^{17}O + e^+ + \nu$

$^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$

CN cycle more significant

NO cycle efficient only when $T > 20 \times 10^6$ K

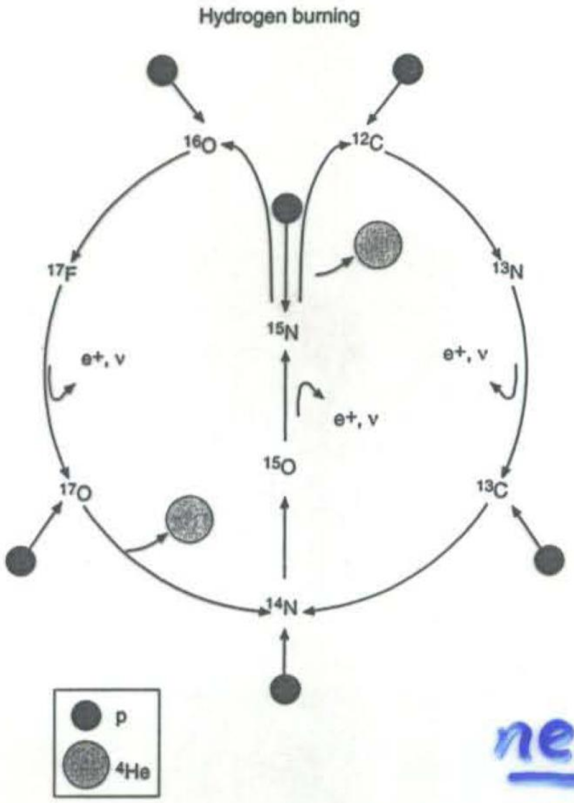


Figure 4.4 The nuclear reactions of the CNO bi-cycle.

Recognized by Bethe and independently by von Weizsäcker

CN cycle + NO cycle

Cycle can start from any reaction as long as the involved isotope is present.

$Q_{CNO} \sim 25$ MeV

after that carried away by the neutrinos

net

$4H \rightarrow {}^{4}He$

$\varepsilon_{CNO} \sim \rho T^{16}$

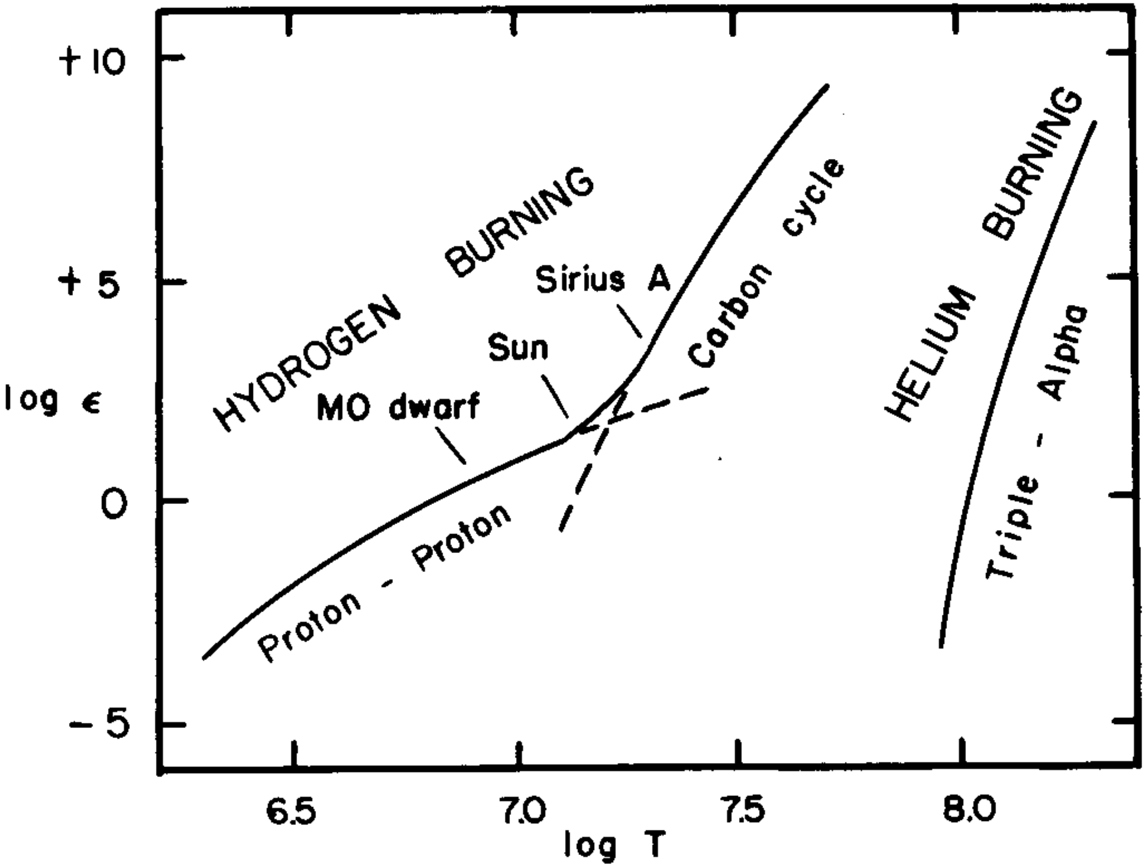


Fig. 10.1. Nuclear energy generation as a function of temperature (with $\rho X^2 = 100$ and $X_{CN} = 0.005X$ for the proton-proton reaction and the carbon cycle, but $\rho^2 Y^3 = 10^8$ for the triple-alpha process).

Schwarzschild

At the center of the Sun,

$$q_{\mathrm{CNO}}/q_{\mathrm{pp}} \approx 0.1$$

CNO dominates in stars $> 1.2\ M_{\odot}$, i.e., of a spectral type F7 or earlier
→ large energy outflux
➔ a convective core

This separates the lower and upper MS.

CN cycle takes over the PP chains near T_6=18.
Helium burning starts ~10^8 K.

The Solar Standard Model (SSM)

Best structural and evolutionary model to reproduce the observational properties of the Sun

- $L_{\odot} = 3.842 \times 10^{33}$ [ergs/s]
- $R_{\odot} = 6.9599 \times 10^{10}$ [cm]
- $M_{\odot} = 1.9891 \times 10^{33}$ [gm]
- Spectroscopic observations → Z/X =0.0245

  (latest value seems to indicate $\mathrm{Z}_{\odot} = 0.013$)

Neglecting rotation, magnetic fields, and mass loss ($dM/dt \sim 10^{-14}\ M_{\odot}/\mathrm{yr}$)

Sun Fact Sheet

http://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html

A He Gas — the triple-alpha process He-burning ignites at Tc ~ 10^8 K

$^4\text{He} + {}^4\text{He} \to {}^8\text{Be}$ (-95 keV, i.e., endothermic)

The lifetime of ^{8}Be is 2.6×10^{-16} s but is still longer than the mean-free time between α particles at T_8 (Edwin Salpeter, 1952)

$^8\text{Be} + {}^4\text{He} \to {}^{12}\text{C} + \gamma$ (7.4 MeV) ← bottleneck

Note: net $3\ {}^4\text{He} \to {}^{12}\text{C}$

$Q_{3\alpha} = 7.275$ MeV

$\hookrightarrow 5.8 \times 10^{17}$ erg g^{-1} ~ 0.1 of H → He

net: $3\ {}^4He \to {}^{12}C$

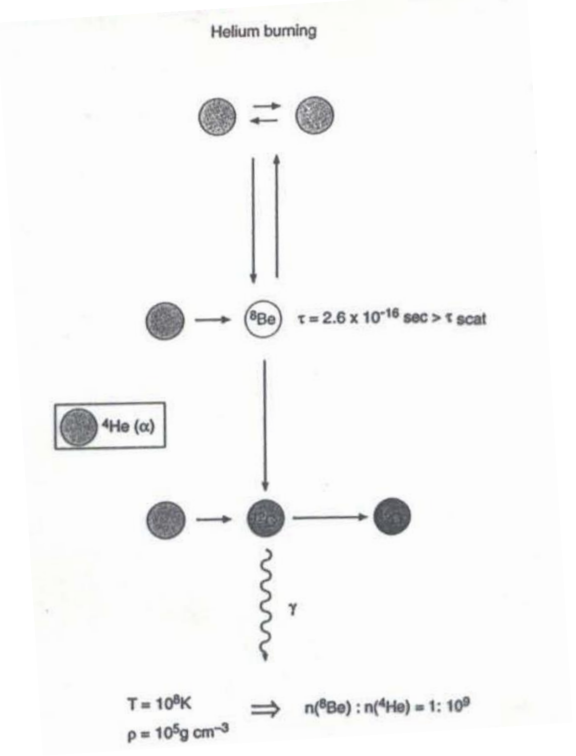
$\varepsilon_{3\alpha} \sim \rho^2 T^{40}$

∵ bottleneck = 2^{nd} reaction
$\leftrightarrow {}^8Be$

Nucleosynthesis during helium burning

$C^{12}(\alpha, \gamma) O^{16}$ Q = 7.162 MeV

$O^{16}(\alpha, \gamma) Ne^{20}$



A succession of (α, γ) processes
→ ^{16}O, ^{20}Ne, ^{24}Mg … (the α-process)

A carbon/oxygen Gas

$^{12}C + ^{12}C$

$T \gtrsim 5 \times 10^8 K$

$Q_{cc} \sim 13$ MeV

$$\begin{aligned} ^{12}\mathrm{C} + ^{12}\mathrm{C} &\to ^{24}\mathrm{Mg} + \gamma \\ &\to ^{23}\mathrm{Mg} + n \\ &\to ^{23}\mathrm{Na} + p \\ &\to ^{20}\mathrm{Ne} + \alpha \\ &\to ^{16}\mathrm{O} + 2\alpha \end{aligned}$$

$$\begin{aligned} ^{16}\mathrm{O} + ^{16}\mathrm{O} &\to ^{32}\mathrm{S} + \gamma \\ &\to ^{31}\mathrm{S} + n \\ &\to ^{31}\mathrm{P} + p \\ &\to \boxed{^{28}\mathrm{Si}} + \alpha \\ &\to ^{24}\mathrm{Mg} + 2\alpha \end{aligned}$$

$^{16}O + ^{16}O$

$T \gtrsim 10^9 K$

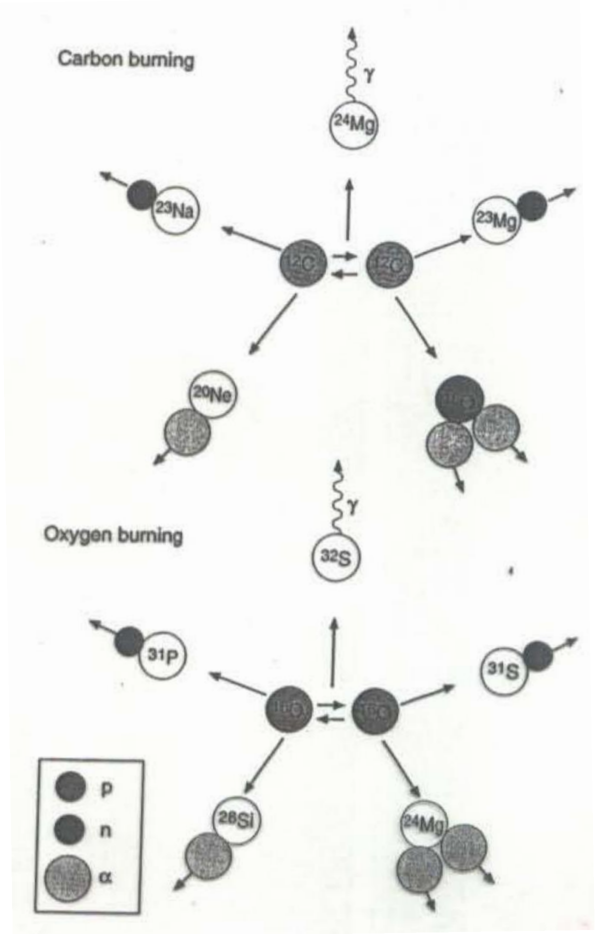
$Q_{oo} \sim 16$ MeV

C-burning ignites when Tc ~ (0.3-1.2) × 10^9 K, i.e., for stars 15-30 $M_\odot$

O-burning ignites when Tc ~ (1.5-2.6) × 10^9 K, i.e., for stars > 15-30 $M_\odot$

The p and α particles produced are captured immediately (because of the low Coulomb barriers) by heavy elements

➔ isotopes



O burning → Si

$$q_{PP} = 2.4 \times 10^6\,\rho\,X^2\,T_6{}^{-2/3}\exp\left[-33.8\,T_6{}^{-1/3}\right]\quad [\mathrm{erg\,g^{-1}\,s^{-1}}]$$

$$q \propto \rho\,X_H^2\;T^{4}$$

$$q_{CN} = 8 \times 10^{27}\,\rho\,X\,X_{CN}\,T_6{}^{-2/3}\exp\left[-152.3\,T_6{}^{-1/3}\right]\quad [\mathrm{erg\,g^{-1}\,s^{-1}}]$$

$$q \propto \rho\,X_H X_{CN}\,T^{16}$$

$\frac{X_{CN}}{X_H} = 0.02$ ok for Pop I

$$q_{3\alpha} = 3.9 \times 10^{11}\,\rho^2 X_\alpha{}^3\,T_8{}^{-3}\exp[-42.9\,T_8]\quad [\mathrm{erg\,g^{-1}\,s^{-1}}]$$
$$\approx 4.4 \times 10^{-8}\;\rho^2 X_\alpha{}^3\,T_8{}^{40}\quad [\mathrm{erg\,g^{-1}\,s^{-1}}]\quad (\text{if } T_8 \approx 1)$$

Clayton

Does ^{28}Si follow the same scenario?

$^{28}Si + {}^{28}Si \underset{?}{\rightarrow} {}^{56}Fe$?

C+C 6×10^{8}K

O+O 10^{9}K

No! Coulomb barrier becomes extremly high; another nuclear reaction takes place

e.g.

Photoionization

Likewise

Photodisintegration

For example, ${}^{16}\mathrm{O} + \alpha \leftrightarrow {}^{20}\mathrm{Ne} + \gamma$

If $T < 10^9$ K $\rightarrow$

but if $T \geq 1.5 \times 10^9$ K (in radiation field) $\leftarrow$

So ${}^{28}\mathrm{Si}$ disintegrates at $\approx 3 \times 10^9$ K to lighter elements

(then recaptured …)

Until a nuclear statistical equilibrium is reached

But the equilibrium is <u>not</u> exact

→ a pileup of the iron group nuclei (Fe, Co, Ni)

which can resist photodisintegration until 7×10^9 K

Nuclear Fuel	Process	$T_{\text{threshold}}$ (10^6 K)	Products	Energy per nucleon (MeV)
H	p-p	~4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C, O	0.61
C	C + C	600	O, Ne, Na, Mg	0.54
O	O + O	1,000	Mg, S, P, Si	~0.3
Si	Nuc. Equil.	3,000	Co, Fe, Ni	<0.18

From Prialnik Table 4.1

$^{56}Fe + 100 \text{ MeV} \rightarrow 13\ ^{4}He + 4\,n$

If T ↑↑↑, even $^{4}He \rightarrow p^{+} + n^{0}$

So stellar interior has to be between a few T_6 and a few T_9.

<u>Lesson</u>: Nuclear reactions that absorb (rather than emit) energy from ambient radiation field (in stellar interior) can lead to catastrophic consequences.

Time Scales

Different physical processes inside a star,
e.g., nuclear reactions (changing chemical composition) are slow (longer time scales); structural adjustments (dP/dt) take places on relatively shorter time scales.

- ✓ Dynamical timescale
- ✓ Thermal timescale
- ✓ Nuclear timescale
- ✓ Diffusion timescale

Dynamical Timescale

$$\text{hydrostatic equilibrium} \xrightarrow{\text{perturbation}} \text{motion} \xrightarrow{\text{adjustment}} \text{hydrostatic equilibrium}$$

Free-fall collapse

Equation of motion $\ddot{r} = -\frac{GM_r}{r^2} - \frac{1}{\varrho}\frac{dP}{dr}$

Near the star's surface $r = R, M_r = M$, so $\ddot{R} = -\frac{GM}{R^2} - \frac{1}{\varrho}\frac{dP}{dR}$

Free-fall means pressure $\ll$ gravity, so $\ddot{R} \approx -\frac{GM}{R^2}$

Assuming a constant acceleration $R = -\left(\ddot{R}/2\right)\tau_{\text{ff}}^2$, so

$$\boxed{\tau_{\text{ff}} = \left(2R^3/GM\right)^{1/2} = \frac{1}{\left(\frac{2}{3}\pi G\bar{\rho}\right)^{1/2}} \approx 0.04\left(\frac{\rho_\odot}{\bar{\rho}}\right)^{1/2} [\text{d}]}$$

Stellar Pulsation

The star pulsates about the equilibrium configuration

→ same as dynamical timescale

$$\boxed{\tau_{\mathrm{pul}} \propto 1/\sqrt{\bar{\rho}}}$$

Propagation of Sound Speed (pressure wave)

Pressure induced perturbation,

$$R/\tau_{\mathrm{ff}}^2 = -\frac{\ddot{R}}{2} = \frac{GM}{R^2} + \frac{1}{\varrho}\frac{dP}{dR} \approx \frac{1}{\varrho}\frac{dP}{dR} \approx \frac{1}{\varrho}\frac{P}{R}$$

so $\frac{R}{\tau_{\mathrm{ff}}} \approx \sqrt{\frac{P}{\rho}} \approx c_s$ (sound speed) $\propto \sqrt{T}$ (for ideal gas)

$$\boxed{\tau_{\mathrm{s}} \approx \frac{R}{c_s}}$$

In general, $\tau_{\mathrm{dyn}} \approx \frac{1}{\sqrt{G\bar{\rho}}} \approx \frac{1.6\times10^{15}}{\sqrt{n}}$ [s] $= 1000\sqrt{\left(\frac{R}{R_\odot}\right)^3\left(\frac{M_\odot}{M}\right)}$ [S]

Thermal Timescale

Kelvin-Helmholtz timescale (radiation by gravitational contraction)

$$E_{\text{total}} = E_{\text{grav}} + \mathrm{E}_{\text{thermal}} = \frac{1}{2} E_{\text{grav}} = -\frac{1}{2} \alpha G M^2 / R$$

This amount of energy is radiated away at a rate L, so timescale

$$\tau_{\text{KH}} = \frac{E_{\text{total}}}{L} = \frac{1}{2} \alpha G M^2 / RL$$

$$= 2 \times 10^7 M^2 / RL \quad \text{[yr] in solar units}$$

$$\boxed{\tau_{\text{KH}} \approx 2 \times 10^7 \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R_\odot}{R}\right) \left(\frac{L_\odot}{L}\right) \ \text{[yr]}}$$

$\boldsymbol{M = 1\,\mathcal{M}_{\odot}, R = 1\,\mathrm{pc}}$	$\boldsymbol{M = 1\,\mathcal{M}_{\odot}, R = 1\,\mathcal{R}_{\odot}}$
$\tau_{\mathrm{dyn}} \approx 1.6 \times 10^{7}\ \mathrm{yr}$	$\tau_{\mathrm{dyn}} \approx 1.6 \times 10^{3}\ \mathrm{s} \approx 30\ \mathrm{min}$
$\tau_{\mathrm{ther}} \approx 1\ \mathrm{yr}$	$\tau_{\mathrm{ther}} \approx 3 \times 10^{7}\ \mathrm{yr}$

Nuclear Timescale

Time taken to radiate at a rate L on nuclear energy

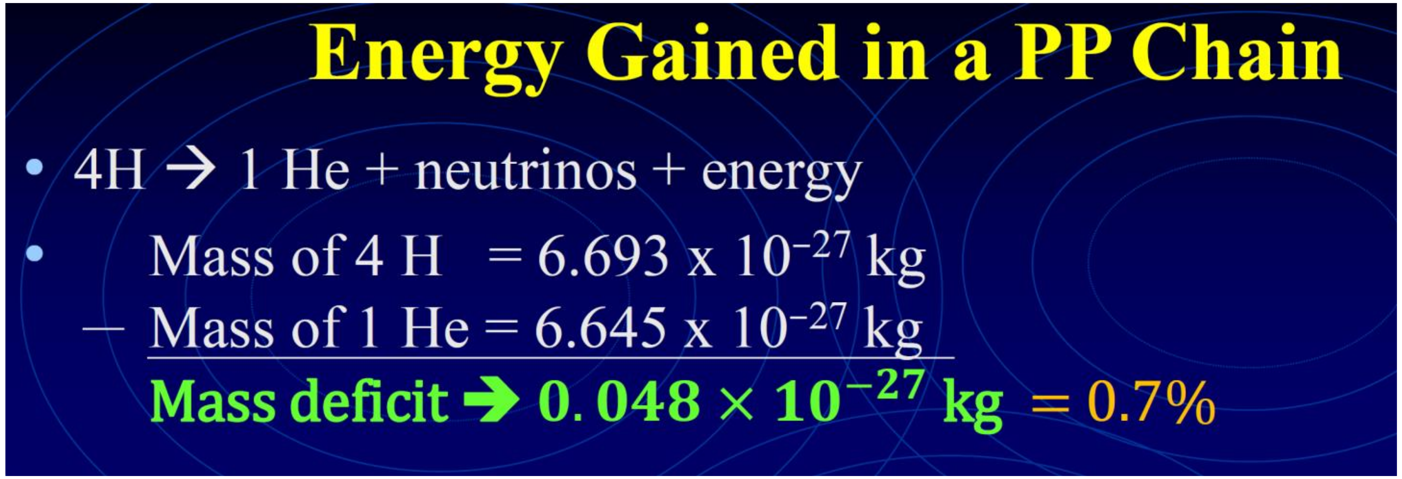
$4\ {}^{1}H \rightarrow\ {}^{4}He\ (Q = 6.3 \times 10^{18}\text{erg/g})$

$$\tau_{\text{nuc}} = \frac{E_{\text{nuc}}}{L} = 6.3 \times 10^{18} \frac{M}{L}$$

$$\boxed{\tau_{\text{nuc}} \approx 10^{11} \left(\frac{M}{M_{\odot}}\right) \left(\frac{L_{\odot}}{L}\right)\ [\text{yr}]}$$

From the discussion above, $\tau_{\text{nuc}} \gg \tau_{\text{KH}} \gg\ \tau_{\text{dyn}}$

Main-Sequence Lifetime of the Sun



Energy Gained in a PP Chain

- 4H → 1 He + neutrinos + energy
- Mass of 4 H = 6.693×10^{-27} kg

— Mass of 1 He = 6.645×10^{-27} kg

Mass deficit ➔ 0.048×10^{-27} kg = 0.7%

$$M_\odot \approx 2 \times 10^{33}\ [\mathrm{g}]$$

$$L_\odot \approx 4 \times 10^{33}\ [\mathrm{ergs/s}]$$

Fusion efficiency

Nuclear physics

Stellar physics

$$\tau_\odot^{\mathrm{MS}} \approx M_\odot \frac{(0.007)(0.1)\ c^2}{L_\odot} = 3.15 \times 10^{17} [\mathrm{s}] = 10^{10}\ [\mathrm{yr}]$$

Given $L_{\mathrm{MS}}/L_\odot \approx \left(M/M_\odot\right)^4 \rightarrow \tau^{\mathrm{MS}} \approx 10^{10} \left(M_\odot/M\right)^3$ [yr]

Diffusion Timescale

Time taken for photons to randomly walk out from the stellar interior to eventual radiation from the surface

$r_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{m_e c^2}$ ("classical" radius of the electron)

$\sigma_{\mathrm{Thomson}} = \frac{8\pi}{3} {r_e}^2 = 6.6525 \times 10^{-29}$ [m^2] for interactions with photon energy $h\nu \ll m_e c^2$ (electron rest energy)

Thus, mean free path $\ell = 1/(\sigma_T n_e)$, where for complete ionization of a hydrogen gas, $n_e = M/(m_p R^3)$.

So, $\ell \approx m_p R^3/\sigma_T M = 4$ [mm] for the mean density.

At the core, it is 100 times shorter.

$\tau_{\mathrm{dif}} \approx 10^4$ [yr] (Exercise: Show this.)

For an isotropic gas

$$P = \frac{1}{3} \int_0^\infty p \, v_p \, n(p) \, dp$$

- p and v_p: relativistic case
- $n(p)$: particle type & quantum statistics

For a photon gas, $p = h\nu/c$, so

$$P_{rad} = \frac{1}{3} \int_0^\infty h\nu \, n(\nu) \, d\nu = \frac{1}{3} \, u = \frac{1}{3} \, aT^4 \, ,$$

$$a = 7.565 \times 10^{15} \text{ ergs cm}^{-3} \text{ K}^{-4}$$

Radiation Pressure

$P_{\mathrm{total}} = P_{\mathrm{radiation}} + \ P_{\mathrm{gas}}$

Since $P_{\mathrm{rad}} \sim T^4 \sim M^4/R^4$

But $P_{\mathrm{tot}} \sim M^2/R^4$

➔ $P_{\mathrm{rad}}/P_{\mathrm{tot}} \sim M^2$

So the more massive of a star, the higher relative contribution by radiation pressure (and γ decreases to 4/3.)

When P_{rad} dominates

$$\mathcal{F} = \frac{-d\,P_{\mathrm{rad}}/dr}{\kappa\rho} = \frac{4ac}{3}\,T^3\,\frac{dT}{dr} = \frac{L}{4\pi r^2}$$

$$\frac{d\mathrm{P}_{\mathrm{rad}}}{dr} \sim \frac{\kappa\rho}{c}\frac{L}{4\pi r^2}$$

On the other hand, by definition

$$\frac{dP_{\mathrm{tot}}}{dr} = -\rho\,\frac{Gm}{r^2}$$

$$\Rightarrow \frac{d\mathrm{P}_{\mathrm{rad}}}{dP_{\mathrm{tot}}} = \frac{\kappa L}{4\pi c G m}$$

Toward the outer layers, both P_{gas} ↘ and $\mathrm{P}_{\mathrm{rad}}$ ↘, so P_{tot} ↘↘, and $dP_{\mathrm{tot}} > dP_{\mathrm{rad}}$. This leads to

$$\kappa L \leq 4\pi c G m$$

At the surface, $m = M, P = 0$, it is always radiative, so

$$\boxed{L < \frac{4\pi c G M}{\kappa}}$$

This is the **Eddington luminosity limit** = Maximum luminosity of a celestial object in balance between the radiation and gravitational force.

Numerically,

$$\boxed{L_{Edd}/L_{\odot} = 3.27 \times 10^{4} \ \mu_e \, M/M_{\odot}}$$

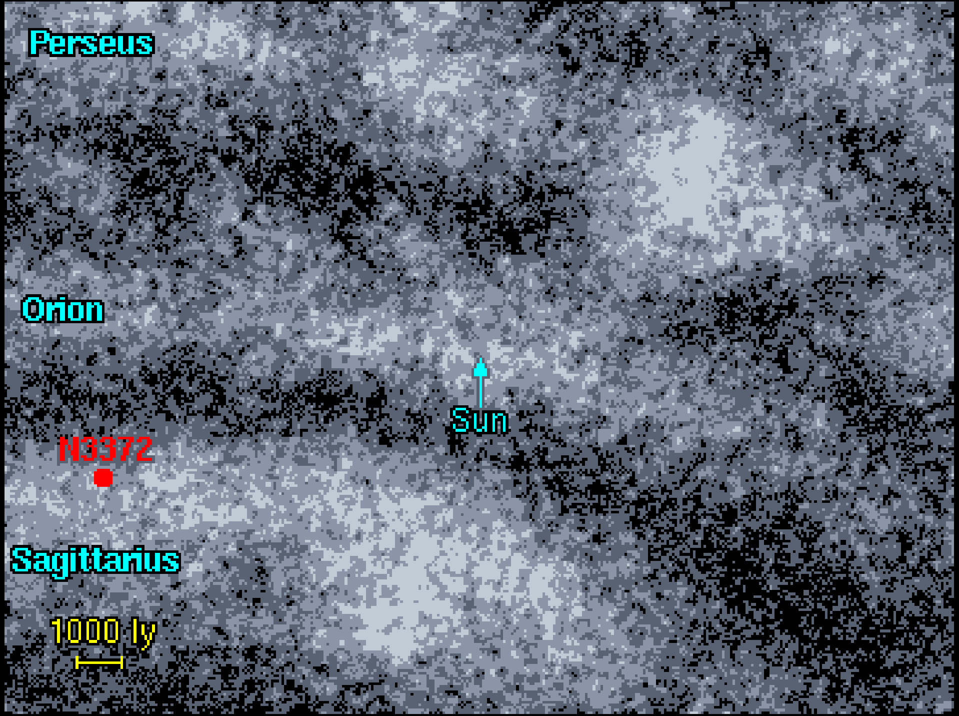
For X-ray luminosity, scattered by electrons in an optically thin gas, $L_X < 10^{38}$ erg sec^{-1}

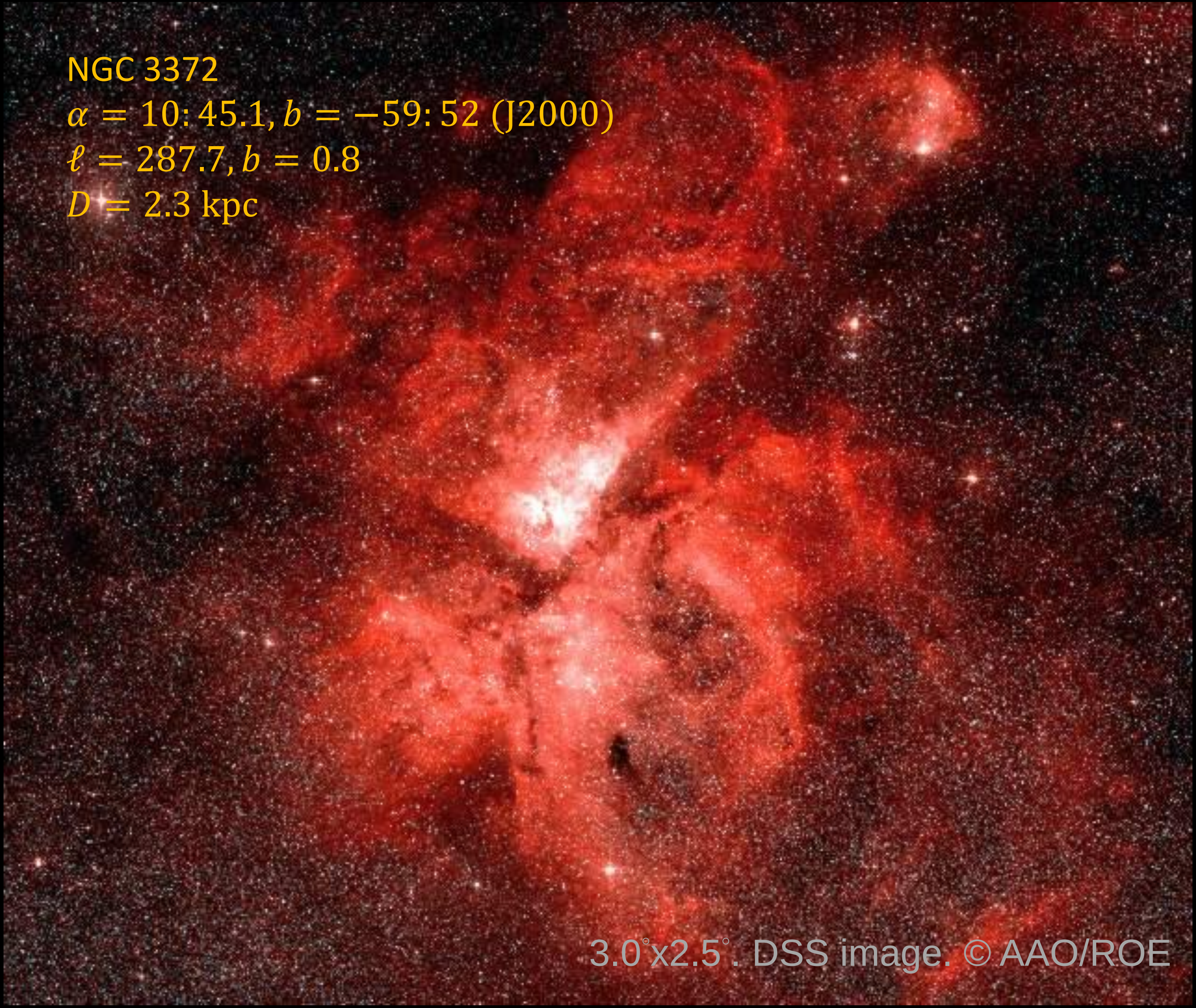
Eddington limit is the upper limit on the luminosity of an object of mass M, $L \leq \left(\frac{4\pi G m_p}{\sigma_T}\right) M$

$$\equiv L_{\mathrm{Edd}} \approx 10^{38} \, {}^{M}\!/_{M_\odot} \; [\mathrm{erg\,s^{-1}}]$$

For 1 $M_\odot$, $L_{\mathrm{Edd}} \approx 5 \times 10^4 L_\odot, M_{\mathrm{bol}} = -7.0$
For 40 $M_\odot$, $M_{\mathrm{bol}} = -11.0$

Eta Carina, $L \approx 5 \times 10^6 L_\odot$, $M_{\mathrm{bol}} = -11.6$, $M \approx 120\, M_\odot$





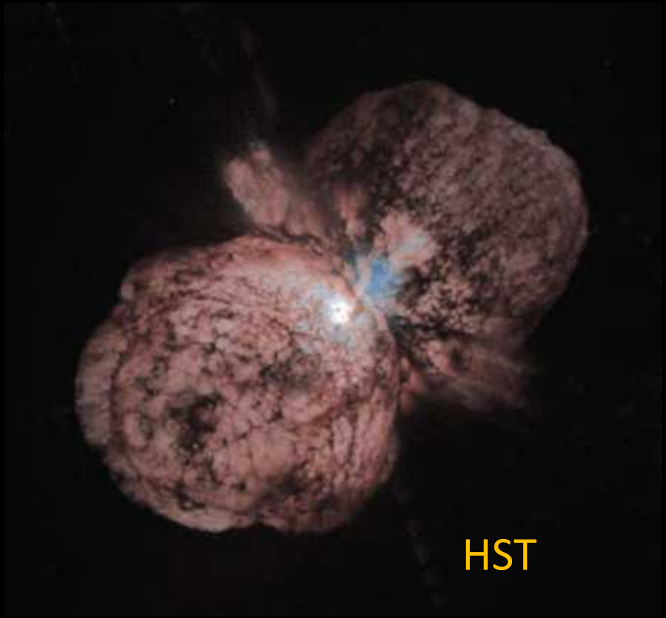
NGC 3372

$\alpha = 10{:}\,45.1, b = -59{:}\,52$ (J2000)

$\ell = 287.7, b = 0.8$

$D = 2.3$ kpc

3.0°x2.5°. DSS image. © AAO/ROE



HST

http://www.atlasoftheuniverse.com/nebulae/ngc3372.html

In general,

$$L_{Edd} \simeq 3.2 \times 10^{4} \frac{M}{M_{\odot}} \frac{\kappa_{e}}{\kappa} \; [L_{\odot}]$$

inequality is violated

L_{Edd} can be exceeded if

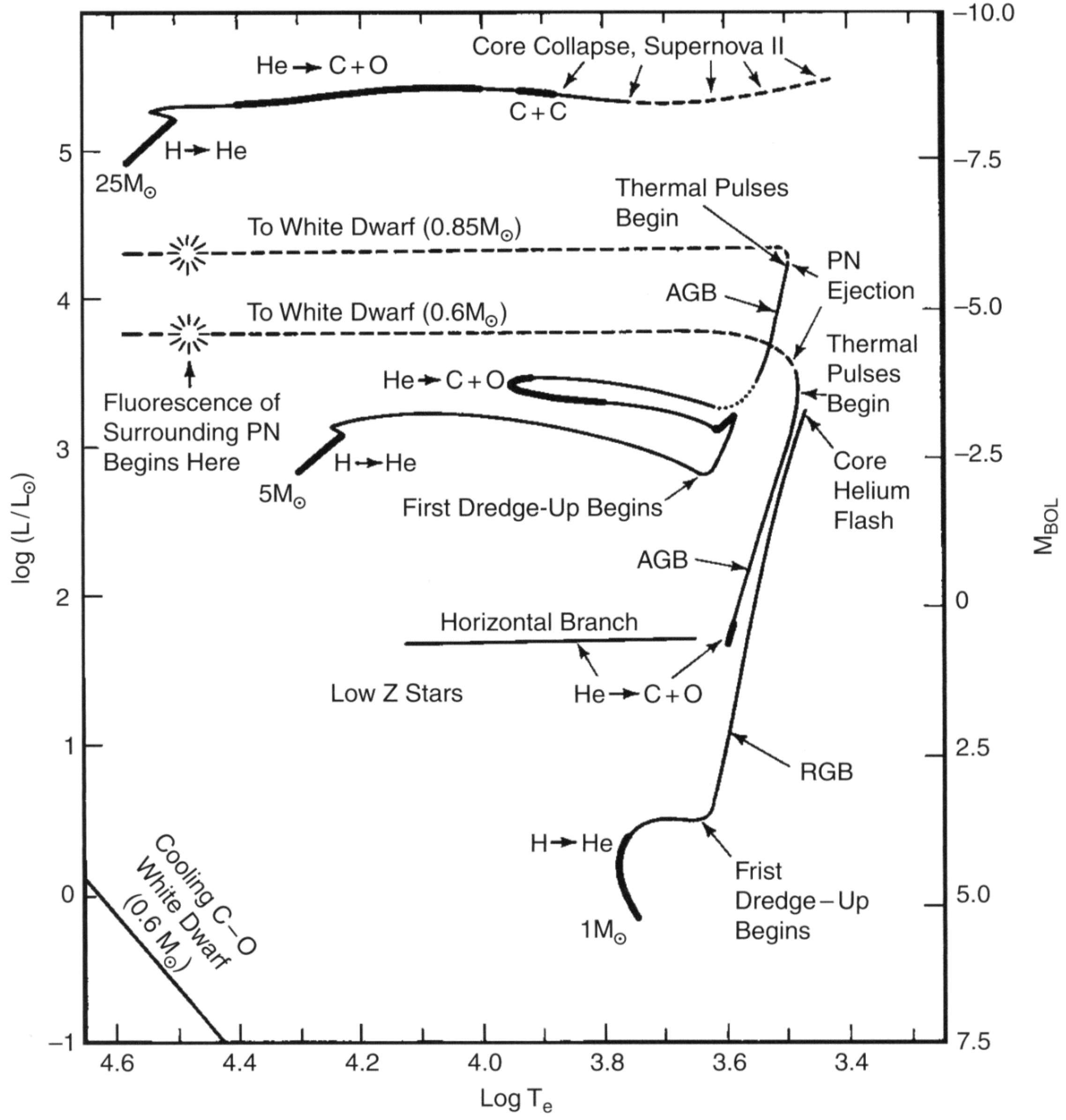
① $L \uparrow\uparrow$, e.g., intense thermonuclear burning

② $\kappa \uparrow\uparrow$, e.g. H or He ionization

⇒ Hydrostatic equilibrium can no longer maintained

∴ need a different heat transfer mechanism

Comparison of 1, 5, and 25 $\mathcal{M}_\odot$ stars



Evolutionary tracks of theoretical model stars in the HR diagram (Iben, 1985)