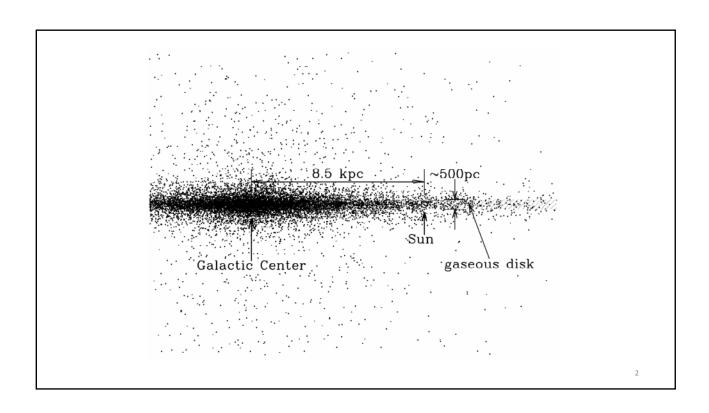
# Molecular Clouds and Star Formation

Stars are formed in molecular cloud cores, whereas planets are formed, contemporaneously, in young circumstellar disks.

http://www.astro.ncu.edu.tw/~wchen/Courses/Stars/Lada1995summerschool.pdf



THE ASTROPHYSICAL JOURNAL, 218: 148-169, 1977 November 15 © 1977. The American Astronomical Society. All rights reserved. Printed in U.S.A.

# A THEORY OF THE INTERSTELLAR MEDIUM: THREE COMPONENTS REGULATED BY SUPERNOVA EXPLOSIONS IN AN INHOMOGENEOUS SUBSTRATE

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### ABSTRACT

Supernova explosions in a cloudy interstellar medium produce a three-component medium in which a large fraction of the volume is filled with hot, tenuous gas. In the disk of the galaxy the evolution of supernova remnants is altered by evaporation of cool clouds embedded in the hot medium. Radiative losses are enhanced by the resulting increase in density and by radiation from the conductive interfaces between clouds and hot gas. Mass balance (cloud evaporation rate = dense shell formation rate) and energy balance (supernova shock input = radiation loss) determine the density and temperature of the hot medium with  $(n,T) = (10^{-2.5}, 10^{5.7})$  being representative values. Very small clouds will be rapidly evaporated or swept up. The outer edges of "standard" clouds ionized by the diffuse UV and soft X-ray backgrounds provide the warm ( $\sim 10^4$  K) ionized and neutral components. A self-consistent model of the interstellar medium developed herein accounts for the observed pressure of interstellar clouds, the galactic soft X-ray background, the O v1 absorption line observations, the ionization and heating of much of the interstellar medium, and the motions of the clouds. In the halo of the galaxy, where the clouds are relatively unimportant, we estimate  $(n,T) = (10^{-3.5}, 10^{6.0})$  below one pressure scale height. Energy input from halo supernovae is probably adequate to drive a galactic wind.

Interstellar Medium (ISM)

- Gas, dust + radiation, magnetic fields, cosmic rays (i.e., charged particles)
- Very sparse ----

[star-star distance] / [stellar diameter]  $\sim 1$  pc/ $10^{11}$  cm  $\sim 3$  x  $10^7$ :1 or  $\sim 1$ :  $10^{22}$  in terms of volume (space)

- Mass: 99% mass in gas, 1% in dust  $\sim$  15% of total MW visible matter
- Of the gas, 90%, H; 10% He
- Hydrogen: mainly H I (atomic), H II (ionized), and H<sub>2</sub> (molecular)
- Studies of ISM ---
  - Beginning of evolution of baryonic matter "recombination"
  - Stars form out of ISM
  - Important ingredient of a galaxy

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# Material Constituents of the ISM

Component	<b>T</b> ( <b>K</b> )	n (cm <sup>-3</sup> )	Properties
Hot, intercloud and coronal gas	$10^{6}$	10-4	
Warm intercloud gas	$10^{4}$	0.1	
Diffuse cloud (H I)	$10^{2}$	0.1	Mostly H I; $n_e/n_0 = 10^{-4}$
H II regions	104	>10	
Dark Molecular Clouds	10	> 10 <sup>3</sup>	Mostly H <sub>2</sub> mol. and dust
Supernova Remnants	10 <sup>4</sup> ~10 <sup>7</sup>	>1	
Planetary Nebulae			

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# **Energy Density in the Local ISM**

Component	u (eV/cm <sup>-3</sup> )	Properties
Cosmic microwave background	0.265	
FIR radiation from dust	0.31	
Starlight	0.54	
Thermal kinetic energy	0.49	
Turbulent kinetic energy	0.22	
Magnetic field	0.89	
Cosmic rays	1.39	

There seems to be equi-partition between these energies. Why? Read Draine's book, page  $10\,$ 

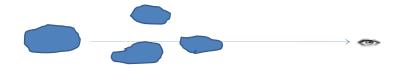
# A "standard" HI cloud

 $D \sim 5 \text{ pc}$ 

 $M\sim 50~{\rm M}_{\odot}$ ;

 $d_{\rm intercloud} \sim 100~{\rm pc}$ 

 $v_{\rm cloud} \sim 10~{\rm km s^{-1}}$ 



Clouds are patchy → extinction depends greatly on the sightline

Extinction = absorption + scattering

Extinction versus reddening

Av=30 toward the Galactic center

In the Galactic plane,  $Av \sim 0.7-1 \text{ mag kpc}^{-1}$ 

Extinction  $\leftarrow \rightarrow$  amounts of dust grains along the line of sight

Reddening  $\leftarrow \rightarrow$  grain properties (size, shape, composition, structure)

# (a) Profile for zero bandwidth **(b)** (c) (d)

Fig. 8-59. Idealized hydrogen-line profiles.

# Different clouds along the line of sight ...

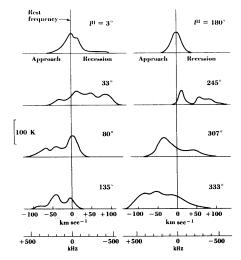
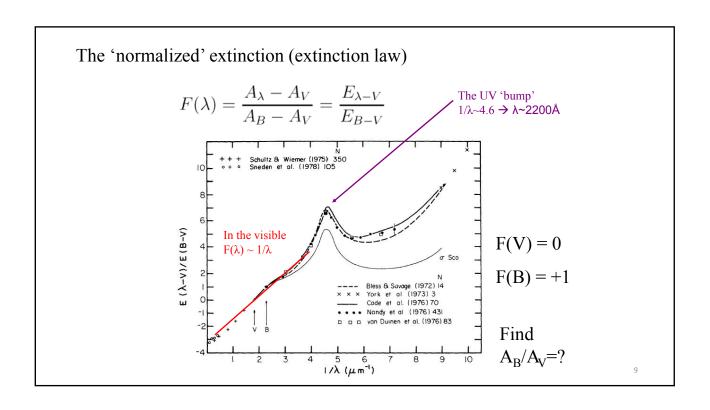
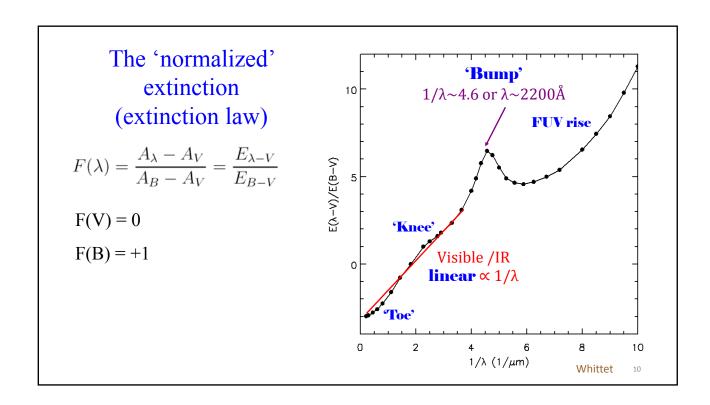
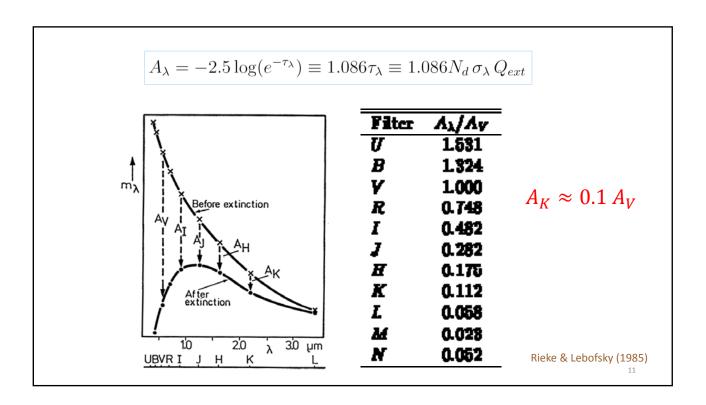


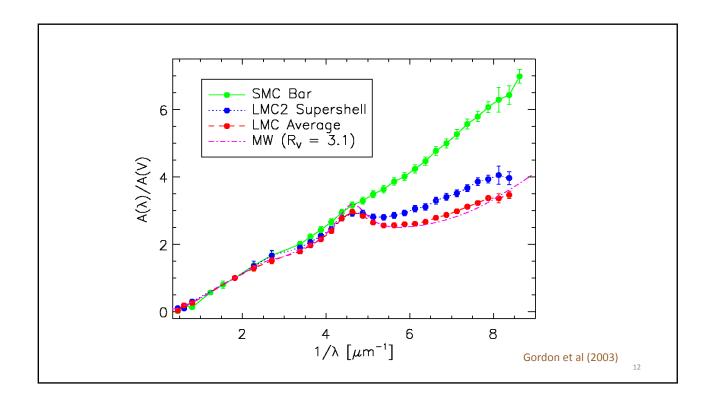
Fig. 8-60. Hydrogen-line profiles at different longitudes in the plane of our galaxy. (After Kerr and Westerhout, 1964).

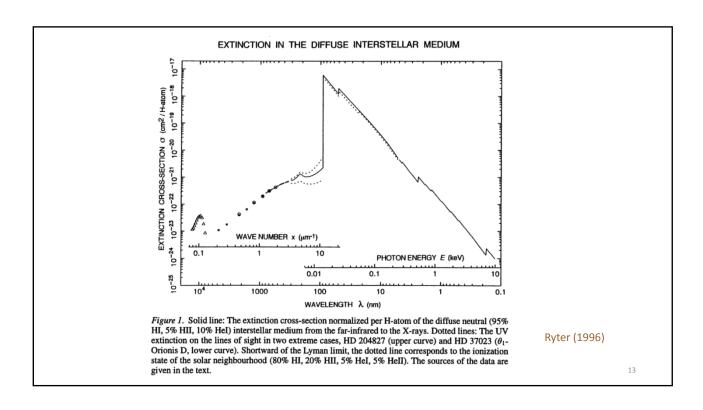
John Kraus "Radio Astronomy"

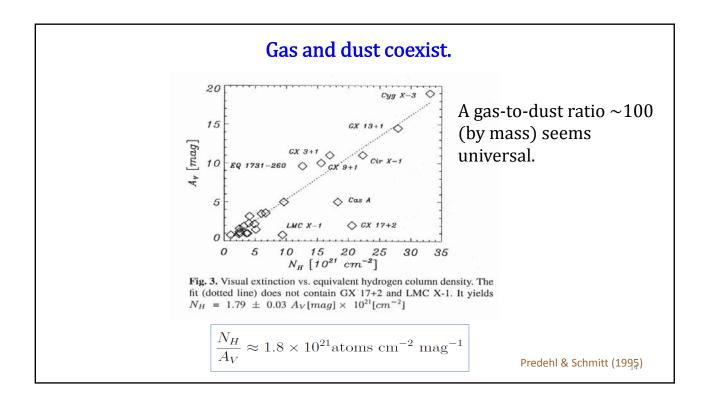












# Exercise

- 1. The star Vega is used to define the zeroth magnitude in all the classical (Vega) photometric systems, e.g., Johnson.
- 2. Plot its spectral energy distribution (SED) from UV to IR.
- 3. What is the spectral type of Vega? What is its effective temperature?
- 4. Compare this in a plot with a blackbody curve of the temperature.
- 5. It was surprising hence when *IRAS* data revealed IR excess of Vega. What are the flux densities observed by *IRAS*? Given the age of Vega, why is this discovery significant?

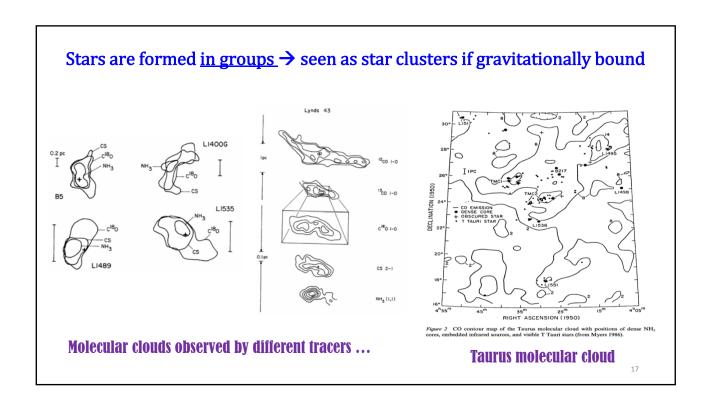
15

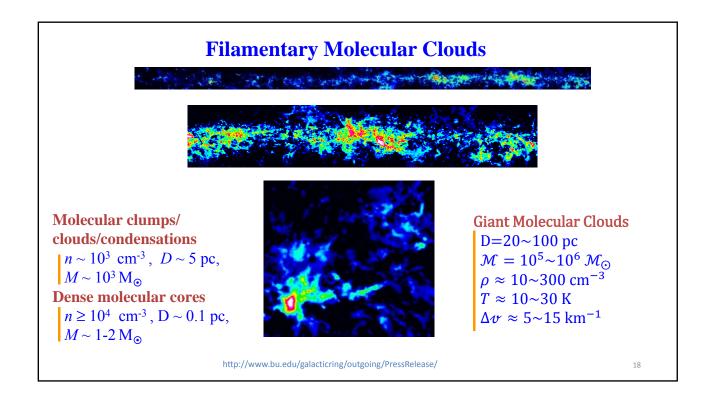
# http://www.astro.utoronto.ca/~patton/astro/mags.html#conversions

Band	lambda_c	dlambda/lambda	Flux at m=0	Reference
	um		Jy	
U	0.36	0.15	1810	Bessel (1979)
В	0.44	0.22	4260	Bessel (1979)
V	0.55	0.16	3640	Bessel (1979)
R	0.64	0.23	3080	Bessel (1979)
I	0.79	0.19	2550	Bessel (1979)
J	1.26	0.16	1600	Campins, Reike, & Lebovsky (1985)
Н	1.60	0.23	1080	Campins, Reike, & Lebovsky (1985)
K	2.22	0.23	670	Campins, Reike, & Lebovsky (1985)

Astronomical Magnitude Systems.pdf

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# **Nearby Examples**

# **Massive Star-Forming Region**

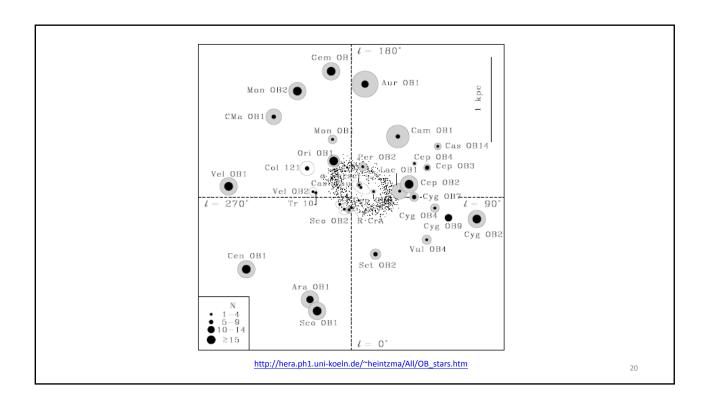
- *Per OB2* (350 pc)
- Orion OB Association (350-400 pc) ... rich

# **Low-Mass Star-Forming Regions**

- Taurus Molecular Cloud (TMC-1) (140 pc)
- Rho Ophiuchi cloud (130 pc)
- Lupus (140 pc)
- Chamaeleon (160 pc)
- Corona Australis (130 pc)

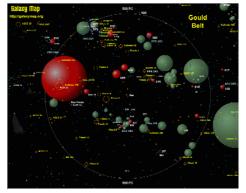
4/5 in the southern sky ... why?

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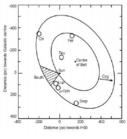


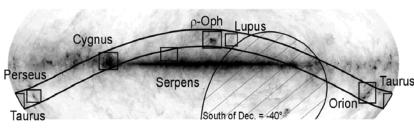
The Gould Belt, a (partial) ring in the sky, ~1 kpc across, centered on a point 100 pc from the Sun and tilted about 20 deg to the Galactic plane, containing star-forming molecular clouds and OB stars = local spiral arm

Origin unknown (dark matter induced star formation?)



 $\underline{http://galaxymap.org/detail\_maps/download\_maps/gould.png}$ 



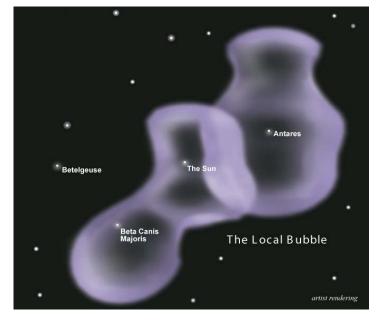


http://www.jach.hawaii.edu/JCMT/surveys/gb/ Gould's Belt superimposed on to an IRAS 100 micron emission map

The **Local Bubble**, a cavity of sparse, hot gas,  $\sim 100$  pc across, in the interstellar medium, with H density of  $0.05 \text{ cm}^{-3}$ , an order less than typical in the Milky Way.

Likely caused by a (or multiple) supernova explosion (10-30 Myr ago).

Where is the supernova (remnant)? Check out the Orion-Eridanus Superbubble

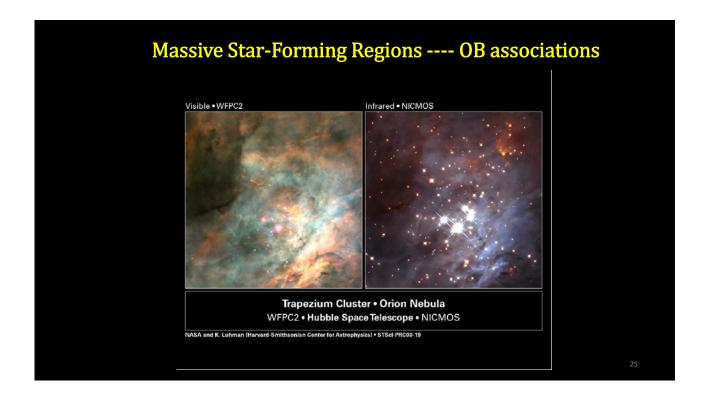


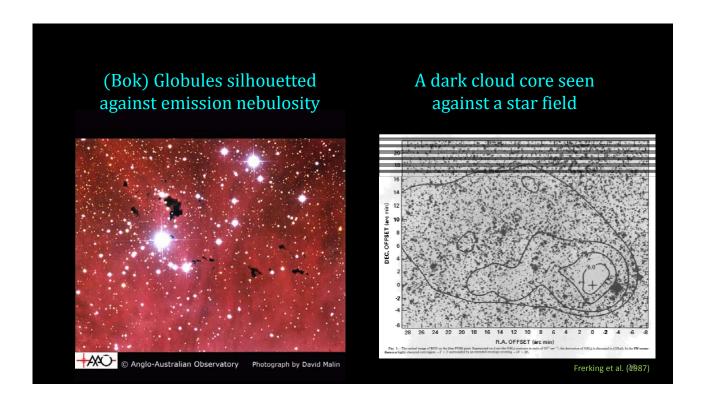
https://en.wikipedia.org/wiki/Local\_Bubble

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# Molecules in space

# H<sub>2</sub> molecules

- the main constituent of cold clouds, but lacking a permanent electric dipole moment, so is very difficult to detect. A rotationally excited molecule would radiate through a relatively slow electric quadrupole transition.
- Only in a hot medium, where stellar radiation or stellar wind excites vibrational and electronic states which then decay relatively quickly.

Refer to the slides for WPC's ISM course <a href="http://www.astro.ncu.edu.tw/~wchen/Courses/ISM/index.htm">http://www.astro.ncu.edu.tw/~wchen/Courses/ISM/index.htm</a>

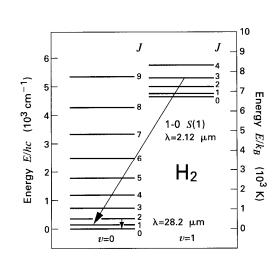


Figure 5.4 Rotational levels of  $H_2$  for the first two vibrational states. Within the v=0 state, the  $J=2\to 0$  transition at 28.2  $\mu$ m is displayed. Also shown is the transition giving the 1-0 S(1) rovibrational line at 2.12  $\mu$ m. Note that two different energy scales are used.

Stahler & Palla

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dipole moment

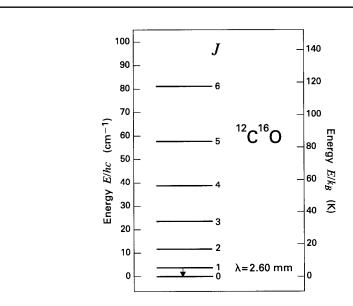
Dipole moment

= 1.85 Debyes

# CO molecules

- simple and abundant. Strong binding energy E=11.1 eV self-shielding against UV field
- with a permanent electric dipole moment; radiating strongly at radio frequencies.
- 12C16O easiest to detect; isotopes 13C16O, 12C18O, 12C17O, 13C18O also useful
- Excitation of CO to the J=1 level mainly through collisions with ambient  $H_2$   $X_{CO}=2\times10^{20}$  cm $^{-2}$  [K km/s] $^{-1}$  (Bolatto et al. 2013, ARAA)
- At low densities, each excitation is followed by emission of a photon. At high densities, the excited CO transfers the energy by collision to another  $H_2$  molecule;  $n_{\rm crit} \approx 3 \times 10^3 \ {\rm cm}^{-3}$ . Low critical density  $\rightarrow$  CO to study <u>large-scale distribution</u> of clouds, as a tracer of  $H_2$
- $^{12}{\rm C}^{16}{\rm O}$  almost always optical thick; same line from other rare isotopes usually not.  $N_H$ =  $10^6~N_{13}_{CO}$

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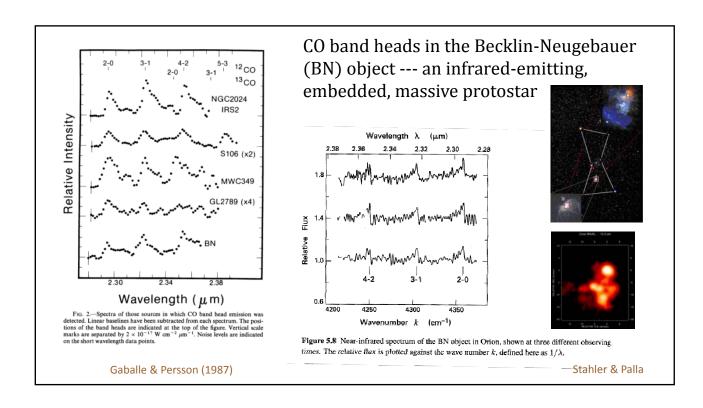


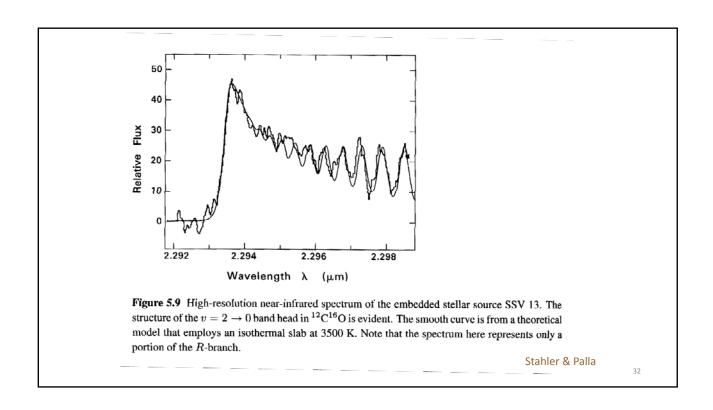
2.6 mm = 115 GHz

Only 5 K above the ground level ... can be excited by collisions with ambient molecules or CMB photons

Figure 5.6 Rotational levels of  $^{12}\text{C}^{16}\text{O}$  within the ground (v=0) vibrational state. The astrophysically important  $J=1\to 0$  transition at 2.60 mm is shown.

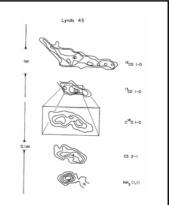
Stahler & Palla



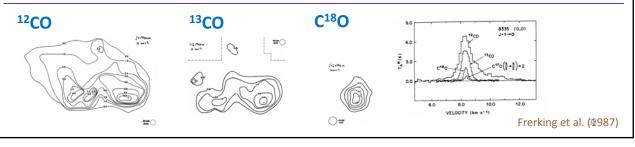


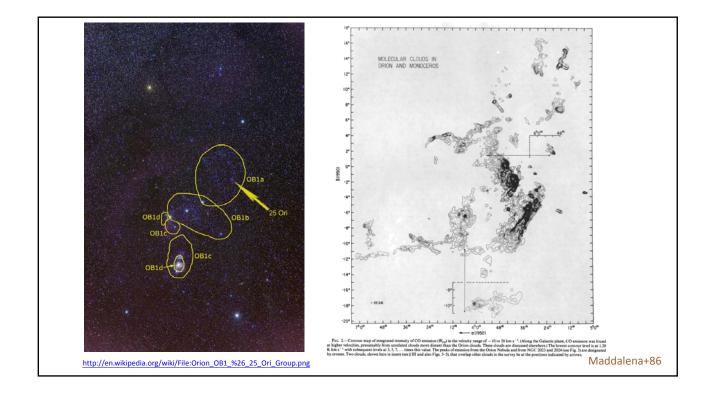
Each species has a different set of excitation conditions (density, temperature; cf. Boltzmann equation)

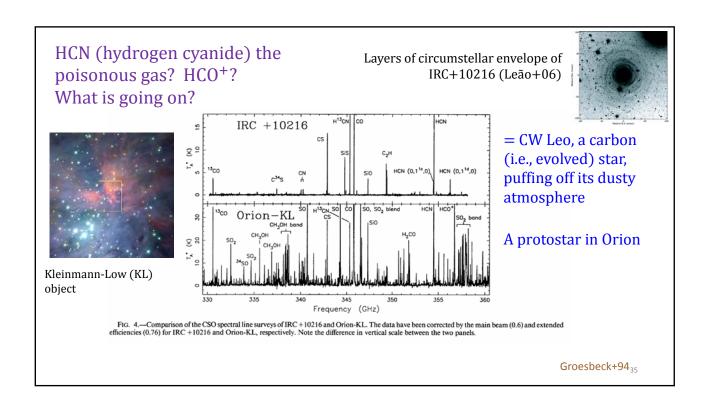
→ Different molecules/isotopes serve as <u>tracers</u> of these conditions, e.g., C180 traces denser parts of a cloud than 12CO does; NH3 maps the dense cores where protostars are located.



Myers et al. 1991







Star formation is not an isolated event.

Massive stars in particular may trigger the birth of next-generation stars → triggered star formation

... also possible by stellar jets, Galactic density waves, cloud-cloud collisions ...

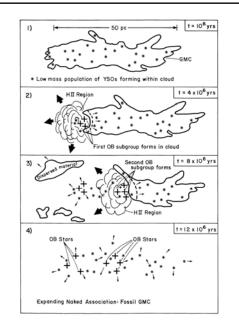


Figure 1. Probable stages in the formation of an expanding OB association from a giant molecular cloud. Low star formation efficiency in conjunction with efficient dispersal of residual, unprocessed molecular gas by OB stars result in a stellar system with positive total energy.

Lada 1987

# Luminous stars → photoionization of a nearby cloud → Radiative driven implosion \*\*Rolecular Cloud\*\* (No young stars) O star CTTSs HAeBe

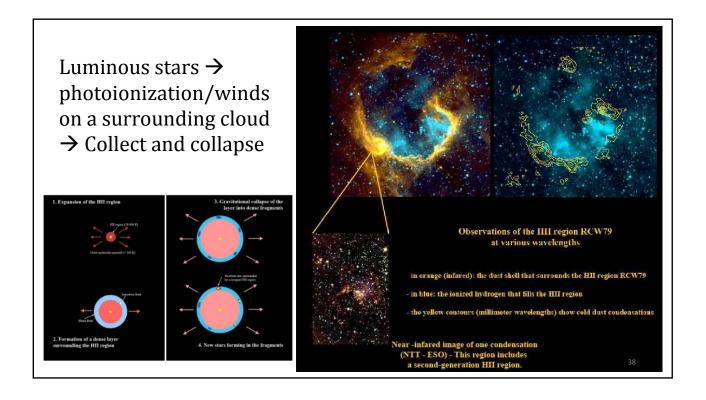
Figure 2. An illustration of a massive star to trigger star formation in a nearby molecular cloud.

Younger

Older

Chen+06

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# Size Scales for Star Formation.

Object	log size scale [cm]
Galactic spiral arm	22
Giant molecular cloud	20
Molecular dense core	17
Protostellar accretion disk	15
Protostar	11

Myers in You & Yuan (1995), p. 47

# Mass Inventory in a Star-Forming Galaxy

Component	log M [M <sub>☉</sub> ]
Molecular clouds	9
$H_2$	9
Не	8
CO	7
Young stars	5

Myers in You & Yuan (1995), p. 47

# **Properties of Giant Molecular Clouds**

Diameter	Mass	Density	T	Velocity Width
[pc]	[M <sub>☉</sub> ]	[cm <sup>-3</sup> ]	[K]	[km/s]
20-100	$10^5 - 10^6$	10-300	10-30	

Myers in You & Yuan (1995), p. 47

# Exercise

- 1. What is the BN object (why is it called an "object")? What is its brightness, distance, luminosity, and mass (how are these known)?
- 2. Answer the same for the KL object. What is the relation between the two?
- 3. There is a class of objects called the "Herbig-Haro objects". What are they?
- 4. "Quasi-Stellar Objects (QSOs)

# Cloud Stability --- The Virial Theorem

Moment of Inertia

$$I = \int r^2 dm = \sum_i m_i r_i^2$$

$$\frac{d^2I}{dt^2} = \frac{d^2}{dt^2}(mr^2)\cdots(\text{if } \dot{m} = 0)\cdots$$

$$= 2m\frac{d}{dt}(r\dot{r}) = 2m(\dot{r}^2 + r\ddot{r})$$

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2E_K + E_P$$



$$\frac{GmM}{r^2} = m\frac{v^2}{r}$$

To be stable, LHS = 0

$$2E_K + E_P = 0 \qquad \qquad 2 \left(\frac{1}{2}\right) mv^2 = GM/r$$

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# **Virial Mass**

MASS, LUMINOSITY, AND LINE WIDTH RELATIONS OF GALACTIC MOLECULAR CLOUDS

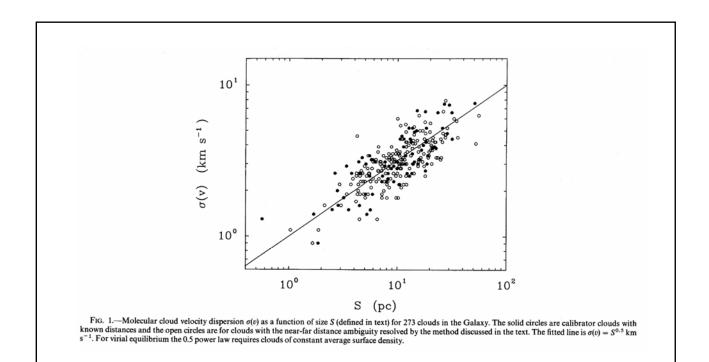
P. M. SOLOMON, A. R. RIVOLO, J. BARRETT, AND A. YAHIL Astronomy Program, State University of New York-Stony Brook Received 1986 October 2; accepted 1987 February 2

THE ASTROPHYSICAL JOURNAL, 319:730-741, 1987 August 15

### ABSTRACT

We present measurements of the velocity line width, size, virial mass, and CO luminosity for 273 molecular clouds in the Galactic disk between longitudes of 8° and 90°. These are obtained from three-dimensional data in the Massachusetts-Stony Brook CO Galactic Plane Survey. From an analysis of these measurements we show that the molecular clouds are in or near virial equilibrium and are not confined by pressure equilibrium with a warm or hot phase of interstellar matter. The velocity line width is shown to be proportional to the 0.5 power of the size,  $\sigma_v \propto 5^{0.5}$ . Combined with virial equilibrium, this shows that the clouds are characterized by a constant mean surface density of 170  $M_{\odot}$  pc<sup>-2</sup> and have a mass  $M \propto \sigma_v^4$ . A tight relationship, over four orders of magnitude, is found between the cloud dynamical mass, as measured by the virial theorem, and the CO luminosity of minosity for individual clouds and for the Galactic disk. The cloud CO luminosity is  $L_{\rm CO} \sigma_v^4$ , which is the molecular cloud analog of the Tully-Fisher or Faber-Jackson law for galaxies. The mass-luminosity law is accounted for by a cloud model consisting of a large number of optically thick clumps in virial equilibrium, each with a thermal internal velocity dispersion, but with the clouds effectively optically thin at a fixed velocity along the line of sight. The typical clump mass is of order a stellar mass and approximately equal to the Jeans mass at the clump density and thermal velocity dispersion.

	TABLE 1 GALACTIC FIRST QUADRANT MOLECULAR CLOUD CATALOG								1					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
No.	$T_{min}$ - $I$	$\ell_p$	$b_p$	$v_p$	$T_p$	R	$\boldsymbol{D}$	z	$\sigma_{\ell}$	$\sigma_b$	$\sigma_v$	$L_{co}/10^{4}$	$M_{v\tau}/10^4$	Flag
	(K)	(Deg.)	(Deg.)	(km·s <sup>-1</sup> )	(K)	(kpc)	(kpc)	(pc)	(Deg.)	(Deg.)	(km·s <sup>-1</sup> )	$(K \cdot km \cdot s^{-1} \cdot pc^2)$	(M <sub>⊙</sub> )	
1	4-3	8.00	-0.50	128.	5.7	1.4	10.1	-89.	0.06	0.07	4.4	7.27	44.4	т
2	5-3	8.20	0.20	20.	10.2	6.2	15.9	56.	0.17	0.21	4.1	140.2	176.3	F,V
3	4-4	8.30	0.00	3.	5.7	4.0	6.2	0.	0.40	0.11	3.8	22.6	65.4	X
4	4-5	8.30	-0.10	48.	8.2	3.6	13.2	-23.	0.05	0.05	2.2	5.02	11.1	F,U
Ē	5.6	9.40	-0.20	27	17.0	4.4	5.7	-30	0.32	0.15	3.9	23.3	66.5	N.H



LHS =  $0 \rightarrow$  stable

LHS  $< 0 \rightarrow$  collapsing

LHS  $> 0 \rightarrow$  expanding

# $\mathbf{E}_{\mathbf{K}}$

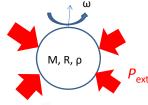
- Kinetic energy of molecules
- Bulk motion of clouds
- Rotation

 $E_{\mathbf{p}}$ 

$$E_{\text{total}} = E_{\text{K}} + E_{\text{P}}$$

- Gravitation
- Magnetic field
- Electrical field

$$E_{\text{total}} = E_{K} + \Omega \text{ (mostly)}$$



Cloud of mass M, radius R, rotating at  $\omega$ 

$$E_{rot} = \frac{1}{2}I\omega^2$$

$$I = \frac{2}{5}MR^3$$

$$I = \frac{2}{5}MR^2 \qquad \qquad \Omega = -\frac{3}{5}\frac{GM^2}{R}$$

# Generalized virial theorem

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2 < E_K > + \int \vec{r} \cdot \vec{F} dm \, + 3 \int P dV - \oint P \vec{r} \cdot d\vec{s}$$

If 
$$\omega=0$$
, and  $P_{\rm ext}=0$   $2\cdot\frac{3}{2}\frac{M}{\mu m_H}kT-\frac{3}{5}\frac{GM^2}{R}=0$ 

$$R_J = rac{1}{5} rac{GM \mu m_H}{kT}$$
 This is the **Jeans length**.

 $\mu \approx 2.37$  for solar abundance with H<sub>2</sub> Jeans length = critical spatial wavelength

If perturbation length scale is longer

→ Medium is decoupled from self-gravity → stable

$$M_J = \frac{4}{3}\pi R_J^3 \rho$$

$$R_J = \left(\frac{15}{4\pi} \frac{kT}{\mu m_H G \rho}\right)^{1/2} \sim \sqrt{\frac{T}{\rho}}$$

$$M_J=(rac{\pi kT}{4\mu m_H G})^{3/2}\sqrt{rac{1}{
ho}}\simrac{T^{3/2}}{
ho^{1/2}}$$
 This is the **Jeans mass** ... the

This is the **Jeans mass** ... the **critical** mass for onset of gravitational collapse

If cloud mass  $M > M_{Jeans}$   $\rightarrow$  cloud collapse

Note the above does not consider external pressure, or other internal supporting mechanisms.

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A non-magnetic, isothermal cloud in equilibrium with external pressure → a Bonnor-Ebert sphere (Bonnor 1956, Ebert 1955)

$$2E_K + E_P - 3P_{\text{ext}}V = 0$$

The potential term can include, other than the gravitational force, also rotation, magnetic field, etc.

At first, the cloud is optically thin.

Contraction  $\rightarrow$  density  $\uparrow \rightarrow$  collisions more frequent

- $\rightarrow$  molecules excited and radiated  $\rightarrow$  radiation escapes
- $\rightarrow$  cooling  $\rightarrow$  less resistance to the contraction
- → collapse (free fall)

 $R_{J} \approx c_{s} \tau_{ff} = [\text{isothermal sound speed}] * [\text{free fall time}]$ 

A spherical symmetric gas cloud with temperature T and external pressure P

For one particle,  $F_i = m_i \ddot{r}_i \leftarrow \frac{\partial}{\partial r}$ 

$$m_i r_i \cdot \ddot{r}_i = m_i \frac{d}{dt} (\dot{r}_i \cdot r_i) - m_i \dot{r}_i \cdot \dot{r}_i$$
$$= \frac{1}{2} m_i \frac{d^2}{dt^2} (r_i^2) - m \dot{r}_i^2$$

Summing over all particles

$$\frac{1}{2}\frac{d^2}{dt^2} \left[ \sum_{i} m_i r_i^2 \right] - 2 \sum_{i} \frac{1}{2} m_i \dot{r}_i^2 = \sum_{i} r_i F_i$$

Moment of inertial

Kinetic energy

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To maintain  $2E_K + E_P = 0$ , the total energy  $E_t = E_K + E_P$  must change. The gravitational energy

$$\Omega \sim -\frac{GM^2}{r} \to d\Omega \sim \frac{dr}{r^2}$$

For contraction, dr < 0, so  $d\Omega < 0 \rightarrow$  Then  $dE_t = dE_K + d\Omega = \frac{1}{2}\Omega = L\Delta t$ 

This means to maintain quasistatic contraction, <u>half</u> of the gravitation energy from the contraction is radiated away.

Eventually the cloud becomes dense enough (i.e., optically  $\underline{\text{thick}}$ ) and contraction leads to temperature increase.

The cloud's temperature increases while energy is taken away → negative heat capacity

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# • H I clouds

$$R_{\rm J} \approx 25$$
 pc;  $M_{\rm J} \approx 120~{\rm M}_{\odot} > M_{\rm obs}$   
So H I clouds are not collapsing.

• Dark molecular clouds

$$M_{\rm obs} \approx 100\text{-}1000 \,\mathrm{M}_{\odot} > M_{\rm J} \approx 10 \,\mathrm{M}_{\odot}$$

So H<sub>2</sub> clouds should be collapsing. But observations show that most are not.

→ There is additional support other than the thermal pressure, e.g., rotation, magnetic field, turbulence, etc.

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Roughly, the requirement for a cloud to be gravitational stable is

$$|E_{\text{grav}}| > E_{\text{th}} + E_{\text{rot}} + E_{\text{turb}} + E_{\text{mag}} + \dots$$

For a spherical cloud,  $E_{\rm grav} = -C_{\rm grav}{}^{GM^2}/_R$ , where  $C_{\rm grav}$  is a constant depending on the mass distribution (=3/5 for uniform density).

The thermal energy,  $E_{\rm th} = \frac{3}{2} \frac{m}{\mu m_H} k_B T$ , where  $\mu$  is the mean molecular weight of the gas in atomic mass units.

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The rotational energy  $E_{\rm rot} = C_{\rm rot} \, M \, R^2 \omega^2$ , where  $C_{\rm rot}$  depends on the mass distribution and is 1/5 for uniform density;  $\omega$  is the (assumed) uniform angular velocity.

The turbulent kinetic energy  $E_{\rm turb} = \frac{1}{2} M \sigma^2$ , where  $\sigma$  is the mean turbulent velocity.

The magnetic energy  $E_{\text{mag}} = \frac{1}{8} \int B^2 \, dV \approx \frac{1}{6} B^2 R^3$ , where B is the uniform magnetic field.

5

For rotational support to be important,

$$\frac{3}{5} \frac{GM}{R} = \frac{1}{2} I \omega^2 = \frac{1}{2} (\frac{2}{5} M R^2) (\frac{v_{\text{crit}}}{R})^2 = \frac{1}{5} M v_{\text{crit}}^2$$

So  $v_{\rm crit}=(3GM/R)^{1/2}$ , where  $v_{\rm crit}$  is the critical rotation velocity at the equator.

Numerically, 
$$v_{\rm crit} = 0.11 \left[ \frac{M}{M_{\odot}} \frac{\rm pc}{R} \right]^{1/2} [{\rm km/s}]$$

For HI clouds,  $v_{\rm crit} = 0.11 \, [\frac{50}{2.5}]^{1/2} \approx 0.5 \, [{\rm km/s}]$ 

Typically,  $\omega \approx 10^{-16} \, \mathrm{s}^{-1}$ , so  $v \approx 0.01 \, \mathrm{to} \, 0.1 \, \mathrm{[km/s]}$ 

→ Clouds are generally **not** rotationally supported.

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# Measuring the ISM Magnetic Fields

Method	Medium	Info
Polarization of starlight	Dust	$B_{\perp}$
Zeeman effect	Neutral hydrogen; a few mol. lines	$B_{\parallel}$
Synchrotron radiation	Relativistic electrons	$B_{\perp}$
Faraday rotation	Thermal electrons	$B_{\parallel}$

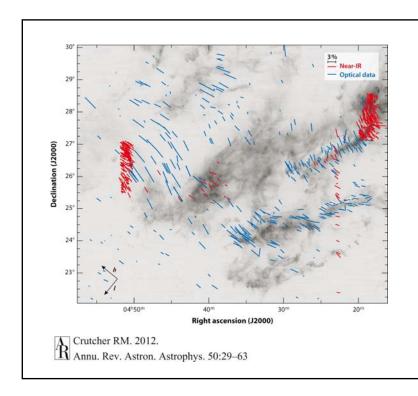
The Zeeman effect is the only technique for <u>direct</u> measurements of magnetic field strengths.

Unsöld, Crutcher (2012) ARAA

Polarization of Starlight

OPTICAL POLARIZATION 8
PCO CONTOLARS (Tn. 3 x 8 6 K)

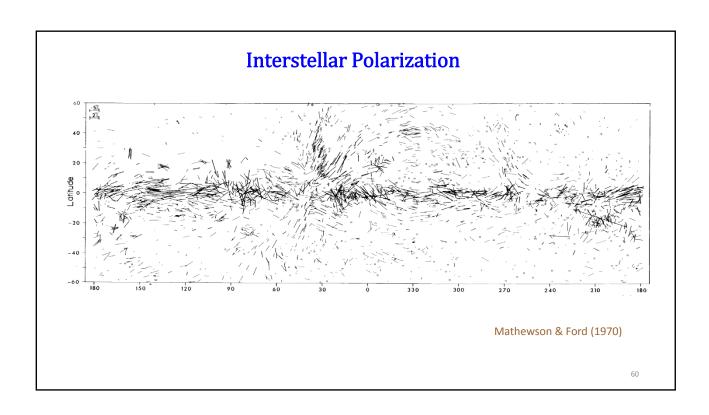
LU730

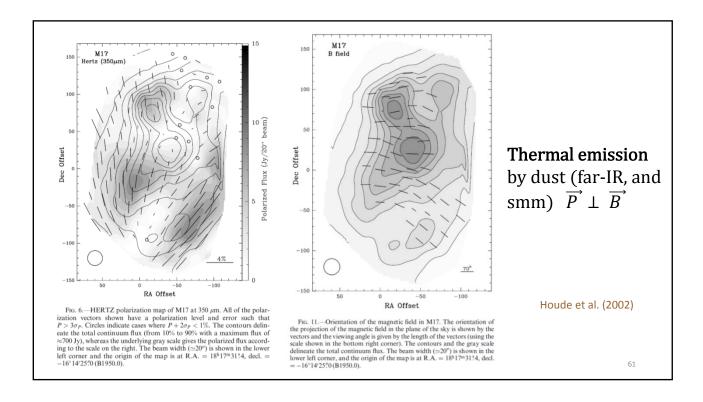


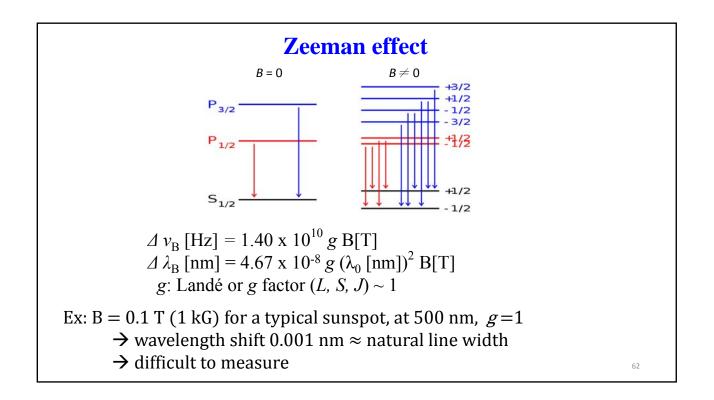
Organized magnetic field morphology in the Taurus dark-cloud complex superposed on a <sup>13</sup>CO map (Chapman et al. 2011). Blue lines show polarization measured at optical wavelengths and red lines show near-IR (H-band and I-band) polarization.

**Dichroic extinction** by dust (optical and near-IR)  $\overrightarrow{P} \parallel \overrightarrow{B}$ 

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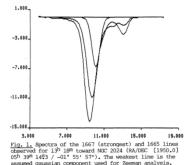
Astron. Astrophys. 125, L 23-L 26 (1983)

### Letter to the Editor

# The magnetic field of the NGC 2024 molecular cloud: detection of OH line Zeeman splitting

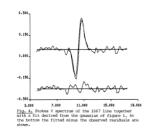
Richard M. Crutcher 1,2 and Ilya Kazès 1

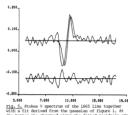
- <sup>1</sup> Department de Radioastronomie, Observatoire de Paris-Meudon, F-92195 Meudon, France
- <sup>2</sup> Department of Astronomy, University of Illinois, Urbana, IL 61801, USA



## Summary

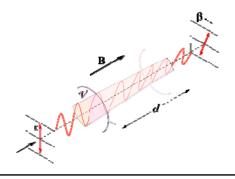
Zeeman splitting of the main lines of OH in absorption has been detected for the first time. The derived magnetic field for a clump in the NGC 2024 molecular cloud is -38 ± 1 microgauss.





**Faraday Rotation** --- rotation of the plane of polarization when light passes through a magnetic field

Circularly polarized light  $\rightarrow E$  field rotates  $\rightarrow$  force on the charged particles to make circular motion  $\rightarrow$  creating its own B field, either parallel or in opposite direction to the external field  $\rightarrow$  phase difference → Change of position angle of the linear polarization



Faraday rotation angle  $\beta = RM \lambda^2$ where the rotation measure (RM) is

$$RM = \frac{e^3}{2\pi m^2 c^4} \int_0^d n_e(s) B_{\parallel}(s) \, ds$$

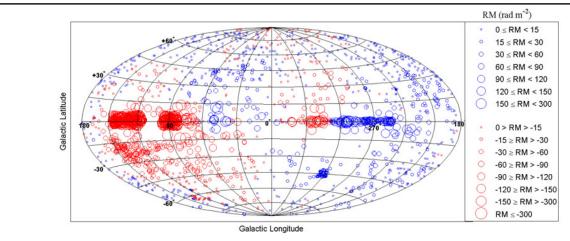


Figure 3. A smoothed representation of 2257 **Faraday rotation** measures in Galactic coordinates with the Galactic center at (0,0). (Kronberg & Newton-McGee, [3]). Blue and red circles represent positive and negative RM's respectively, and the circle size is proportional to RM strength.

http://ned.ipac.caltech.edu/level5/Sept10/Kronberg/Figures/figure3.jpg

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For magnetic support to be important,

$$\frac{3}{5} \frac{GM^2}{R} = \frac{B^2}{8\pi} \left( \frac{4}{3} \pi R^3 \right) = \frac{1}{6} B^2 R^3$$

So,  $M \propto BR^2$ , and since  $M \propto \rho R^3$ , we get  $R \propto \frac{B}{\rho}$ 

The magnetic Jeans mass becomes

$$M_{\rm Jeans}^B \propto BR^2 \propto {}^{B^3}/{}_{\rho^2}$$

Numerically,  $M_{\rm Jeans}^B \approx 2.4 \times 10^4 \ B_{\mu \rm G}^3 \ n_H^{-2} \ [M_{\odot}]$ 

and 
$$B_{\rm crit} = 0.1 \frac{M}{M_{\odot}} (\frac{\rm pc}{R})^2 [\mu \rm G]$$

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If the magnetic flux is conserved,  $\mathbf{B} \propto \frac{1}{R^2}$ 

Because  $M \propto R^3 \rho = \text{constant}$ , the **frozen-in** (i.e., flux conservation) condition would have led to  $\mathbf{B} \propto R^{-2} \sim \rho^{2/3}$ 

If flux is conserved,  $\boldsymbol{B}_0$  (ISM)  $\sim 10^{-6}$  [G]

$$R_0 \approx 0.1 \text{ [pc]} \rightarrow R = R_{\odot} \rightarrow B \approx 10^7 \text{ [G]}$$

But what has been actually observed is

$$\mathbf{B} \propto \rho^{1/3}$$
 to  $\rho^{1/2}$ ,

Implying magnetic flux loss.

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### THE ASTROPHYSICAL JOURNAL, 301:339-345, 1986 February 1

INTERSTELLAR MAGNETIC FIELD STRENGTHS AND GAS DENSITIES: OBSERVATIONAL AND THEORETICAL PERSPECTIVES

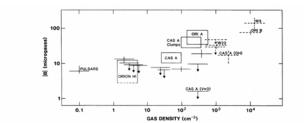
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Received 1985 January 31, accepted 1985 July 16

### ABSTRACT

ABSTRACT

We present an updated compilation of observational data concerning the relationship between the interstellar magnetic field strength and the gas density. Pulsar and Zeeman-effect data provide the only reliable information about the (B, n) relationship, and they now span nearly six orders of magnitude in gas density. Field strengths show no evidence of increase over the density range  $0.1-\sim100~\text{cm}^{-3}$ . At higher densities, a modest increase in field strength is observed in some regions, in line with theoretical expectations for self-gravitating clouds. In two regions of the interstellar medium, the magnetic field is unusually high; however, these are not locales where self-gravitation is important. Despite the consistency between observations and theory, questions still exist about how the magnetic field strength remains constant for densities up to  $\sim100~\text{cm}^{-3}$ . Further Zeeman effect studies and a better theoretical understanding of the formation of interstellar clouds and complexes will be necessary to answer these questions.



A&A 484, 773-781 (2008) DOI: 10.1051/0004-6361:200809447

### A new probe of magnetic fields during high-mass star formation

Zeeman splitting of 6.7 GHz methanol masers

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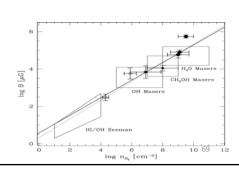
Received 24 January 2008 / Accepted 30 March 2008

Abstract

Context. The role of magnetic fields during high-mass star formation is a matter of fierce debate, yet only a few direct probes of magnetic field strengths are available.

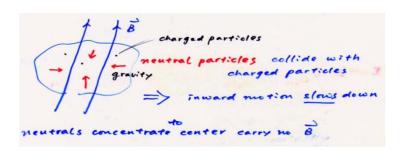
Aims. The magnetic field is detected in a number of massive star-forming regions through polarization observations of 6.7 GHz methanol masers. Although these masers are the most abundant of the maser species occurring during high-mass star formation, most magnetic field measurements in the high-density gas currently come from OH and H<sub>2</sub>O maser observations.

come from OH and H<sub>2</sub>O maser observations. Methods. The 100-m Effelsberg telescope was used to measure the Zeeman splitting of 6.7 GHz methanol masers for the first time. The observations were performed on a sample of 24 bright northern maser sources. Results. Significant Zeeman splitting is detected in 17 of the sources with an average magnitude of 0.56 m s<sup>-1</sup>. Using the current best estimate of the 6.7 GHz methanol maser Zeeman splitting coefficient and a geometrical correction, this corresponds to an absolute magnetic field strength of 23 mG in the methanol maser region. Conclusions. The magnetic field is dynamically important in the dense maser regions. No clear relation is found with the available OH maser magnetic field measurements. The general sense of direction of the magnetic field is consistent with other Galactic magnetic field measurements, although a few of the masers display a change of direction between different maser features. Due to the abundance of methanol masers, measuring their Zeeman splitting provides the opportunity to construct a comprehensive sample of magnetic fields in high-mass star-forming regions.



 $\vec{B}$  confines motion of charged particles.

Molecular clouds  $\rightarrow$  most neutral with only a tiny fraction of particles; ionized by cosmic rays or by natural radioactivity



= decoupling of neutral particles from plasma in the initial stage of star formation

 $\rightarrow$  1. leakage of  $\vec{B}$ 

2. charged particles escaped from magnetic poles (ampipolar diffusion=plasma drift)

If  $\mathcal{M}_{\text{cloud}} > \mathcal{M}_{\text{crit}} \rightarrow \underline{\text{supercritical}} \rightarrow \text{Cloud will collapse dynamically}$ 

→ Massive star formation

If  $\mathcal{M}_{\text{cloud}} < \mathcal{M}_{\text{crit}} \rightarrow \underline{\text{subcritical}} \rightarrow \text{Cloud collapses, if ever, quasi-statically}$ 

→ Low-mass star formation

Clouds tend to condense with  $\mathcal{M} \sim 10^4 M_{\odot}$ , but observed stellar mass ranges  $0.05 \leq \mathcal{M}/M_{\odot} \leq 100$ 

Why is there a lower mass limit and an upper mass limit for stars?

Cloud collapse  $\rightarrow$  (local) density increase  $\rightarrow$  (local)  $M_J$  decrease  $\rightarrow$  easier to satisfy  $M > M_J$ , i.e., cloud becomes more unstable

→ fragmentation

Formation of a cluster of stars ~~



Recall Jeans mass 
$$M_J \approx 1.2 \times 10^5 \left(\frac{T}{100 \, \text{K}}\right)^{3/2} \left(\frac{\rho_0}{10^{-24} \, \text{g cm}^{-3}}\right)^{-1/2} \frac{1}{\mu^{3/2}} \left[M_{\odot}\right]$$

$$\propto T^{2/3} / \rho^{1/2}$$

If during collapse,  $M_J \downarrow \rightarrow$  subregions become unstable and continue to collapse to smaller and smaller scales (fragmentation).

Since during collapse  $\rho$  always  $\uparrow$ , the behavior of  $M_J$  depends on T. If gravitational energy is radiated away, i.e.,  $\tau_{\rm cooling} \ll \tau_{\rm ff}$  and collapse is **isothermal**,  $T = {\rm const.}$  so  $M_J \propto \rho^{-1/2} \longrightarrow {\rm collapse}$  continues

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However, once the isothermal condition is no longer valid, e.g., when the cloud becomes optically thick, the collapse is **adiabatical**.

$$T \propto P^{2/5} \propto \rho^{2/3}$$

So  $M_J \propto \frac{\rho}{\rho^{1/2}} = \rho^{1/2}$ , i.e., grows with time (ever more difficult to overcome/collapse), so the collapse halts

For a monatomic idea gas, the adabatic index

$$\gamma \equiv c_p/c_v = \frac{f+2}{f} = \frac{5/2}{3/2} = 5/3$$

$$PV^{\gamma} = \text{const}; TV^{\gamma-1} = \text{const};$$

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**Equation of motion** for a spherical surface at r is

$$\frac{d^2r}{dt^2} = -\frac{Gm}{r^2}$$

with initial condition  $r(0) = r_0$ ,  $\frac{dr}{dt}(0) = 0$ ,  $m = 4\pi r_0^3 \rho_0/3$ .

Multiplying both sides by dr/dt, and since  $\frac{d}{dt}(\frac{dr}{dt})^2 = 2\frac{dr}{dt}\frac{d^2r}{dt^2}$ ,

$$\frac{d}{dt}(\frac{dr}{dt})^2 = -\frac{2Gm}{r^2}\frac{dr}{dt}$$

Integrating both sides, we get

$$\left(\frac{dr}{dt}\right)^2 = 2Gm\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

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Substituting *m*, we get

$$\frac{dr}{dt} = -\left[\frac{8\pi G \rho_0 r_0^2}{3} \left(\frac{r_0}{r} - 1\right)\right]^{\frac{1}{2}}$$

Define a new variable  $\theta$ , so that  $r(t) = r_0 \cos^2 \theta$ ,  $(\theta = 0 \text{ at } t = 0)$  then

$$\frac{d\theta}{dt}\cos^2\theta = \frac{1}{2} \left(\frac{8\pi G\rho_0}{3}\right)^{1/2}$$

Integrating this, we obtain  $\theta + \frac{1}{2}\sin 2\theta = \left(\frac{8\pi G\rho_0}{3}\right)^{1/2}t$ 

The free-fall time is when 
$$\theta=\pi/2$$
,  $t_{\rm ff}=\left(\frac{3\pi}{32~G\rho_0}\right)^{\frac{1}{2}}=\frac{3.4\times10^7}{\sqrt{n_0}}~{\rm [yr]}$ 

Bodenheimer p.34 ... when density becomes  $\infty$  for all m.

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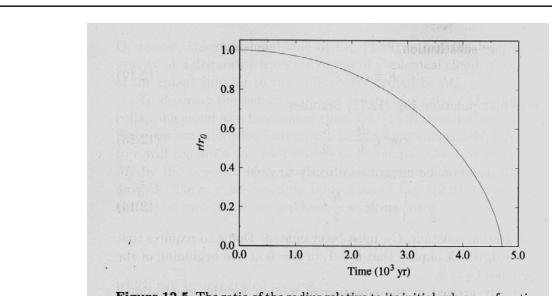
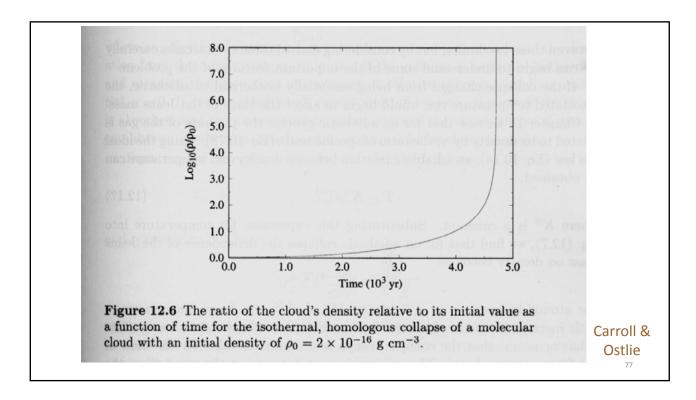


Figure 12.5 The ratio of the radius relative to its initial value as a function of time for the homologous collapse of a molecular cloud. The collapse is assumed to be isothermal, beginning with a density of  $\rho_0 = 2 \times 10^{-16} \, \mathrm{g \ cm^{-3}}$ .

Carroll & Ostlie

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Chap 2



Note that  $t_{\rm ff} \propto \frac{1}{\sqrt{G\rho_0}}$  has no dependence on  $r_0$ .

If  $ho_0$  is uniform, all  $\emph{m}$  collapse to the center at the same time

→ homologous collapse

If  $ho_0$  is somewhat centrally condensed, as observed, e.g.,  $ho_0 \propto r^{-1}$  to  $r^{-2}$ , inner region (small r),  $t_{\rm ff} \downarrow \downarrow$ 

→ inside-out collapse

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Gravitational energy available 
$$E_{\rm g} \sim \frac{4m^2}{R}$$
 which is released during the contraction  $R$  mass  $M$  brown so to  $R$ 

$$t_{\rm KH} \sim \frac{6m^2}{R} / L \sim R^{-3} \quad (\because L \sim R^2 T^4)$$

$$t_{\rm ff} \sim \frac{1}{16\rho} \sim R^{3/2}$$

For an elyest already on the main sequence
$$t_{\rm ff} << t_{\rm KH}$$

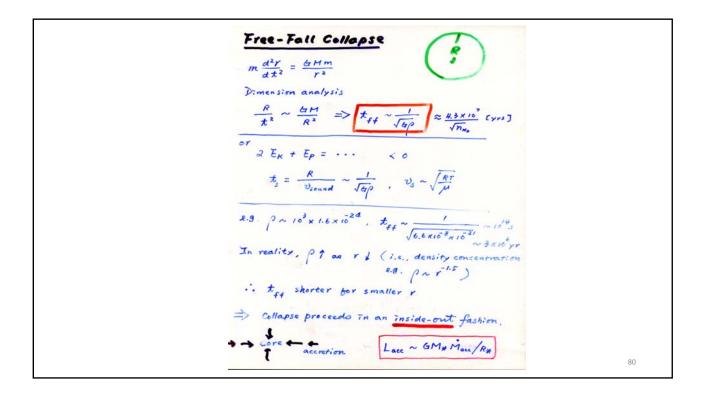
$$Ex. For 1Mo 11Lo t_{\rm ff} \sim 10^4 {\rm yr}$$

$$t_{\rm KH} \sim 2\times10^7 {\rm yr} \quad t_{\rm ff} << t_{\rm KH}$$

$$\therefore When R \ge 3 \text{ so } Ro \quad t_{\rm ff} \ge t_{\rm KH}$$

$$\Rightarrow protestellar collapse is a dynamical process.$$

$$T_{\rm ff} \sim 66120/\sqrt{\rho_{\rm MKS}} \sim 35/\sqrt{\rho_{\rm cgs}} [{\rm min}]$$

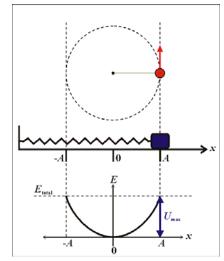


Recall the relation between a circular motion and a simple harmonic motion.

Acceleration to the center Time scale =  $\frac{1}{4}$  period

#### **Applications**:

- Gas in a collapsing cloud
- Stars in a globular cluster
- Galaxies in a galaxy cluster



http://prism.texarkanacollege.edu/physicsbook/shm-ucm.gif

81

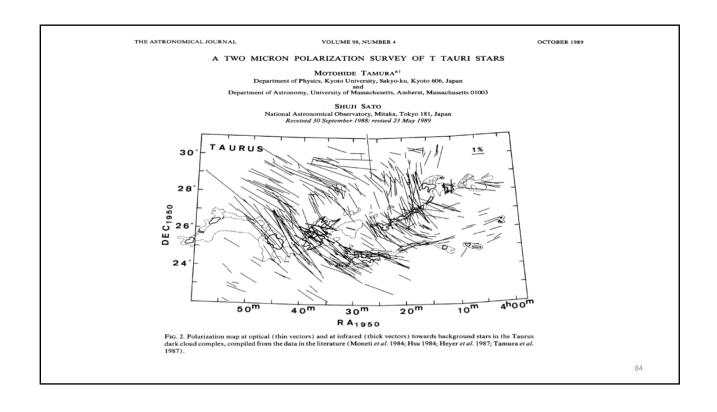
#### Exercise

- 1. For a the sun, i.e., a mass  $\mathcal{M}=1$   $\mathcal{M}_{\odot}$ , a luminosity  $\mathcal{L}=1$   $\mathcal{L}_{\odot}$ , and a radius  $\mathcal{R}=1$   $\mathcal{R}_{\odot}$ , compute the free-fall time scale  $\tau_{\rm ff}$  and the Kevin-Helmholtz time scale  $\tau_{\rm KH} \approx {}^{G}\mathcal{M}^2/_{RL}$ . Which time scale is longer?
- 2. Note that both time scales have different dependence on the size scale. At what size, do the two time scales equal?

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# Ann. Rev. Astron. Astrophys. 1987. 25: 23-81 STAR FORMATION IN **MOLECULAR CLOUDS: OBSERVATION AND THEORY** Frank H. Shu, Fred C. Adams, and Susana Lizano Astronomy Department, University of California, Berkeley, California 94720 Figure 7 The four stages of star formation. (a) Cores form within molecular clouds as magnetic and turbulent support is lost through ambipolar diffusion. (b) A protostar with a surrounding nebular disk forms at the center of a cloud core collapsing from inside-out. (c) A stellar wind breaks out along the rotational axis of the system, creating a bipolar flow. (d) The infall terminates, revealing a newly formed star with a circumstellar disk.



42 Chap 2

### Evolution from a circumstellar toroid (geometrically thick) to a disk; opening angle of the outflow widened

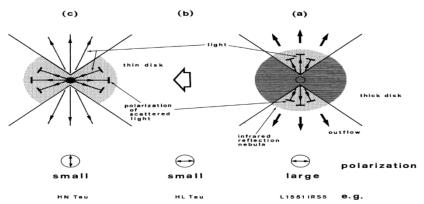


Fig. 10. Model of infarred solarization of (a) youngs tellar objects with mass outflows, (b) T Tauri stars with extreme mass-outflow objects and (c) frame stars without fact the property of the property of

85

# ApJ. . . 239L. . 17

#### OBSERVATIONS OF CO IN L1551: EVIDENCE FOR STELLAR WIND DRIVEN SHOCKS

#### RONALD L. SNELL

Astronomy Department and Electrical Engineering Research Laboratory, University of Texas at Austin; and Five College Radio Astronomy Observatory, University of Massachusetts at Amherst

#### ROBERT B. LOREN

Electrical Engineering Research Laboratory and McDonald Observatory, University of Texas at Austin

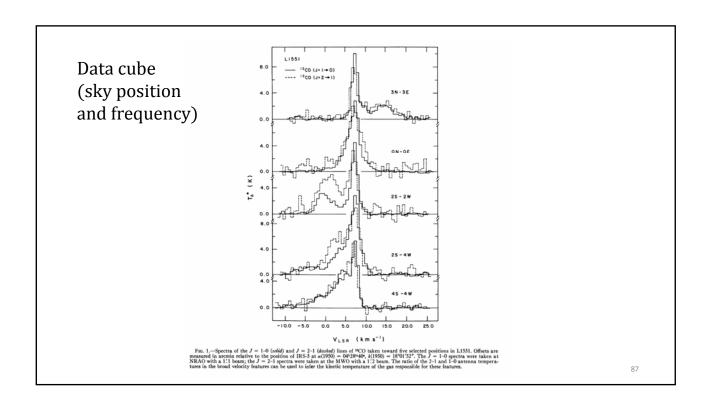
#### RICHARD L. PLAMBECK

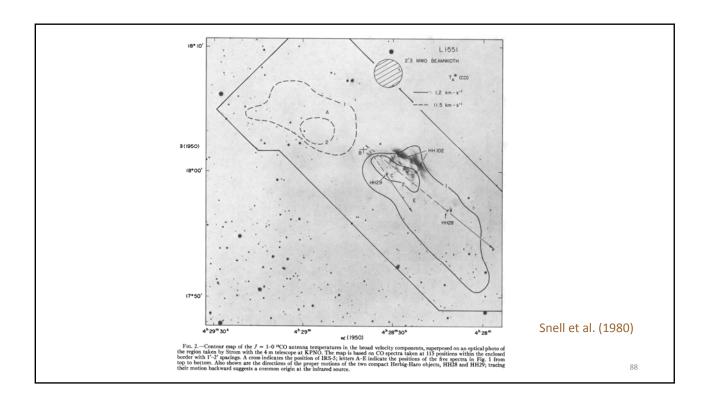
Radio Astronomy Laboratory, University of California at Berkeley

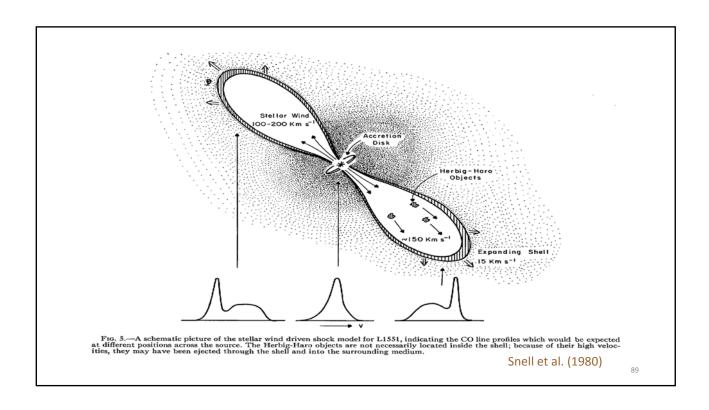
#### ABSTRACT

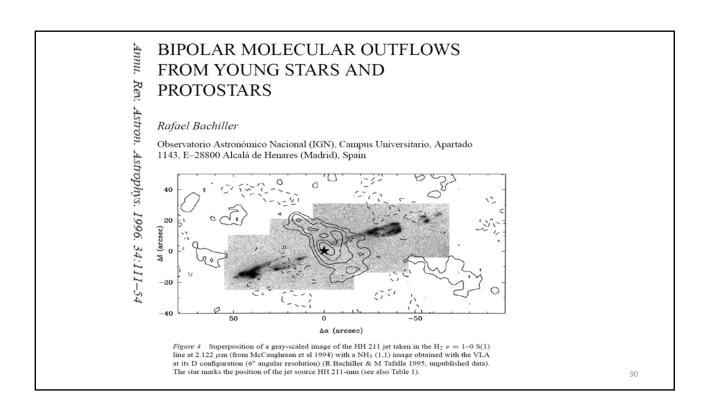
CO observations reveal the presence of a remarkable, double-lobed structure in the molecular cloud L1551. The two lobes extend for  $\sim$ 0.5 pc in opposite directions from an infrared source buried within the cloud; one lobe is associated with the Herbig-Haro objects HH28, HH29, and HH102. We suggest that the CO emission in the double-lobed structure arises from a dense shell of material which has been swept up by a strong stellar wind from the infrared source. This wind has a velocity of  $\sim$ 200 km s<sup>-1</sup>, and evidently is channeled into two oppositely directed streams. The CO observations indicate that the shell has a velocity of  $\sim$ 15 km s<sup>-1</sup>, a mass of 0.3  $M_{\odot}$ , and a kinetic temperature of 8-35 K. Its age is roughly 3  $\times$  10<sup>4</sup> years. A stellar mass-loss rate of  $\sim$ 8  $\times$  10<sup>-7</sup>  $M_{\odot}$  yr<sup>-1</sup> would be sufficient to create such a shell.

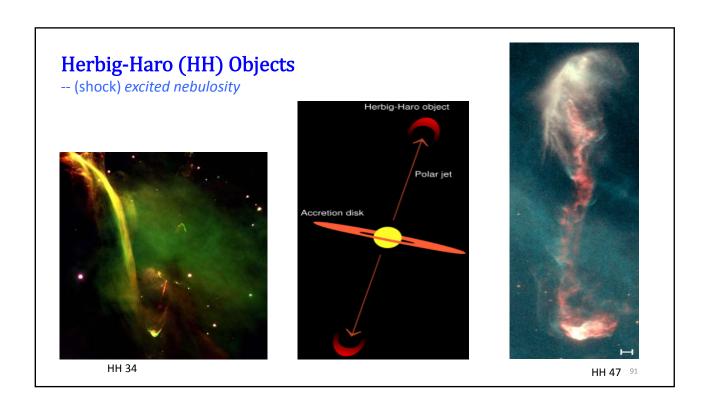
86

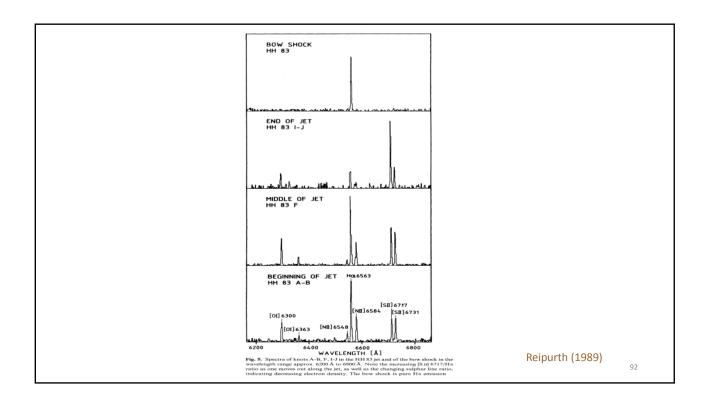


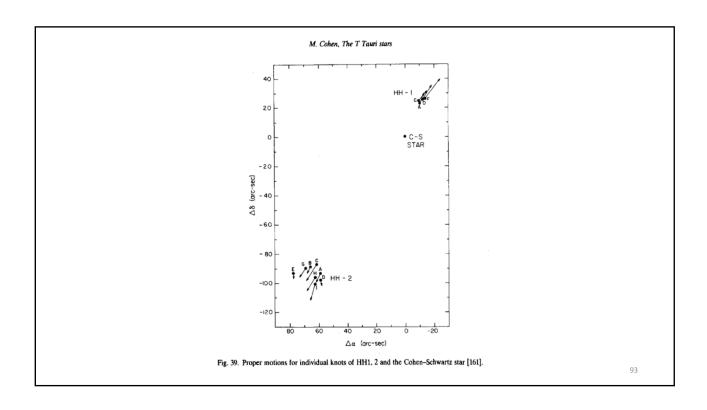




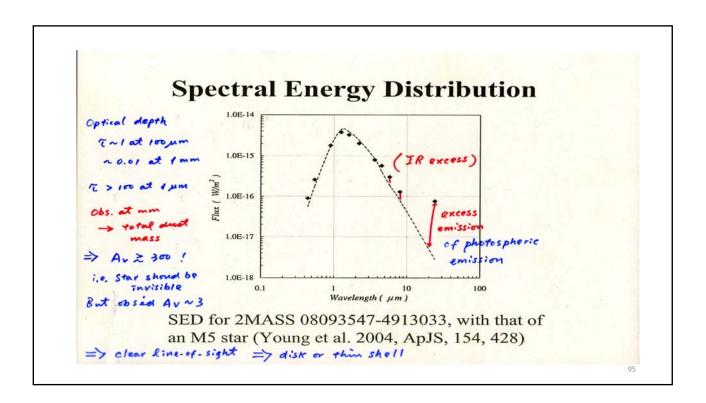


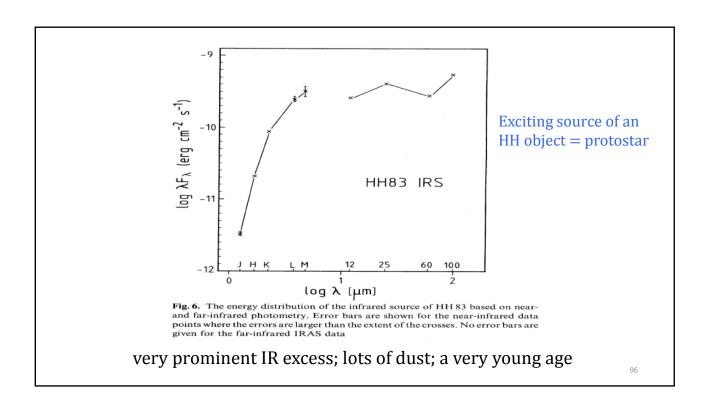


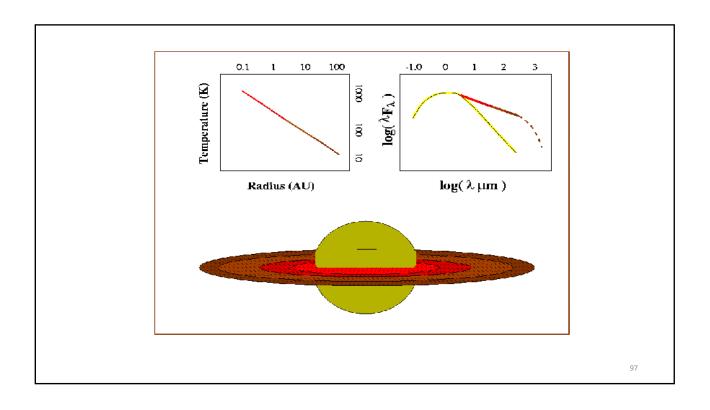




# Molecular Hydrogen Objects (MHOs) 1000+ now known Infrared image of molecular bow shocks (MHO 27) associated with bipolar outflows in Orion. Credit: UKIRT/Joint Astronomy Centre







#### **Accretion Disks**

- Found in YSOs, supermassive BHs in AGB, binaries, Saturnian rings
- Turbulent viscosity important
  - generating heat
  - transporting angular momentum <u>outwards</u>
  - transporting matter <u>inwards</u>

<u>Fact</u>: The Sun has > 99% of the total mass in the solar system, but accounts for  $\sim$ 3% of the total angular momentum (rotation), whereas Jupiter's orbital angular moment accounts for 60%.

<u>Fact</u>: Outer planets rotate fast (thus are flattened.)

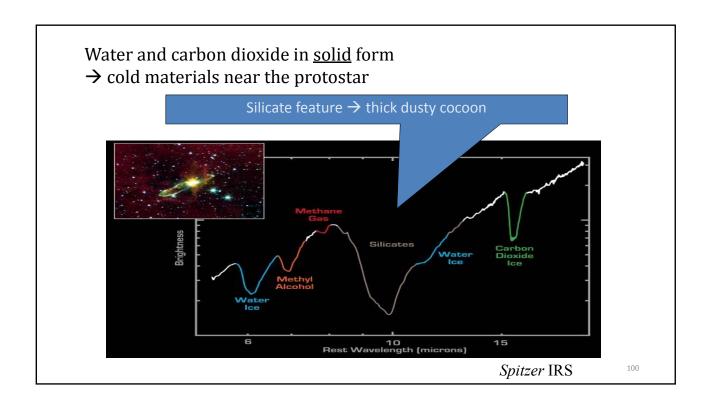
98

#### Exercise

- 1. Compare the angular momenta of the Sun, Jupiter, and Earth.
- 2. What is the specific angular momentum of the Earth versus Jupiter?
- 3. How round (or flat) is the shape of the Earth, of Jupiter, and of the Sun?

http://www.zipcon.net/~swhite/docs/astronomy/Angular\_Momentum.html

go



## **T Tauri stars** (= PMS sun-like stars) are seen against dark nebulosity and characterized by <u>emission-line spectra</u>.

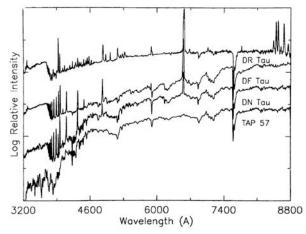
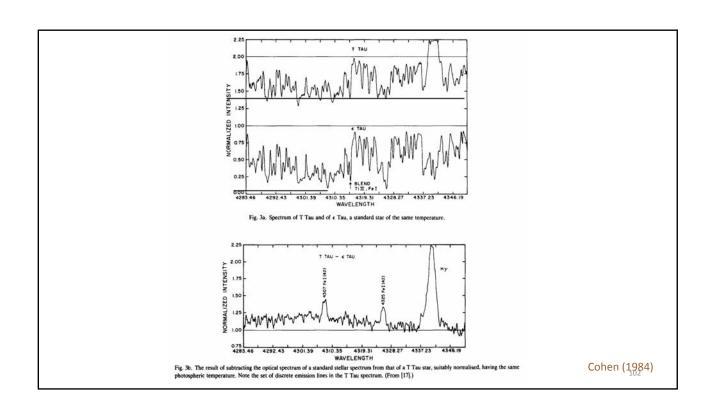
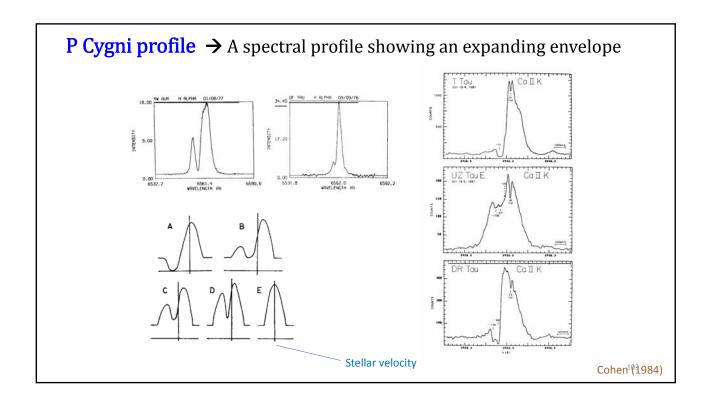
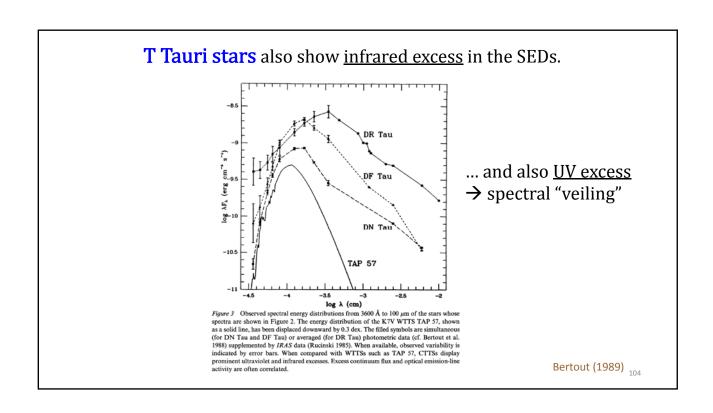


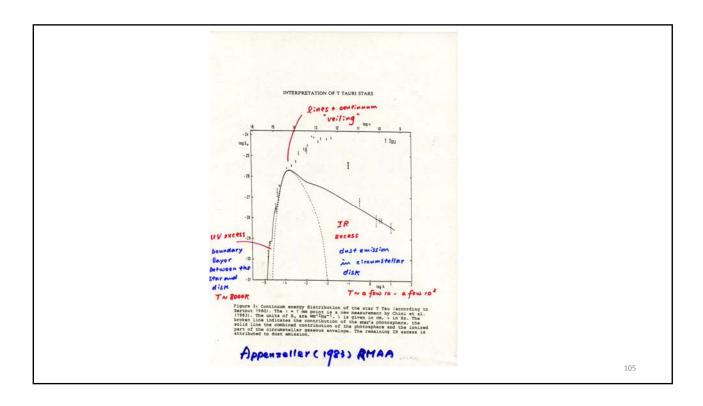
Figure 2 Medium-resolution spectrograms covering the spectral range 3200–8800 Å of four late-K or early-M T Tauri stars, shown in order of increasing emission levels. The relative intensity is displayed in wavelength units.

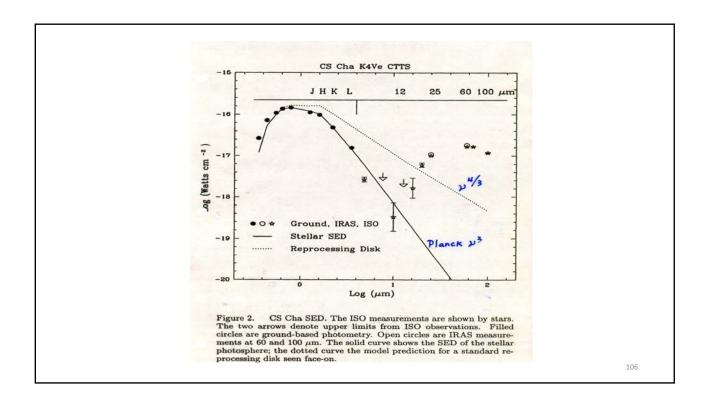
Bertout (1989)

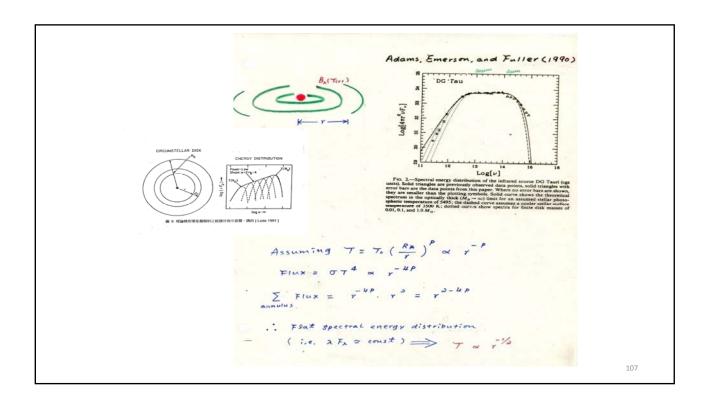












#### Spectral index useful to classify a young stellar object (YSO)

$$\alpha = \frac{d \log(\lambda F_{\lambda})}{d \log(\lambda)}$$
 Where  $\lambda =$  wavelength, between 2.2 and 20 μm;  $F_{\lambda} =$  flux density

**Class 0** sources --- undetectable at  $\lambda$  < 20  $\mu$ m

Class I sources ---  $\alpha > 0.3$ 

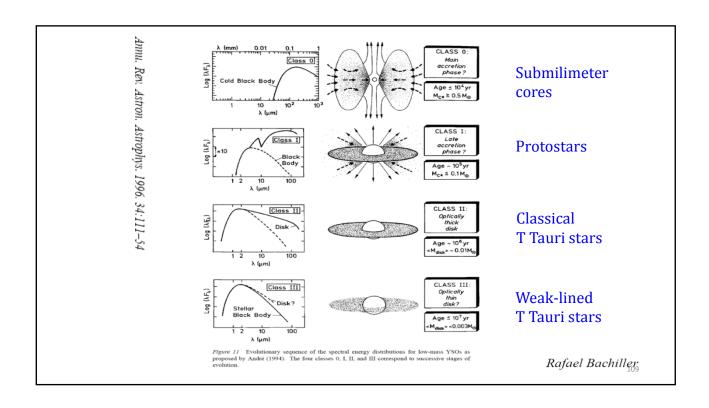
**Flat spectrum** sources ---  $0.3 > \alpha > -0.3$ 

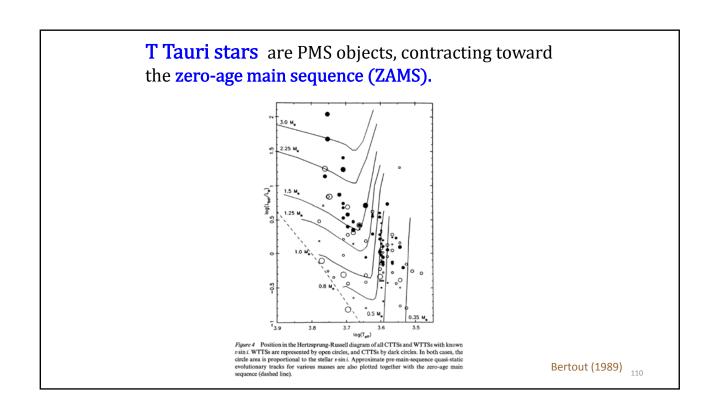
**Class II** sources ---  $0.3 > \alpha > -1.6$ 

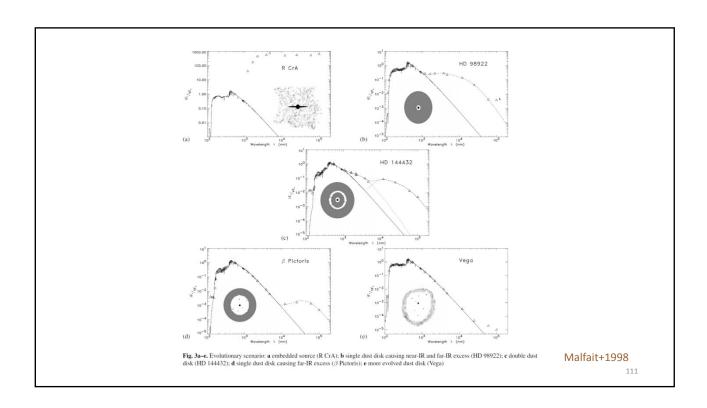
Class III sources ---  $\alpha$  < - 1.6

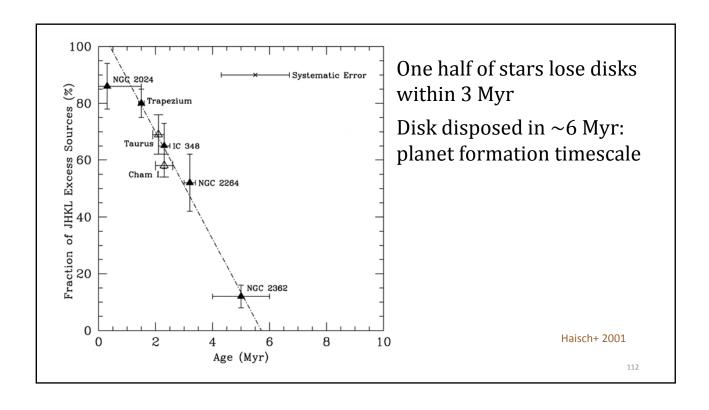
→ Evolutionary sequence in decreasing amounts of circumstellar material (disk clearing)

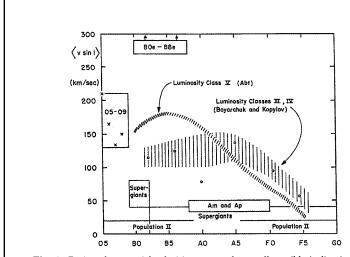
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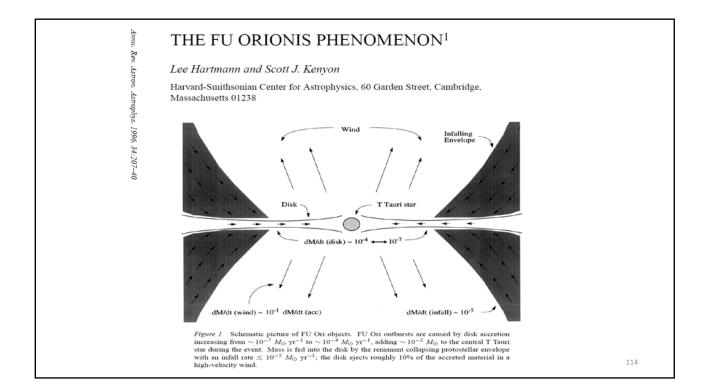


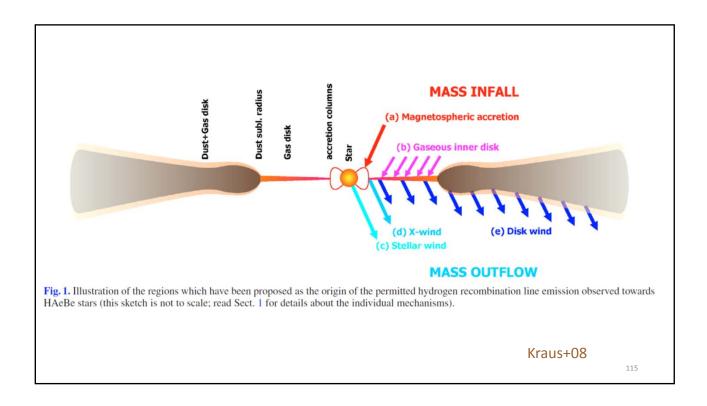


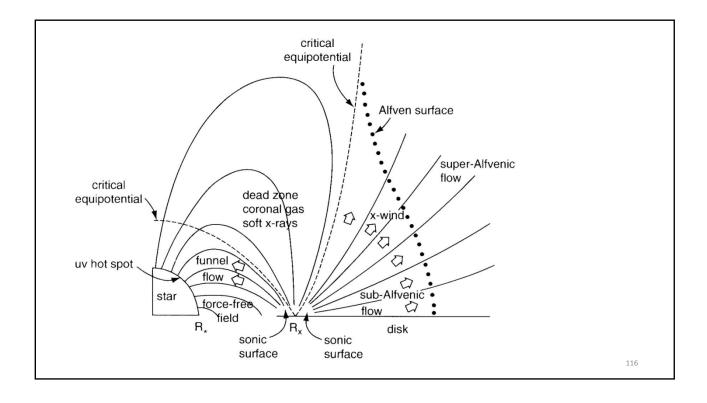
- Early-type stars are fast rotators
- ☐ Stars later than ~F5 rotate very slowly
- □ Disk/planet formation?

Fig. 3. Projected equatorial velocities, averaged over all possible inclinations, as a function of spectral type. On the main sequence (luminosity class V), early-type stars have rotational velocities that reach and even exceed 200 km/s; these velocities drop to a few km/s for late-type stars, such as the Sun (type G2) (Slettebak [20]; courtesy Gordon & Breach)

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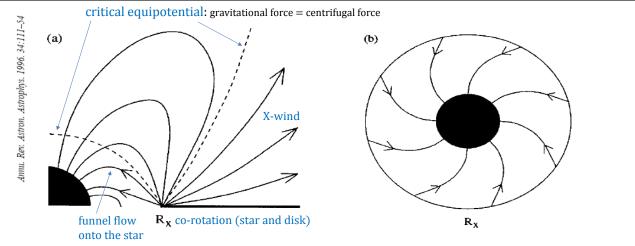
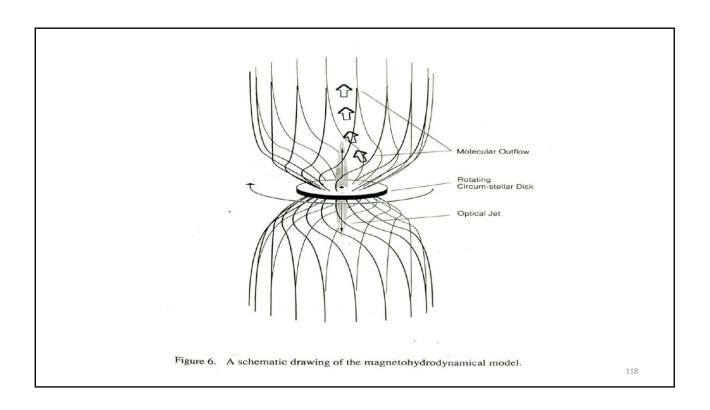
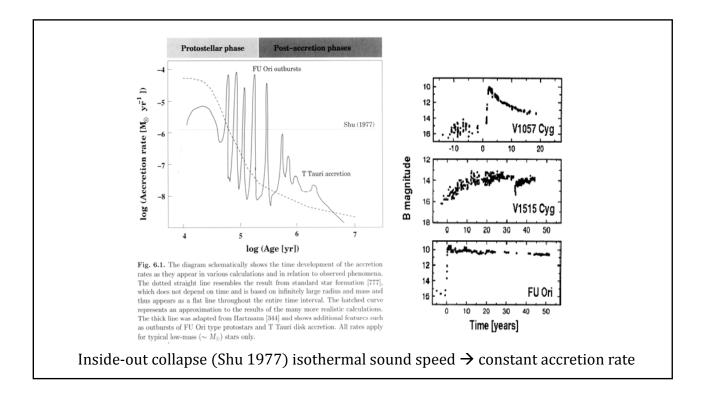


Figure 13 Schematic views of the (a) meridional plane and (b) equatorial plane of the configuration modeled by Shu et al (1994a,b) for the origin of bipolar outflows. The circumstellar disk is truncated at a distance  $R_X$  from the star. Both energetic outflows and funnel flows emerge from the disk truncation region. Gas accreting from the disk onto the star in a funnel flow drags the stellar field into a trailing spiral pattern. (From Najita 1995.)

Rafael Bachiller

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#### Collision

Gas (hydrogen atoms) root-mean-squared speed

For H I regions,

$$m_{\rm H}\sqrt{\langle v^2 \rangle} = 3kT$$

$$T \sim 100~\mathrm{K}, < v >_{\mathrm{HI}} \sim 1~\mathrm{km~s^{-1}}$$

For 
$$e^-$$
,  $< v >_{e^-} \sim 50 \text{ km s}^{-1}$ 

#### Cross sections σ



 Hard sphere OK for neutral atoms, i.e., 'physical' cross section

$$\sigma = \pi(a_1 + a_2)^2$$

$$\sigma_{\rm HI,HI} \leftarrow a \sim 5.6 \times 10^{-9} {\rm cm}$$

c.f., Bohr radius (first orbit) = 
$$5.3 \times 10^{-9}$$
 cm

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#### Cross sections σ

• For free e<sup>-</sup>, p<sup>+</sup>

 $\sigma >> \sigma_{\rm physical}$  because of Coulomb force, need QM

$$a \sim \frac{2.5 \times 10^{-2}}{v^2} \text{ cm } (v \text{ in km})$$

If  $v_{e^-} \sim 50 \text{ km s}^{-1}$ ,  $a \sim 10^{-5} \text{ cm for } e^-\text{-}e^- \text{ collision}$ 

$$T = 3 \times 10^4 \ \mathrm{K}, < v > \sim 10^3 \ \mathrm{km \ s^{-1}}$$

$$\longrightarrow$$
  $a \sim 2.5 \times 10^{-8} \text{ cm}$ 

c.f., classical electron radius  $\sim 2.8 \times 10^{-13} \ \mathrm{cm}$ 

$$\frac{e^2}{r_0} = m c^2$$

$$r_0 = \frac{e^2}{m c^2} \sim 2.8 \times 10^{-13} \text{ cm}$$

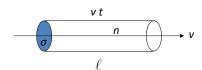
Conventional unit for cross section

 $1 \text{ barn} = 10^{-24} \text{ cm}^2$ 

 $\sigma_{\rm HI,HI} = \sim 10^8~\rm barns~(\sim 10^{-16}~\rm cm^2)$ 

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#### Collision



# of collisions = # of particles in the (moving) volume

$$N = n\sigma v t$$

# of collisions per unit time =  $N/t = n \sigma v$ 

Time (mean-free time) between 2 consecutive collisions (N=1) =  $t_{\rm collision} = \frac{1}{n \, \sigma \, v}$ 

Mean-free path  $\ell = v t_{\rm col}$ , i.e.,  $\ell = \frac{1}{N \sigma}$ 

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#### Ex 1 between hydrogen atoms in an H I region

$$n_{\rm HI} \sim 10~{\rm cm^{-3}};~v_{\rm HI} \sim 1~{\rm km~s^{-1}};~\sigma_{\rm HI,HI} \sim 10^{-16}~{\rm cm^2}$$
  
 $t_{\rm HI,HI} \sim 10^{10}~{\rm s} \sim 300~{\rm years}$   
 $\ell \sim 10^{15}~{\rm cm} \sim 100~{\rm AU}$ 

: Collisions are indeed very rare.

#### Ex 2 between a hydrogen atom and an electron

$$\sigma_{\rm e^-,HI} \sim 10^{-15} {\rm ~cm^2~(polarization)}$$
  
 $t_{\rm e^-,HI} \sim \frac{1}{10 \times 10^{-15} \times 10^5} \sim 30 {\rm ~years}$ 

#### Ex 3 between electrons

$$\begin{split} &\sigma_{\rm e^-,e^-} \sim 10^{-12}~{\rm cm^2}; \, n_e \sim 0.2~{\rm cm^{-3}} \\ &t_{\rm e^-,e^-} \sim \frac{1}{0.2 \times 10^{-12} \times 50 \times 10^5} \sim 10~{\rm days} \end{split}$$

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Chap 2