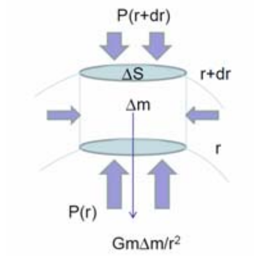
Stellar Structure

Structure Equations

What does each of these equations means?



$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2}$$ Hydrostatic equilibrium

$$\frac{dm}{dr} = 4\pi r^2 \rho$$ Mass continuity (distribution)

$$\frac{dL}{dr} = 4\pi r^2 \rho q$$ Energy generation

$$\frac{dT}{dr} = \frac{-3\kappa\rho L}{4ac4\pi r^2 T^3}$$ by radiation

Energy transport

$$\frac{dT}{dr} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$ by convection

$$P = P(\rho, T, \mu)$$ Equation of state

$$\kappa = \kappa(\rho, T, \mu)$$ Opacity

$$q = q(\rho, T, \mu)$$ Nuclear reaction rate

Variables (7): *m, ρ, T, P, κ, L*, and *q*

Vogt-Russell theorem
the structure of a star is uniquely determined by its mass and the chemical abundance.

In fact, … by any two variables above, cf. the HRD. It is not really a "theorem" in the mathematical sense, i.e., not strictly valid. It is a "rule of thumb".

In general, the equation of motion is

$$\ddot{r} = -\frac{Gm}{r^2} - \frac{1}{\rho}\frac{\partial P}{\partial r} = -\frac{Gm}{r^2} - 4\pi r^2 \frac{\partial P}{\partial m}$$

Mean molecular weight

In a fully ionized gas (in stellar interior),
$\mu = 1/2$ (H) … 2 particles per m_H
$= 4/3$ (He) … 3 particles per 4 m_H
$\cong 2$ (metals) … 2 particles per m_H

$\mu = 4/(6X + Y + 2)$ for a fully ionized gas

Adopting the solar composition,
$X_\odot = 0.747, Y_\odot = 0.236, Z_\odot = 0.017$
→ $\mu \simeq 0.6$

Note recent revision $Z_\odot = 0.0152$ (Caffau+11)

At the center of a star in hydrostatic equilibrium

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

Integrating from the center to the surface

$$P(M) - P(0) = -\int_0^M \frac{Gm\,dm}{4\pi r^4}$$

With the boundary conditions,

$$P(M) \approx 0 \quad P(0) = P_c$$

Thus,

$$P_c = \int_0^M \frac{Gm\,dm}{4\pi r^4} > \int_0^M \frac{Gm\,dm}{4\pi R^4} = \frac{GM^2}{8\pi R^4} = 4.4\times 10^{13} (\frac{M}{M_\odot})^2 (\frac{R_\odot}{R})^4 \text{ N m}^{-2}$$

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm(r)}{r^2}\rho, \text{ so } \frac{P}{R} = \frac{GM}{R^2}\frac{M}{R^3} \rightarrow P = \frac{GM^2}{R^4}$$

Ideal gas law $P = \frac{\rho}{\mu m_H} kT; \ \rho = \frac{M}{R^3}$

So $P = \frac{M}{R^3}\frac{T}{\mu}$, and $T \sim \frac{\mu GM}{R}$

This should be valid at the star's center, thus

$$\boxed{T_* \sim \frac{\mu\, GM_*}{R_*}}$$

Luminosity

Ohm's law *in circuit $I = V / R$, hydraulic analogy*

[flow] ∝ [pressure gradient] / [resistance]

(unit) Pressure = [energy] / [volume]

$$L \sim 4\pi R^2 \frac{d\left(\frac{1}{3}aT^4\right)/dr}{\kappa\rho}$$

$$\sim 4\pi R^2 \frac{4}{3}\frac{aT^3}{\kappa\rho}\frac{dT}{dr}$$

$$\sim \frac{R^2T^3}{\kappa\rho}\frac{dT}{dr}$$

Blackbody radiation

Energy density $u = aT^4$

Radiation pressure $P = (1/3)\,u$

Exercise: Derive Ohm's law.

For a given structure,

$$T \sim T_c,\ \frac{dT}{dr} \sim \frac{T_c}{R},\ T_c \sim \frac{\mu GM}{R}.$$

$$L \sim \frac{R^2T^4/R}{\kappa(M/R^3)} \sim \frac{R^4T^4}{\kappa M} \sim \frac{R^4}{\kappa M}\left(\frac{\mu GM}{R}\right)^4$$

$$\boxed{L \sim \frac{\mu^4G^4M^3}{\kappa}}$$

$$L \sim \frac{\mu^4 G^4 M^3}{\kappa}$$

The opacity $\kappa = \kappa(\rho, T, \mu)$

- For solar composision, Kramers opacity

$$\kappa \sim \rho T^{-3.5} \quad \text{valid for } 10^4\text{–}10^6 \text{ K.}$$

$$T \sim \frac{\mu\, GM}{R}$$

So $\kappa \sim \mu^{-3.5} G^{-3.5} M^{-2.5} R^{0.5}$

and $\boxed{L \sim \mu^{7.5} G^{7.5} M^{5.5} R^{-0.5}}$

- For high-mass stars, i.e., high temperature and low density, opacity by electron scattering

$$\kappa = 0.2\,(1 + X)\ \mathrm{cm^2 g^{-1}} = \text{const.}$$

and $\boxed{L \sim \mu^4 G^4 M^3}$

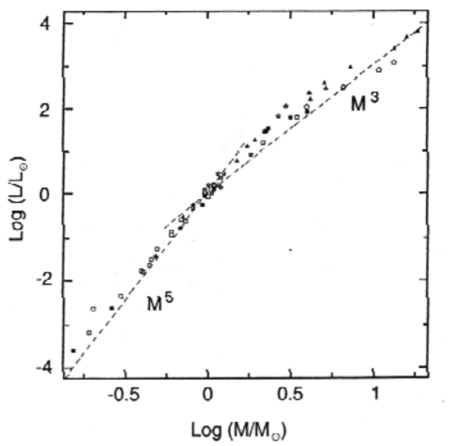
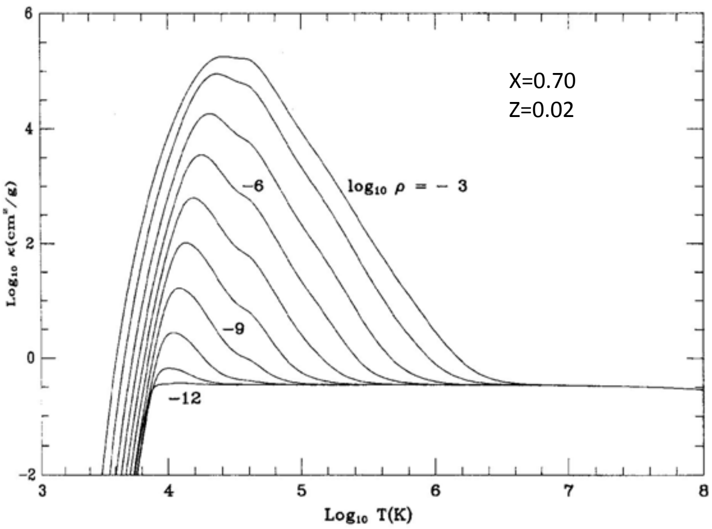


Figure 1.6 The mass–luminosity relation for main-sequence stars. Symbols denote ordinary binary stars (squares); eclipsing variables (triangles); Cepheids (pentagons); double-star statistics (stars).

Opacity

- <u>Bound-bound absorption</u> Excitation of an electron of an atom to a higher energy state by the absorption of a photon. The excited atom then will be de-excited spontaneously, emitting a photon, or by collision with another particle.
- <u>Bound-free absorption</u> Photoionization of an electron from an atom (ion) by the absorption of a photon. The inverse process is radiative recombination.
- <u>Free-free absorption</u> Transition of a free electron to a higher energy state, via interaction of a nucleus or ion, by the absorption of a photon. The inverse process is bremsstrahlung.
- <u>Electron scattering</u> Scattering of a photon by a free electron, also known as Thomson (common in stellar interior) or Compton (if relativistic) scattering.
- <u>H^{-} absorption</u> Important when $< 10^4$ K, i.e., dominant in the outer layer of low-mass stars (such as the Sun)

- Bound-bound, bound-free, and free-free opacities are collectively called Kramers opacity, named after the Dutch physicist H. A. Kramers (1894-1952).
- All have similar dependence $\kappa \propto \rho\, T^{-3.5}$.
- Kramers opacity is the main source of opacity in gases of temperature $10^4 \sim 10^6$ K, i.e., in the interior of stars up to $\sim 1\ M_{\odot}$.
- In a star much more massive, the electron scattering process dominates the opacity, and the Kramers opacity is important only in the surface layer.

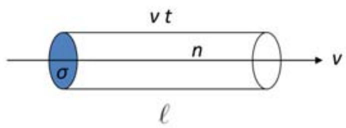


Data from Iglesias & Rogers (1996)

Opacity

κ in cm^2 g^{-2}

$$\boxed{\kappa\rho = \Sigma_i n_i \sigma_i}$$



$\int \kappa\rho ds$ gives the optical depth

The Rossland mean opacity

$$\frac{1}{<\kappa>} = \frac{1}{B}\int_0^\infty \frac{B_\nu}{\kappa_\nu} d\nu$$

For Kramers opacity

$$\kappa_{Kr} \approx 4 \times 10^{25}\,(1+X)(Z+0.001)\,\rho\,T^{-3.5}\ \ [\text{cm}^2\text{g}^{-1}]$$

For Thomson scattering,

$$\kappa_\nu = \frac{8\pi}{3}\frac{r_e^2}{\mu_e m} = 0.20\,(1+X)\ \ [\mathrm{cm^2 g^{-1}}]$$

is frequency independent, so is the Rossland mean.

$$\kappa_{es} = 0.20\,(1+X)\ \ [\mathrm{cm^2 g^{-1}}]$$

Here r_e is the electron classical radius, X is the H mass fraction, and $\mu_e = 2/(1+X)$

the electron cross section $\sigma = 0.665 \times 10^{-24}$ [cm^2]

- For H^- opacity, $E_{\rm ion} = 0.754$ eV
  important for $4 \times 10^3 < T < 8 \times 10^3$ K

$$\boxed{\kappa_{H^-} \approx 2.5 \times 10^{-31} \left(\frac{Z}{0.02}\right) \rho^{0.5} T^9\ [\mathrm{cm^2\ g^{-1}}]}$$

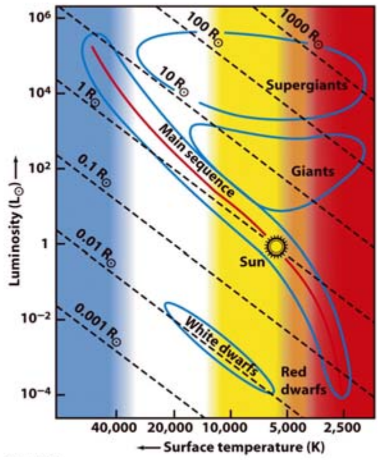
is temperature and metallicity (providing electrons) dependent.

- For $T > 10^4$ K, H^- is ionized.

- At $T < 3500$ K, molecular opacity dominates.

Main sequence = a mass sequence defined by hydrogen fusion at the center of a star

Radius does not vary much; but the luminosity does.



$\log L \propto \log T$

$$T_c \approx \frac{\mu\, GM}{R}$$

So for a given T_c, M $\left.\begin{array}{l} \to R \\ \to L \end{array}\right\}$ $L\,(\propto R^2\, T^4)$ and T

Main sequence is a run of L and T_c as a function of stellar mass, with T_c nearly constant.

Why $T_c \approx$ constant?
Because H burning at $\sim 10^7$ K regardless of the stellar mass

Gas Thermodynamics

Heat capacity: heat supplied to increase one degree in temperature; C_P and C_V

Specific heat capacity (=per unit mass), c_P and c_V

$c_P - c_V = k_B$

$c_P \,/\, c_V = \gamma$

γ: the adiabatic index or heat capacity ratio

e.g., dry air, =1.403 (0°C), =1.400 (20°C)

O_2, =1.400 (20°C), =1.397 (200°C)

H_2O, = 1.330 (20°C), =1.310 (200°C)

To Determine γ of a Star

For an ideal gas, $u_i = \frac{1}{2}kT$ per degree of freedom

Equipartition of energy $\rightarrow u = \Sigma u_i = \frac{n}{2}kT$ for n dof

Since $c_V = \left(\frac{\partial u}{\partial T}\right)_V = \frac{n}{2}k$, and $\frac{c_P}{c_V} \equiv \gamma = \frac{nk/2+k}{nk/2} = 1 + \frac{2}{n}$

For an ideal gas, $n = 3, \gamma = 5/3$

For a photon gas, $n = 6, \gamma = 4/3$

(3 propagation directions, each with 2 polarizations)

For a monatomic gas, dof=3 $\rightarrow \gamma = 5/3 = 1.67$

For a diatomic gas, dof =5 $\rightarrow \gamma = 7/5 = 1.4$

<u>Equation of State</u>

Stability of a star: $2E_K + E_P = 0$

$$\begin{aligned} E_{\text{thermal}} &= \frac{3}{2}kT = \frac{3}{2}(c_P - c_V)T \\ &= \frac{3}{2}(\gamma - 1)\, c_V T \\ &= \frac{3}{2}(\gamma - 1)\, U \end{aligned}$$

$$E_P = \Omega$$

So, $3(\gamma - 1)U + \Omega = 0$

$$E_{\text{total}} = U + \Omega = \left[\frac{-1}{3(\gamma-1)} + 1\right]\Omega = \frac{3\gamma - 4}{3(\gamma - 1)}\Omega$$

Because $\Omega < 0$, in order to be stable, $E_{\text{total}} < 0 \rightarrow \gamma > 4/3$

In general, for a stable star with a mixture of gas and radiation,

$$\frac{4}{3} \leq \gamma \leq \frac{5}{3}$$

$\gamma \rightarrow 4/3$, radiation pressure dominates.

$\gamma \rightarrow 5/3$, gas pressure dominates.

For an ideal gas, $P = \frac{N}{V}kT = \frac{\rho}{\mu m_H}kT$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \quad \text{and} \quad PdV + VdP = NkdT$$

First law of thermodynamics (conservation of energy)

$$dQ = dU + PdV$$

For constant V, $c_V = \left(\frac{dQ}{dT}\right)_V = \frac{dU}{dT}$

$$dQ = dU + NkdT - VdP = \left(\frac{dU}{dT} + Nk\right)dT - VdP$$

So for constant P, $c_P = \left(\frac{dQ}{dT}\right)_P = \frac{dU}{dT} + Nk = c_V + Nk$

Hence $c_P = c_V + Nk$,

and $\gamma = c_P/c_V = (Nk + c_V)/c_V$

An isothermal (= constant in temperature) process:

internal energy does not change

An adiabatic process: $dQ = 0$

$$dQ = c_V dT + PdV = c_V dT + (NkT/V)dV$$

$$= dT/T + (c_P - c_V)/c_V(dV/V) = 0$$

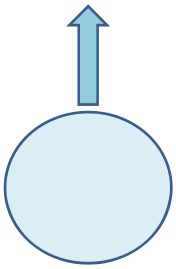
$$\log T + (\gamma - 1)\log V = \text{constant}$$

$TV^{\gamma-1} = \text{constant}$

$PV^{\gamma} = \text{constant}$

$P^{1-\gamma}T^{\gamma} = \text{constant}$

Convective equilibrium (stability vs instability)



A fluid convective "cell" is buoyed upwards.

If temperature inside is higher than surroundings, the cell keeps rising. E_{kin} of particles higher → dissipates

Otherwise it sinks back (convectively stable).

The rising height is typified by the mixing length ℓ, or parameterized as the scale height H, defined as the pressure (or density) varies by a factor of e. Usually

$$0.5 \lesssim \ell/H \lesssim 2.0$$

Convective stability/instability

How good is energy transportation by radiation?

cf. Schwarzschild

Convective stability: a fluid resisting vertical motion

atmosphere in radiative equilibrium $T\downarrow$ as $r\uparrow$

Consider a mass of gas

Rises → expands adiabatically

$\therefore T\downarrow$

⇒ denser than the surroundings

→ sinks back

$\therefore$ Stable in rad. equil.

So, vertical perturbation dampens out

But if rises, adiabatically cooling, but still warmer than the surroundings

⇒ less denser than surr.

→ keeps rising

⇒ Convection sets in when the adiabatic temp. gradient is smaller than temp. gradient by radiative equil.

i.e., $\boxed{\left(\frac{dT}{dr}\right)_{ad} < \left(\frac{dT}{dr}\right)_{rad}}$

Compared with surrounding temperature gradient

Radiation can no longer transport the energy efficiently enough
➔ Convective instability

For an adiabatic process, PV^{γ} = *constant*

Since $\frac{dP}{dr} = -\rho g$ and $P = \rho k T$

$dT \cdot \frac{dP}{dr}\frac{1}{P} \propto \frac{1}{T} \cdot dT$

$\therefore \quad \frac{dT}{dr} \propto \frac{dT/T}{dP/P} = \frac{d\ln T}{d\ln P}$

⇒ Criterion for convection equilibrium becomes

$$\boxed{\left(\frac{d\ln T}{d\ln P}\right)_{ad} < \left(\frac{d\ln T}{d\ln P}\right)_{rad}}$$

With the notation ∇ (nabla)

$$\boxed{\nabla_{ad} < \nabla_{rad}}$$

Convection takes place when the temperature gradient is "sufficiently" high (compared with the adiabatic condition) or the pressure gradient is low enough.

Such condition also exists when the gas absorbs a great deal of energy without temperature increase, e.g., with phase change or ionization
➔ when c_V is large or γ is small

In meteorology, dry and cool air tends to be stable, whereas wet and warm air (smaller gamma values) is vulnerable to convection → thunderstorm

How to calculate ∇_{rad} ?

$$\frac{dT}{dr} = -\frac{3}{4ac} \cdot \frac{\kappa\rho}{T^3} \frac{L_r}{4\pi r^2} \qquad \text{but } \frac{dP}{dr} = -g\rho$$

$$\therefore \frac{dT}{dP} \propto \frac{\kappa}{T^3} \frac{L_r}{r^2}$$

$$\nabla_{rad} \equiv \left(\frac{d\ln T}{d\ln P}\right)_{rad} = \frac{dT/T}{dP/P} = \cdots = \frac{3\kappa}{16\pi ac} \frac{P}{T^4} \frac{L_r}{GM_r}$$

For an adiabatic process for an ideal gas

① $P = nkT \propto \rho T$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

② $nR = c_p - c_v$

③ $\gamma = \frac{c_p}{c_v} = \frac{1+c_v}{c_v}$

$= \frac{1+n/2}{n/2} = 1 + \frac{2}{n}$

n: d.o.f.

$$dQ = c_v\,dT + P\,d\left(\frac{1}{\rho}\right) = c_v\,dT - \frac{P}{\rho^2}\,d\rho = 0$$

$$\therefore\ c_v\,dT = \frac{P}{\rho^2}\,d\rho$$

$$c_v\frac{dT}{T} = \frac{P}{\rho T}\cdot\frac{d\rho}{\rho}$$

$$c_v\frac{dT}{T} = (c_p - c_v)\left(\frac{dP}{P} - \frac{dT}{T}\right)$$

$$\Rightarrow\ c_p\frac{dT}{T} = (c_p - c_v)\frac{dP}{P}$$

n: d.o.f

$$\nabla_{ad} \equiv \left(\frac{d\ln T}{d\ln P}\right)_{ad} = \left(\frac{dT/T}{dP/P}\right)_{ad} = 1 - \frac{c_v}{c_p} = 1 - \frac{1}{\gamma}$$

e.g., for monatomic gases, $\gamma = 5/3$ $\quad\nabla_{ad} = 0.4$

In practice, if $\gamma = {}^{5}/_{3}$, the condition for convective stability (no convective) is $\left(\frac{d\log T}{d\log P}\right) < 0.4$

Note. $\nabla_{rad} \propto P$

At surface $\nabla_{rad} \to 0$

$\therefore\ \nabla_{ad} \overset{\text{always}}{>} \nabla_{rad} \Rightarrow$ no convection!

The outermost layers of a star are always in radiative equilibrium.

$\therefore$ Convection occurs either

① large temperature gradient for radiative equilibrium

② small adiabatic temperature gradient

Ionization satisfies both conditions because

① Opacity ↑

② e^- receive energy → d.o.f. ↑, so γ ↓ → ∇_{ad} ↓

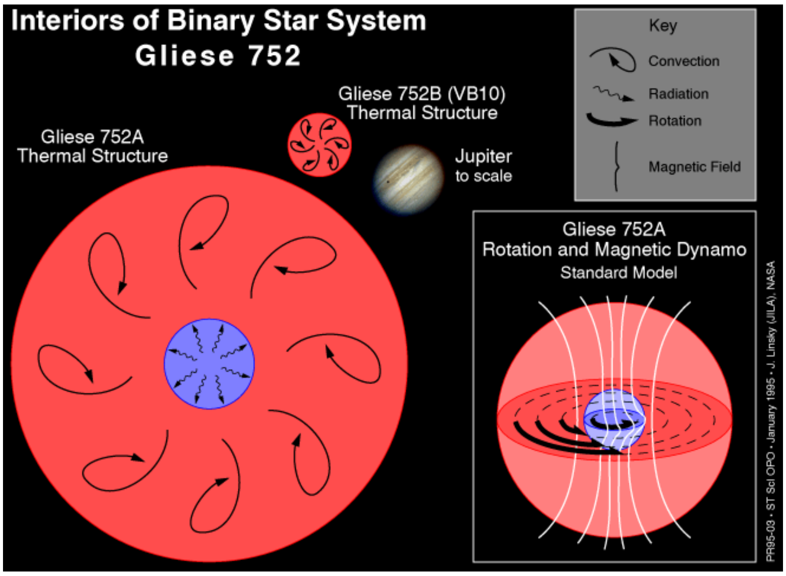
→ Development of hydrogen convective zones

Similarly, there are 1st and 2nd helium convective zones.

For a **very low-mass star**, ionization of H and He leads to a fully convective star → H completely burns off.

For a **sun-like star**, ionization of H and He, and also the large opacity of H^- ions → a convective envelope (outer 30% radius).

For a **massive star**, the core produces fierce amount of energy → convective core
→ a large fraction of material to take part in the thermonuclear reactions



A binary system at 5.74 pc. Gliese 752A (=Wolf 1055) is an M2.5 red dwarf (mass ~0.46 solar, m_V~9.13), whereas Gliese 752B (VB 10) is an M8V (mass ~0.075 solar, m_V~17.30).

Energy Transport

By radiation

$$\frac{dT}{dr} = \frac{-3}{4ac}\,\frac{\kappa\rho}{T^3}\,\frac{L_r}{4\pi r^2}$$

L_r: luminosity

κ: opacity

(electron scattering, b-f, f-f, H^-)

Note For radiative transport

$$\nabla_{rad} \equiv \left(\frac{d\ln T}{d\ln P}\right)_{rad} = \frac{3\kappa}{16\pi a c}\,\frac{P}{T^4}\left(\frac{L_r}{G M_r}\right)$$

If temperature gradient is too large, then

By convection (unstable to convection; convective instability)

criterion $\boxed{\nabla > \nabla_{ad}}$

$$\nabla \equiv \frac{d\ln T}{d\ln P} \qquad \nabla_{ad} \equiv \left(\frac{d\ln T}{d\ln P}\right)_{ad} = \frac{\gamma-1}{\gamma}$$

γ: adiabatic index

In case of convection

$$\nabla \equiv \frac{d\ln T}{d\ln P} = \frac{P}{T}\frac{dT/dr}{dP/dr} \overset{\text{hydrostatic equil}}{=} -\frac{r^2}{GM_r}\left(\frac{P}{\rho T}\right)\frac{dT}{dr} \simeq \nabla_{ad}$$

$$\therefore \frac{dT}{dr} = -\nabla_{ad}\frac{GM_r}{r^2}\frac{\rho T}{P} = -\frac{\gamma-1}{\gamma}\frac{GM_r}{r^2}\frac{\rho T}{P}$$

Hayashi Track

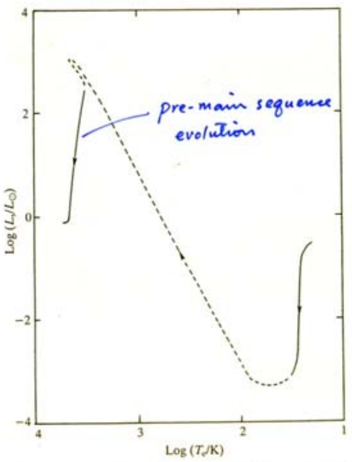
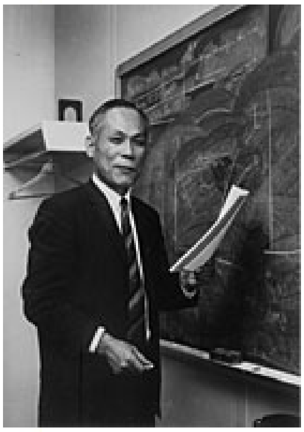


Fig. 53. The complete pre-main-sequence evolution of a star of solar mass.

Chushiro HAYASHI 1920-2010



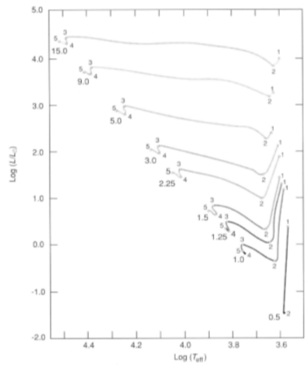
A convection evolutionary track for low-mass pre-main sequence stars

When a protostar reaches hydrostatic equilibrium, there is a minimum effective temperature (~4000 K) cooler than which (the **Hayashi boundary**) a stable configuration is not possible (Chushiro Hayashi 1961).

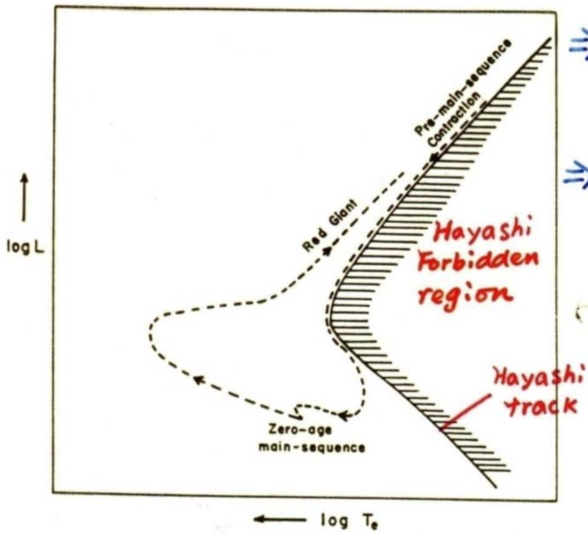
A protostar

- contracts on the Kevin-Helmholtz timescale
- is cool and highly opaque → fully convective
  → homogenizes the composition

A star < 0.5 $M_\odot$ remains on the Hayashi track throughout the entire PMS phase.



Convective instability



⇒ R_* ↓ than pure radiative case

⇒ A radius maximum for a given mass and luminosity

⇒ A temperature min

$T < T_{min}$

convectively unstable

(Hayashi 1966)
Chushiro H.
林忠四郎

Figure 1. Schematic evolutionary path of a star of 0.8 $M_\odot$.

Convection occurs when $\nabla_{\text{rad}} > \nabla_{\text{ad}}$

That is, when ∇_{rad} is large, or when ∇_{ad} is small.

Recall $\nabla_{\text{rad}} = \frac{dT}{dr} = \frac{L_r}{r^2} \frac{\kappa\rho}{\sigma T^3}$

$\nabla_{\text{ad}} = 1 - \frac{1}{\gamma}$ where $\gamma = {}^{c_p}/_{c_v}$

$\rightarrow \nabla_{\text{ad}}$ small $= c_v$ large $\rightarrow$ H_2 dissociation
H ionization, T~6,000 K
He ionization, T~20,000 K
He II ionization, T~50,000 K

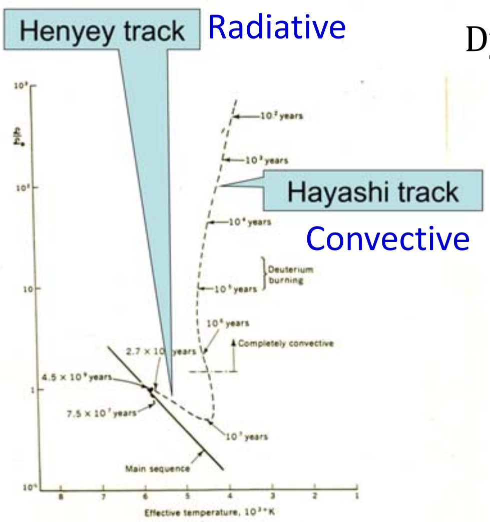


Fig. 5-1 The path on the H-R diagram of the contraction of the sun to the main sequence. The interior has become sufficiently hot to burn deuterium after about 10⁵ years. The contraction ceases near the main sequence when the core has become hot enough to replenish the solar luminosity with the thermonuclear power generated by the fusion of hydrogen into helium. [*After D. Ezer and A. G. W. Cameron, The Contraction Phase of Stellar Evolution, in R. F. Stein and A. G. W. Cameron (eds.), "Stellar Evolution," Plenum Press, New York, 1966.*]

Dynamical collapse → quasi-static contraction

- ☐ half of E_{grav} becomes particle E_{kin} (internal energy); half radiated away
- ☐ Matter optically thin to thick → thermodynamical equilibrium, $T_{\text{rad}} = T_{\text{kin}}$
- ☐ Energy used for ionization (for H, $T < 10^4$ K), so surface temperature remains almost constant. Protosun now size was 60 $R_\odot$
- ☐ Star fully convective

→ Hayashi track

Check out http://www.peripatus.gen.nz/Astronomy/HerRusDia.html for a good summary

Astron. & Astrophys. 40, 397—399 (1975)

On the Luminosity of Spherical Protostars

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Summary. Hydrodynamic model computations have been carried out for a spherically symmetric $1\,M_\odot$ protostar. Compared to similar computations by Larson (1969) we used a different treatment of the accretion shock front. Our computations basically confirm Larsons results and show that Larson's disputed shock jump conditions have little influence on the protostellar models.

Key words: star formation — protostars — YY Orionis stars

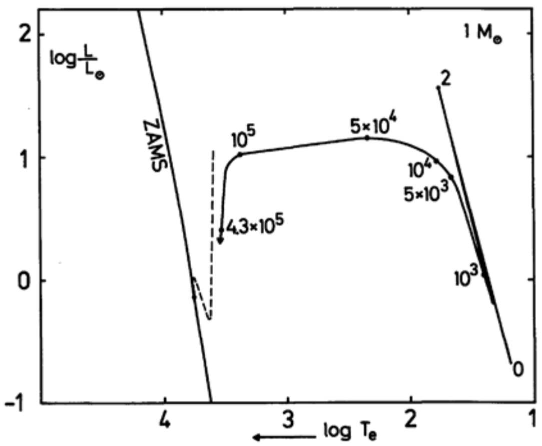


Fig. 1. Evolutionary path of a $1\,M_\odot$ protostar in an infrared HR diagram (solid line). The numbers indicate the time (in years) since the formation of the (final) hydrostatic core. For comparison, the evolutionary path of a conventional fully hydrostatic $1\,M_\odot$ pre-main sequence star is also included (broken line)

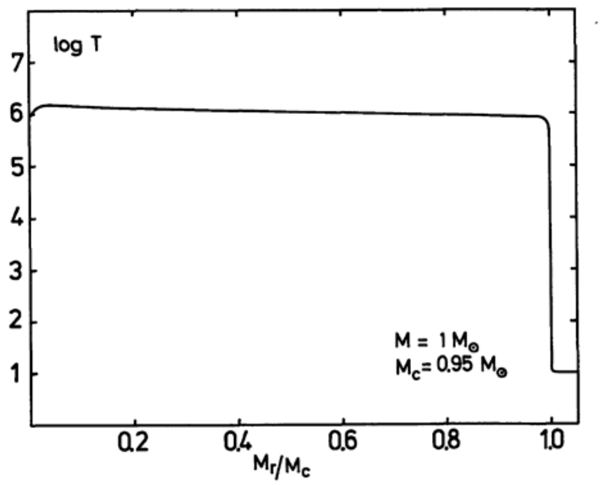


Fig. 2. Temperature distribution in the hydrostatic core of a $1\,M_\odot$ protostellar model after 95% of the total mass has accumulated in the core

The Evolution of a Massive Protostar

I. Appenzeller and W. Tscharnuter
Universitäts-Sternwarte Göttingen

Astron. & Astrophys. 30, 423 – 430 (1974)

Summary. The hydrodynamic evolution of a massive protostar has been calculated starting from a homogeneous gas and dust cloud of $60\,M_\odot$ and an initial density of $10^{-19}\,\mathrm{g\,cm^{-3}}$. Initially the collapsing gas cloud evolved similar to protostar models of lower mass. About 3.6×10^5 years after the beginning of the collapse a small hydrostatic core was formed. About 2×10^4 years later hydrogen burning started in the center of the hydrostatic core. After another 2.5×10^4 years the collapse of the envelope was stopped and reversed by the heat flow from the interior and the entire envelope was blown off, leaving behind an almost normal main-sequence star of about $17\,M_\odot$. During most of the core's evolution the central region of the protostar would have looked like a cool but luminous infrared point source to an outside observer.

Read this paper!

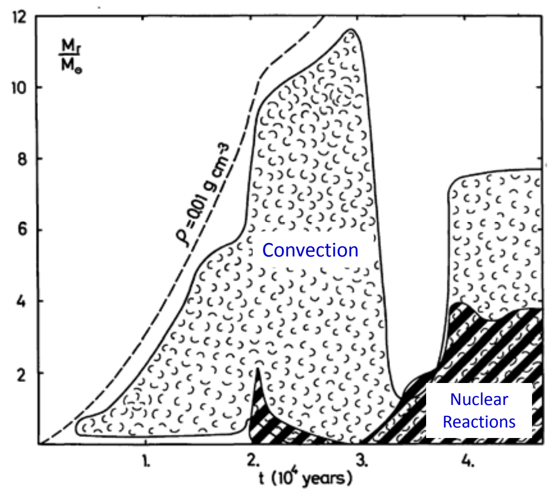


Fig. 5. The variation of the internal structure of the evolving hydrostatic core. The abscissa gives the time since the formation of the (final) hydrostatic core. ($t = 0$ corresponds to an age of the protostar of 361473 years.) "Cloudy" regions represent convection. Cross-hatched regions represent nuclear energy generation at a rate exceeding 10^3 erg g^{-1} s^{-1}. The approximate extent of the hydrostatic core is indicated by the line $\varrho = 0.01$ g cm^{-1}

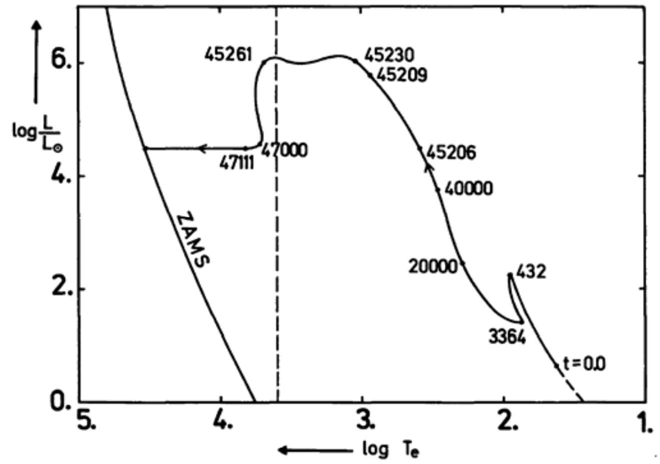
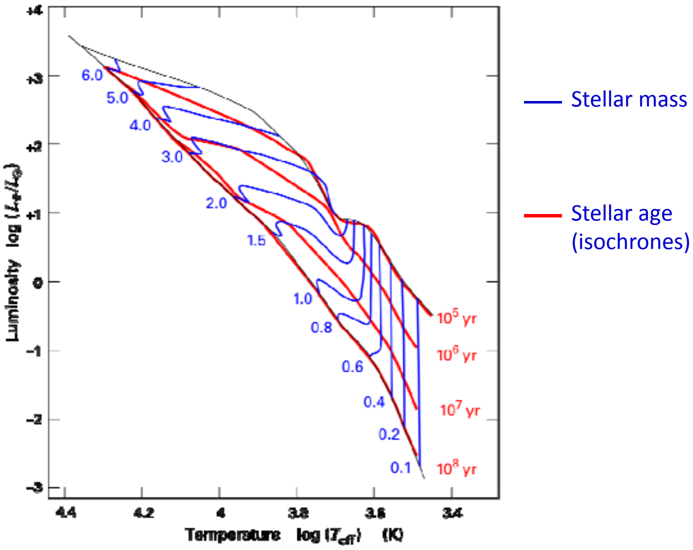


Fig. 6. Approximate evolutionary path of the optically thick central region of the 60 $M_\odot$ protostar in an infrared HR diagram. The numbers indicate the time t since the formation of the final hydrostatic core (c.f. Fig. 5). For comparison we also included the position of the zero-age main-sequence (ZAMS). The broken line gives the approximate lower limit of the effective temperature of hydrostatic configurations (Hayashi *et al.*, 1962)

Pre-Main Sequence Evolutionary Tracks



http://en.wikipedia.org/wiki/File:PMS_evolution_tracks.svg

Theoretical evolutionary tracks

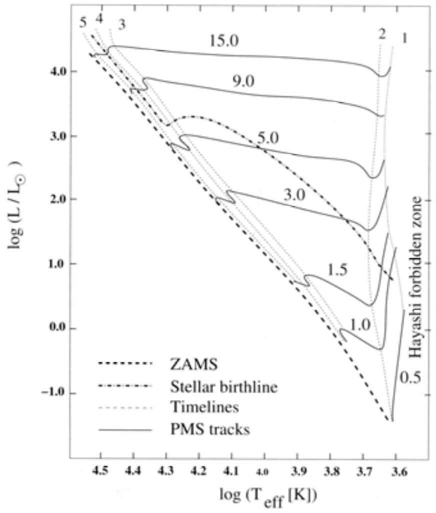


Fig. 6.6. Evolutionary paths in the HR-diagram for stellar masses ranging from 0.5 to 15 $M_\odot$ (solid tracks, adapted from Iben [420]). These paths are marked by thin hatched lines marking time periods labeled 1 to 5. The thick hatched line to the left approximately indicates the location of the ZAMS. The line across the tracks is the stellar birthline approximated from [76] for an accretion rate of $\dot{M}_{acc} = 10^{-5} M_\odot\ yr^{-1}$.

1965ApJ...141..993

STELLAR EVOLUTION. I. THE APPROACH TO THE MAIN SEQUENCE*

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ABSTRACT

The manner in which nuclear reactions replace gravitational contraction as the major source of stellar luminosity is investigated for model stars of population I composition in the mass range $0.5 < M/M_{\odot} <$ 15 0. By following in detail the depletion of C^{12} from high initial values down to values corresponding to equilibrium with N^{14} in the C-N cycle, the approach to the main sequence in the Hertzsprung-Russell diagram and the time to reach the main sequence, for stars with $M \geq 1.25\ M_{\odot}$, are found to differ significantly from data reported previously.

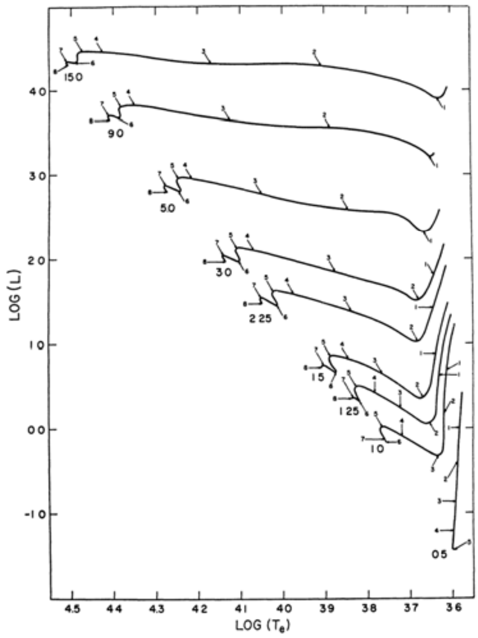

Fig. 17.—Paths in the Hertzsprung-Russell diagram for models of mass $(M/M_{\odot})$ = 0.5, 1.0, 1.25, 1.5, 2.25, 3 0, 5.0, 9.0, and 15.0. Units of luminosity and surface temperature are the same as those in Fig. 1

Effects of chemical abundances and "metals" in determination of stellar structure

Metal poorer → hotter

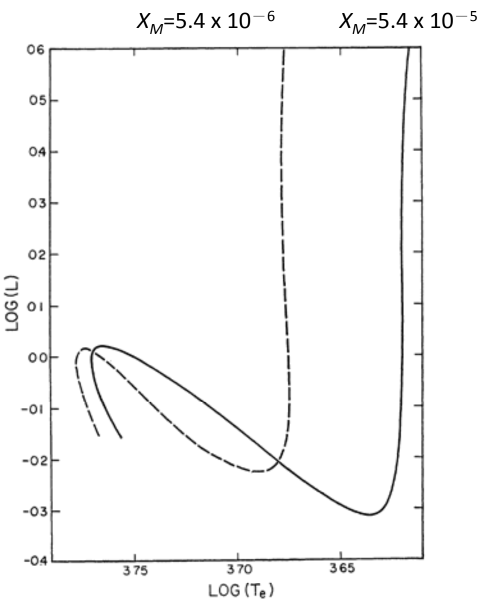

Fig. 1.—Paths in the theoretical Hertzsprung-Russell diagram for $M = M_{\odot}$. Luminosity in units of $L_{\odot} = 3.86 \times 10^{33}$ erg/sec and surface temperature T_e in units of °K. Solid curve constructed using a mass fraction of metals with 7.5-eV ionization potential, $X_M = 5.4 \times 10^{-5}$. Dashed curve constructed with $X_M = 5.4 \times 10^{-6}$.

Exercise

A useful site to download theoretical evolutionary tracks (the "Padova tracks") is the CMD/PARSEC isochrones
http://stev.oapd.inaf.it/cgi-bin/cmd

As homework

1. Plot V versus $(B-V)$ for an ensemble of stars (i.e., a star cluster) of ages 1 Myrs, 10 Myr, 100 Myr, and 1 Gyr.
2. Compare the V versus $(B-V)$ CMDs of two 100 Myr old star clusters, one with Z=0.01 and the other with Z=0.0001 (extremely metal poor).