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On the ejection velocity of meteoroids from comets

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ABSTRACT

The ejection of meteoroids from comets has been discussed by many authors and is a problem that is important both for a full understanding of cometary processes and for the evolution of meteoroid streams. We reinvestigate the problem here, starting from simple physical principles, and compare the results that we obtain with those of other authors, in particular Whipple.

Key words: comets: general – meteors, meteoroids.

1 INTRODUCTION

The most prominent feature of any comet is its dust tail, which increases as the comet approaches the Sun. It is therefore obvious that cometary nuclei lose dust grains to form this tail as the comet approaches perihelion. The basic reason for this was explained in the *icy conglomerate model* proposed by Whipple (1950), in which the primary component of the nucleus is assumed to be ice but with dust particles embedded within it. (Within this context, dust is taken to mean any chemical element or compound that remains in the solid state at temperatures of a few hundred kelvins. In reality it is likely to be dominated by those elements that condensed out during the formation phase.) As the ice sublimes due to the increasing effect of solar heating as the comet approaches perihelion, so these grains are released. Their subsequent motion is governed by interaction with the gas outflow, radiation pressure and, of course, gravity. For the smaller grains, radiation pressure dominates, and these move outwards to form the well-known dust tail. This is not the case for larger grains, and these move on orbits that are similar to that of the parent comet, producing a meteoroid stream, which in turn produces meteor showers when the grains hit the Earth's atmosphere and ablate. The exact nature of a meteoroid orbit depends on its energy and angular momentum. Both of these quantities are determined by the initial velocity of the meteoroid, or, in other words, its velocity relative to the cometary nucleus when it is free to move on an independent orbit. The process of forming a meteoroid stream was first investigated by Whipple (1951), who produced a formula for the ejection speed that has been very widely used in recent investigations of meteoroid stream evolution.

Understanding this mechanism is important for the scientific study of the subject, but it may also have wider implications. The increase in activity of the Perseid meteor shower in the first half of the 1990s, which manifested itself as a second peak in the activity curve, and the recent remarkable outburst of the Leonid meteor shower, have made it clear that space platforms could be at risk during times of enhanced meteor shower activity, although such activity seems harmless to our life on Earth. For example, cosmonauts in the *Mir-1 Space Station* reported audible meteoroid strikes, and the *Mir-1* solar panels were damaged on the night of the Perseid meteor shower maximum (Lenorovitz 1993). It is also believed that the *Olympus* communications satellite operated by the European Space Agency (ESA) lost pointing control as a result of the impact of a Perseid meteoroid with its solar array (Caswell, McBride & Taylor 1995). In 1993, NASA postponed its space shuttle launch in order to avoid the possible Perseid meteor storm, and changed the pointing direction of the *Hubble Space Telescope* for fear of lens damage by high-speed particles. So far, there have been no reports of damage to spacecraft by the Leonids, but safeguards have been taken during the Leonids activity in November every year since 1998. For example, the Chinese space navigation ministry not only took safeguards on the orbiting space-flights, but also modified the launch time for the space shuttle *Heavenly Boat-1* in 1999 November.

There have been many investigations into the evolution of meteoroid streams by numerically integrating the equations of motion of test particles (for example, Hughes, Williams & Murray 1979; Fox, Williams & Hughes 1982; Jones 1985; Williams & Wu 1994). Recently, many investigations have been carried out in order to predict the activity level in the Leonids in the period surrounding the return of the parent comet to perihelion, such as Kondrat'eva & Reznikov (1985), Wu & Williams (1996), Kondrat'eva, Murav'eva & Reznikov (1997), McNaught & Asher (1999), Brown (2000) and Göckel & Jehn (2000). For all of these investigations, the initial position and velocity of the test meteoroids are crucial, and these are defined by the orbital elements of the comet and the ejection velocity of the meteoroid relative to the comet. In particular, the stream profile is dictated primarily by the ejection process, at least for young meteoroid streams. Hence it is important to understand this ejection process and to be able to produce realistic values for the ejection velocity.

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The basic notion of an *icy conglomerate* cometary nucleus as originally proposed by Whipple (1950) is now generally accepted. As this nucleus approaches the Sun, as was explained by Whipple (1951), the absorbed solar radiation heats it up until a temperature is reached that enables the ice to sublime. The resulting gas flows away from the nucleus, and gas drag exerts an outward force on any released small grains, causing them also to flow away from the nucleus. Assuming that the grains are spherical and that the nucleus is also spherical and attains a uniform temperature across its whole surface, Whipple (1951) obtained the following expression for the grain velocity v (m s^{-1}) relative to the nucleus of radius R_c (km) and density ρ_c at a heliocentric distance r (astronomical units, au) as

$$v^2 = 43R_c\left(\frac{1}{ns\sigma r^{2.25}} - \frac{8\pi}{3}\frac{G}{43}\rho_c R_c\right), \qquad (1)$$

where s and σ are the radius and bulk density of a spherical grain (cgs units) and $1/n$ is the fraction of solar radiation available for sublimation, generally taken as unity.

If we consider a meteoroid of radius 1 mm and bulk density 0.8 g cm^{-3} (typical for a visible meteoroid), ejected at 1 au from a comet nucleus of radius 5 km, then Whipple's formula gives an ejection speed of about 52 m s^{-1}. Of course, there are variations from comet to comet and from stream to stream in the numerical values inserted above, but it is difficult to change the ejection velocity so that it is outside the range of about 20–100 m s^{-1}.

However, Harris, Yau & Hughes (1995) and Wu & Williams (1996) considered the spread found in the orbital elements of the observed Perseid meteor stream, and concluded that an ejection speed in the range 600–1000 m s^{-1} was required. Similarly Williams (1996) considered the observations of six other streams (Geminids, Quadrantids, Perseids, Taurids, Eta Aquarids and Orionids), concluding that, for both the Quadrantids and the Perseids, significantly higher velocities than that given above were indicated.

A number of people modified the assumptions in Whipple's formula in order to increase the ejection velocity. For example, Gustafson (1989) considered grains that were flakes rather than spherical, while Jones (1995) dispensed with the assumption that the whole surface was active. Other models were produced by Hughes (1977), Brown & Jones (1998) and Göckel & Jehn (2000). Hughes (2000) also considered the possibility that grains could contain small amounts of ice after leaving the nucleus and that the sublimation of this could act like a small rocket, accelerating the grains.

The intention of this paper is to start again and to visit the whole problem of grain loss from the nucleus, maintaining the assumption of spherical grains.

2 THE BASIC MODEL

The basic model is not controversial. We assume that a predominantly icy cometary nucleus is moving on an elliptical orbit about the Sun. As it approaches perihelion, the increasing energy flux received through solar radiation heats up this nucleus, or at least those parts of it that are being directly heated. When the relevant part reaches a known critical temperature, the ice sublimes, forming a gas. This gas will flow away from the surface, at least in the sense that the gas will occupy an ever-expanding volume of space. There will be interactions between this gas and small dust grains that were originally embedded in the ice, which results in a force on these grains in the direction of the outflowing gas. This interaction force can overcome the gravitational attraction of the nucleus and so cause the grains to be accelerated away from the nucleus. We wish to determine the final velocity relative to the nucleus achieved by these grains.

This is essentially the situation investigated by Whipple (1951) and other authors since then. It is very easy to describe the situation, as we have done above. It is harder to produce a quantitative physical model, which is why models are still being proposed half a century after the first model by Whipple.

One obvious apparent difficulty is in defining what we mean by final velocity. We could take it to mean the velocity where the gravitational field of the Sun becomes equal to that of the cometary nucleus. A simple calculation shows that, for a comet of mass 10^{14} kg, when at 1 au from the Sun, this point of balance is slightly less than 1 km away from the cometary surface. Clearly, from observations of comets, dust and gas are found moving essentially away from the nucleus at very much greater distances. A better answer might be the velocity of the grain when it ceases to be accelerated by the gas. The strict answer to that is probably never. In reality, the acceleration of the grains due to the gas decreases very rapidly through two effects: the drop in density of the gas, and the decrease in relative speed between the gas and grain. Hence we can take the final velocity to be the grain velocity relative to the nucleus at any sufficiently large distance from the nucleus.

A much more serious problem is the outflow speed of the gas – indeed, what we mean by such terminology. It has to be realized that the grains that are of interest to us are much smaller than the mean free path on the gas. Consequently we cannot use a *fluid* model for the gas – we need to use an approach based on the kinetic theory of gases. (A fluid picture may be perfectly adequate for describing many of the phenomena associated with cometary gas tails, but for individual grain interactions, it is not adequate. The grain gets hit by individual gas molecules at irregular time intervals.)

Let us then picture the process of gas production. As the temperature of the ice molecules increases, a critical value is reached where the vibrating molecules can escape. This takes place at a well-determined temperature and requires a well-known amount of energy (the latent heat of vaporization) to be available. The gas molecule is now free and can be imagined to move away from the solid surface with some well-determined (from the energy level) speed c in some random direction within a hemisphere with its axis of symmetry normal to the surface. The mean outward velocity of the collection of released gas molecules at this instant is the mean value over the hemisphere of $c\cos\theta$, that is $c/2$. We call this initial state 'A', and imagine it as a coherent stream moving out with a mean speed of $c/2$. Of course, other adjacent molecules are behaving in a similar fashion, and so molecules will collide, and in time the gas will have become thermalized, that is the molecules obey a Maxwellian velocity distribution law relative to some moving frame. The velocity of this moving frame can be regarded as the bulk velocity, V say, of the gas, and the molecules have some random velocity with a mean equal to the mean thermal velocity, W say, in a frame moving with the bulk velocity. In this final equilibrium stage, the *leading* molecules can be pictured as moving with a speed c still, while the average tangential speed is $c/2$. Hence we can picture the final stage as having a bulk motion outwards of value $c/2$ and a Maxwellian component with a mean thermal velocity of $c/2$. We call this final state 'B'.

In reality, there is also the intermediate, non-equilibrium state, after the molecules have lost their predominantly coherent motion as represented in state A, but before equilibrium has been attained as in state B. This state cannot be modelled; and we will assume that its effect, as far as accelerating any grains is concerned, lies somewhere in between the effects attained in the two end states, A and B. We shall assume that $W = c/2$ is given by the mean thermal velocity at the ambient gas temperature.

2.1 Gas–grain interaction

We shall assume throughout this discussion that the grains are spherical, of radius s and bulk density σ. Hence if a dust grain has a mass m_d, then

$$m_\mathrm{d} = \tfrac{3}{4}\pi s^3 \sigma.$$

Non-spherical grains produce a much more complex situation because the effective surface area to mass ratio is variable depending on the grain orientation relative to its direction of motion. We do not discuss this complication in this paper.

Whenever the velocity of a dust grain is different from the bulk velocity of a gas, there is a transfer of momentum, or a *drag* force, between the two, which attempts to equalize the two velocities. In the situations that are of interest to us, the effect will be to increase the velocity of the grains away from the cometary nucleus, since initially the gas has a velocity W as described above and the grain is essentially stationary. If the grain were larger than the mean free path of the gas, then the drag force would be given by the well-known Stokes' law (e.g. Basset 1888)

$$\mathcal{R} = 6\pi\eta s(v_\mathrm{g} - v_\mathrm{d}),$$

where η is called the coefficient of viscosity, and v_d and v_g are respectively the velocity of the dust grain and gas relative to the cometary nucleus, so that $v_\mathrm{g} - v_\mathrm{d}$ is the relative velocity. For the situation we are discussing, the grain is smaller than the mean free path of the gas and, provided the gas is thermalized, with a mean thermal velocity W, the situation has also been discussed in the literature. If the relative velocity, $v_\mathrm{g} - v_\mathrm{d}$ is small, an expression was derived by Cunningham (1910), while Epstein (1924) slightly extended the results by considering both specular and diffuse reflection of gas molecules off the grains. (In specular reflection, the molecules are perfectly reflected; while in diffuse reflection, the molecules are assumed to be temporarily absorbed by the grain and re-emitted with the velocity corresponding to the grain temperature in random directions.) The situation was further investigated by Baines, Williams & Asebiomo (1965) where the need for the relative velocity to be small was relaxed. They gave the corresponding resistances for specular and diffuse reflections, $\mathcal{R}_\mathrm{s}$ and $\mathcal{R}_\mathrm{d}$, as

$$\mathcal{R}_\mathrm{s} = \tfrac{4}{3}\pi s^2 \rho W(v_\mathrm{g} - v_\mathrm{d})$$

and

$$\mathcal{R}_\mathrm{d} = \tfrac{4}{3}\pi s^2 \rho W(v_\mathrm{g} - v_\mathrm{d})\left(1 + \tfrac{1}{8}\pi\right),$$

where ρ is the density of the gas.

Hence, the final state of the gas flow in the situation that we have described in relation to the comet nucleus is covered by these works. For the initial situation where the gas molecules are regarded as moving on parallel paths as a radial flow, the drag can be simply calculated and is given by multiplying the momentum change per collision by the number of collisions per unit time, that is

$$\mathcal{R} = \pi s^2 \rho (v_\mathrm{g} - v_\mathrm{d})^2$$

if specular reflection is assumed, or

$$\mathcal{R} = \tfrac{4}{3}\pi s^2 \rho (v_\mathrm{g} - v_\mathrm{d})^2$$

on assuming diffuse reflection.

2.2 Equations of motion for the dust grains

In the above section, we have discussed the transfer of momentum to the dust grains. This momentum has come from the gas, and so we must consider the equations of motion for both the dust particle and the gas under the influence of the gravitational field of the nucleus and the momentum exchange. To recapitulate on the situation, we assume that the gas initially leaves the nucleus with all the molecules moving outwards with a mean speed W and that eventually this flow becomes thermalized with a mean thermal velocity W and a bulk outward speed v_g. In the absence of the interaction with the dust grains, v_g would be equal to W, but because of the interaction, v_g is decreasing. Hence, W is a constant, but v_g is variable.

For the first stage, the grain collisions are part of the thermalizing process and so it seems best to assume that the flow is radially outwards with a mean speed W. Hence we only need to consider the equation of motion for a dust grain and this is

$$m_\mathrm{d}\frac{\mathrm{d}v_\mathrm{d}}{\mathrm{d}t} = \pi s^2 \rho (W - v_\mathrm{d})^2 - m_\mathrm{d} G\frac{M_\mathrm{c}}{R^2}. \tag{2}$$

Here, G is the universal gravitational constant, M_c is the mass of the cometary nucleus, and R is the distance from the centre of the comet (throughout we shall use R for distance relative to the comet and r for the corresponding heliocentric distance).

We see that dust particles can only begin to accelerate off the cometary surface if $\mathrm{d}v_\mathrm{d}/\mathrm{d}t > 0$, that is if

$$\pi s^2 \rho_0 W^2 > m_\mathrm{d}\frac{GM_\mathrm{c}}{R_\mathrm{c}^2}, \tag{3}$$

where ρ_0 is the initial gas density at the surface of the comet and R_c is the radius of the comet.

From the equation of continuity for the gas we have

$$\frac{\mathrm{d}M_\mathrm{c}}{\mathrm{d}t} = 4\pi\alpha\rho_0 W R_\mathrm{c}^2 = 4\pi\alpha\rho v_\mathrm{g} R^2, \tag{4}$$

where α is the fraction of the surface area that is active. Because sublimation mainly occurs on the Sun-facing hemisphere, α_max will have a value close to 0.5. Hence only dust grains smaller than s_0 can get off the cometary surface, where

$$s_0 = \frac{3\rho_0 W^2 R_\mathrm{c}^2}{4\sigma GM_\mathrm{c}} = \frac{3\dot{M}_\mathrm{c} W}{16\pi\alpha\sigma GM_\mathrm{c}}. \tag{5}$$

Typical production rates of water in comets are of the order of 10^{28} molecules per second and so $\dot{M}_\mathrm{c}/M_\mathrm{c}$ is of the general order of $3 \times 10^{-10}\ \mathrm{s}^{-1}$. Thus we find that $s_0 \sim 50$ cm, a result that is fairly insensitive to the details of the model.

For the final stage, where the gas is thermalized, the equations of motion become

$$m_\mathrm{d}\frac{\mathrm{d}v_\mathrm{d}}{\mathrm{d}t} = \frac{4}{3}\pi s^2 \rho W(v_\mathrm{g} - v_\mathrm{d}) - m_\mathrm{d} G\frac{M_\mathrm{c}}{R^2}, \tag{6}$$

$$\rho\frac{\mathrm{d}v_\mathrm{g}}{\mathrm{d}t} = -n_\mathrm{d}\frac{4}{3}\pi s^2 \rho W(v_\mathrm{g} - v_\mathrm{d}) - \rho G\frac{M_\mathrm{c}}{R^2}. \tag{7}$$

The dust density in space ρ_d is given by $\rho_\mathrm{d} = n_\mathrm{d} m_\mathrm{d}$. Let $\rho_\mathrm{d}/\rho = \mu$, then $n_\mathrm{d} = \mu\rho/m_\mathrm{d}$.

Also, we are interested in particles that will produce visible meteors, and so are under about 1 cm in radius. This is a factor of 20 down on the size where gravity is similar to gas drag, and so for these grains the gravitational term can be ignored in the above equations of motion. These equations now become, for the initial non-thermalized flow

$$m_\mathrm{d}\frac{\mathrm{d}v_\mathrm{d}}{\mathrm{d}t} = \pi s^2 \rho (W - v_\mathrm{d})^2, \tag{8}$$

and for the thermalized flow

$$m_\mathrm{d}\frac{\mathrm{d}v_\mathrm{d}}{\mathrm{d}t} = \frac{4}{3}\pi s^2 \rho W (v_\mathrm{g} - v_\mathrm{d}), \tag{9}$$

$$m_\mathrm{d}\frac{\mathrm{d}v_\mathrm{g}}{\mathrm{d}t} = -\frac{4}{3}\pi s^2 \mu \rho W (v_\mathrm{g} - v_\mathrm{d}). \tag{10}$$

2.3 Solutions of the equations of motion

Using $\mathrm{d}v_\mathrm{d}/\mathrm{d}t = v_\mathrm{d}\,\mathrm{d}v_\mathrm{d}/\mathrm{d}R$ and the equation of continuity as $\rho_0 {R_\mathrm{c}}^2 = \rho R^2$, the first of these equations integrates simply to give the velocity v at distance R from the nucleus centre as

$$\ln\left(\frac{W - v}{W}\right) + \frac{W}{W - v} - 1 = \frac{\pi s^2 \rho_0}{m_\mathrm{d}}\frac{R_\mathrm{c}(R - R_\mathrm{c})}{R}. \tag{11}$$

If it is assumed that the grain velocity is significantly smaller than W, then we obtain

$$v^2 = W^2\frac{2\pi s^2 \rho_0 R_\mathrm{c}(R - R_\mathrm{c})}{R m_\mathrm{d}} = W^2\frac{3\rho_0 R_\mathrm{c}(R - R_\mathrm{c})}{2 s\sigma R}, \tag{12}$$

on assuming a spherical particle and substituting for m_d. This is the same expression as is obtained if $(W - v_\mathrm{d})$ is replaced by W in the initial equation of motion and then integrating with the same boundary conditions.

Turning now to consider the thermalized flow, by addition of equations (9) and (10), we obtain

$$\mu\frac{\mathrm{d}v_\mathrm{d}}{\mathrm{d}t} + \frac{\mathrm{d}v_\mathrm{g}}{\mathrm{d}t} = 0, \tag{13}$$

which integrates, with the initial conditions $v_\mathrm{g} = W$, $v_\mathrm{d} = 0$, to give

$$\mu v_\mathrm{d} + v_\mathrm{g} = W. \tag{14}$$

Also, we note the obvious, namely that $\mathrm{d}v_\mathrm{d}/\mathrm{d}t = 0$ when $v_\mathrm{g} = v_\mathrm{d}$. Hence, we have the following inequality

$$0 \leqslant v_\mathrm{d} \leqslant \frac{W}{1+\mu} \leqslant v_\mathrm{g} \leqslant W. \tag{15}$$

Using $\mathrm{d}v_\mathrm{d}/\mathrm{d}t = v_\mathrm{d}\,\mathrm{d}v_\mathrm{d}/\mathrm{d}R$ and $v_\mathrm{g} = W - \mu v_\mathrm{d}$, equation (9) becomes

$$\int_0^v v_\mathrm{d}\left[1 + \frac{v_\mathrm{d}}{W - (1+\mu)v_\mathrm{d}}\right]\mathrm{d}v_\mathrm{d} = \int_{R_\mathrm{c}}^R \frac{W^2 \rho_0 R_\mathrm{c}^2}{s\sigma}\frac{\mathrm{d}R}{R^2}. \tag{16}$$

All the integrals can be evaluated using standard methods to give

$$\frac{\mu v^2}{2(1+\mu)} - \frac{W v}{(1+\mu)^2} - \frac{(W)^2}{(1+\mu)^3}\ln\left[1 - \frac{(1+\mu)v}{W}\right] = \frac{W^2 \rho_0 R_\mathrm{c}(R - R_\mathrm{c})}{s\sigma R}, \tag{17}$$

using the same boundary conditions as used when integrating equation (8). The function $\ln[1 - (1+\mu)v/W]$ in equation (17) can be expanded because of the inequality (15). Keeping the first two terms of the expansion, we obtain from equation (17)

$$v^2 = \frac{2W^2 \rho_0 R_\mathrm{c}(R - R_\mathrm{c})}{R s\sigma}. \tag{18}$$

In principle, since equation (12) gives the solution for the initial state before the gas becomes thermalized and equation (18) gives the solution for the final stages with a thermalized flow, the end boundary conditions used in equation (12) should become the initial conditions for equation (18). However, as can be seen, the two equations are essentially identical, differing only by a small amount in the numerical coefficient. Since the thermalized flow regime is likely to last longer than the initial stage, we take the ejection speed, that is the speed when R is large, to be given by

$$v^2 = \frac{2W^2 \rho_0 R_\mathrm{c}}{s\sigma}. \tag{19}$$

This gives a very robust expression for the ejection speed of meteoroids with only one parameter in this expression that is not well determined, namely ρ_0, the initial gas density. The equation of continuity, equation (4), allows us to express ρ_0 in terms of the gas outflow rate, $\dot{M}_\mathrm{c}$, that is

$$\frac{\mathrm{d}M_\mathrm{c}}{\mathrm{d}t} = 4\pi\alpha\rho_0 W R_\mathrm{c}^2. \tag{20}$$

Hence, the ejection speed is given by

$$v^2 = \frac{\dot{M}_\mathrm{c} W}{2\pi\alpha s\sigma R_\mathrm{c}} \tag{21}$$

For a well-observed comet, the mass outflow rate may be well determined, as may α, as discussed for example by Lowry et al. (1999), and so we may be able to use the expression in this form. However, in general for the parent comets of meteoroid streams, $\dot{M}_\mathrm{c}$ is hardly better determined than ρ_0, especially at the epoch when ejection actually took place. Thus, in order to produce a useful formula, we need to consider the physics of the ejection process further.

2.4 The mass-loss rate from comets

As already mentioned, the basic physics is simple: Solar radiation is absorbed by the nucleus, which results in the nucleus heating up as the comet approaches the Sun. When a critical temperature is reached, the ice in the nucleus sublimes to produce the gas outflow that we have described. Changing the state of ice requires input of energy, the latent heat of vaporization, H say, which is much greater than the amount of energy required to change the temperature of the nucleus by a small amount. Hence, a reasonable model is to assume that, once the sublimation temperature is reached, the nucleus remains at this temperature and the excess input energy from the Sun is used to vaporize the ice so that more gas is produced as the comet approaches the Sun. Hence the mass flow rate is governed by the radiation energy input rate. The energy flux falling on the cometary nucleus due to Solar radiation is $R_\mathrm{c}^2 L_\odot / 4r^2$, where $L_\odot$ is the solar luminosity and r is the heliocentric distance of the comet. Not all of this energy can be used for sublimation since the nucleus is also radiating, so that some of the input radiation is lost through this. However, comets appear to have very low albedos of the order of 4 per cent (Fernandez, Jewitt & Sheppard 2001). This leaves 96 per cent of incident energy available for sublimation and re-radiation in the infrared. Hence, we ignore the reflected part and assume that

$$\frac{R_\mathrm{c}^2 L_\odot}{4r^2} = H\dot{M}_\mathrm{c} + 4\pi\sigma R_\mathrm{c}^2 T_0^4, \tag{22}$$

where here and in the next equation σ is the Stefan–Boltzmann radiation constant and T_0 is the temperature at which sublimation begins. (In general, we have used σ for the grain density, but it is such a standard notation for the Stefan–Boltzmann constant that we feel that using it in these two equations will not cause confusion.) But

$$4\pi\sigma R_\mathrm{c}^2 T_0^4 = \frac{R_\mathrm{c}^2 L_\odot}{4r_\mathrm{s}^2}, \tag{23}$$

where r_s is the heliocentric distance at which sublimation begins. Thus, the mass-loss rate becomes

$$\dot{M}_c = \frac{R_c^2 L_\odot}{4H}\left(\frac{1}{r^2} - \frac{1}{r_s^2}\right). \tag{24}$$

Thus, on substituting for $\dot{M}_c$ into equation (21), the expression for the ejection speed becomes

$$v^2 = \frac{W R_c L_\odot}{8\pi H \alpha s \sigma}\left(\frac{1}{r^2} - \frac{1}{r_s^2}\right), \tag{25}$$

an expression that is also fairly robust.

2.5 Effect of the gravitational field of the nucleus

We can slightly modify this equation in two ways. First, the expression obtained assumed specular reflection of gas molecules off the dust grains. For diffuse reflection, the drag is increased by a factor $(1 + \frac{1}{8}\pi)$ and the same factor should be applied to v^2. If we assume that the initial flow model is dominant, then the diffuse reflection gives the same expression for v as above; while for the specular reflection case, the value is reduced by $3/4$. Hence all the cases are covered if we assume that the above expression can be multiplied by a small factor in the range 0.75–1.4.

Secondly, we note that the effect of the gravitational field of the comet can be included. What we have calculated is in effect twice the kinetic energy given to the dust grain by gas interactions. Hence we can simply subtract twice the potential energy change to obtain the correct expression as

$$v^2 = \frac{W R_c L_\odot}{8\pi H \alpha s \sigma}\left(\frac{1}{r^2} - \frac{1}{r_s^2}\right) - \frac{2GM_c}{R_c} \tag{26}$$

3 DISCUSSION

In the above equation, $L_\odot$ and H have well-established values (approximately 4×10^{26} J s^{-1} and 2×10^6 J kg^{-1} respectively). It is also convenient, both for general use and for comparison with other authors, to express heliocentric distances in astronomical units, cometary radii in kilometres, and grain radii in centimetres, with σ also in cgs units. Inserting these into equation (26) gives

$$v^2 = \frac{3.5 \times 10^{-2}\, W R_c}{\alpha s \sigma}\left(\frac{1}{r^2} - \frac{1}{r_s^2}\right) - 0.56 R_c^2 \rho_c, \tag{27}$$

where ρ_c is also in cgs units.

W is also a determinable quantity. It is assumed by us to be the mean thermal velocity of water vapour at the sublimation temperature. For water molecules at 273 K, this gives $W = 580$ m s^{-1}, so that the above equation becomes

$$v^2 = 20 R_c\left[\frac{1}{\alpha s \sigma}\left(\frac{1}{r^2} - \frac{1}{r_s^2}\right) - 0.028 R_c \rho_c\right]. \tag{28}$$

We also need to discuss r_s, the heliocentric distance at which sublimation starts. If the whole of the nucleus is in thermal equilibrium with solar radiation, then this is at about 1.25 au. However, if only a fraction of the surface is active, with the remainder not radiating, sublimation can start at larger distances. Also ices other than water will sublime earlier, and the analysis above will hold, essentially unaltered, for CO_2 sublimation say. Comets are observed to become active at around 3 au, and we shall adopt this figure.

The fraction of the cometary surface that is active, α, was discussed in Lowry et al. (1999). There is a wide range from about 10 to 40 per cent, with a mean of 14 per cent. However, the range is such that it may not be sensible to take any single value other than for illustrative purposes, where we shall take 0.15, the rounded value for the above mean.

It is instructive to compare our expression with that of Whipple (1951), given as equation (1). The main difference at first sight is in terms of the dependence on heliocentric distance. However, since the majority of the activity is at around 1 au, in reality this makes no difference to the value of the ejection velocity. In terms of modelling meteoroid streams, however, it may be important, because the ejection velocity drops off far more sharply in our expression as the heliocentric distance approaches the distance where sublimation starts.

In terms of actual values, we concentrate on visible meteors so that radius $\sim$1 mm and bulk density 0.8 g cm^{-3} (Hughes 1977), so $s\sigma = 0.08$ g cm^{-2}. Cometary radii and densities are not very well determined. However, for the parent of the Perseid meteoroid stream, comet 109P/Swift-Tuttle, O'Ceallaigh, Fitzsimmons & Williams (1995) obtained a value of 11.8 km. This is also not dissimilar to the radius of comet 1P/Halley, the parent of the η Aquarid and Orionid streams. Hence, we consider such a parent and take, in round numbers, R_c as 10 km. The second term [the gravitational term in equation (28)] is relatively unimportant, but we take ρ_c as 0.15, the value given by Rickman et al. (1987). For illustrative purposes, we shall calculate the ejection velocity at 1 and 2.5 au.

Inserting these values, our expression gives an ejection speed of 120 m s^{-1} at 1 au and 28 m s^{-1} at 2.5 au. For the same values, Whipple's formula gives 73 m s^{-1} at 1 au and 26 m s^{-1} at 2.5 au, illustrating the faster decay in our model. From observations of the Perseids, Ma & Williams (2001) deduced an ejection velocity at perihelion of 120 m s^{-1}.

Both the above-mentioned comets are large, and a *typical* comet would be expected to have a much smaller radius, and a value of 1 km, as deduced for example by Hughes (1990), would appear to be much closer to the truth. Adopting this radius, our formula gives the ejection velocity at 1 au of 39 m s^{-1} while Whipple's formula gives 23 m s^{-1}. For the Leonid shower and its parent, comet 55P/Tempel-Tuttle, Ma & Williams (2001) deduced a perihelion ejection velocity of 70 m s^{-1}. Asher (1999), in his model that successfully predicted the Leonids storm in 1999, used a perihelion ejection speed of 25 m s^{-1}. Göckel & Jehn (2000) investigated various models and concluded that a speed of 40 m s^{-1} gave the best fit with observations.

Our formula gives an ejection speed that is about 33 per cent higher than Whipple's value at 1 au but is very similar to Whipple's at 2.5 au. The difference at 1 au is within the noise of the input data; selecting α to be 0.5 for example, as Whipple in effect did, virtually eliminates the difference, as would selecting the first (state A) rather than the second (state B) of our gas flow models.

The difference in the dependence on heliocentric distance is, however, more fundamental, and arises from the assumption that, once sublimation starts, the ice remains at the sublimation temperature because all the incident radiation is used for sublimation rather than heating the nucleus. Whipple assumed that the temperature was the local radiation temperature, thus increasing with decreasing distance.

This is the most interesting aspect of the work and has implications for models of meteor stream formation, where high meteoroid velocities when they leave the comet near perihelion can cause significant spreading in the orbital elements.

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