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Simplified formulae system for resonant inverse Compton scattering of a fast electron in an intense magnetic field

J. H. You,[1⋆] W. P. Chen,[2⋆] S. N. Zhang,[3] L. Chen,[1] D. B. Liu[1⋆] and C. K. Chou[2]

[1] *Institute for Space and Astrophysics, Department of Physics, Shanghai Jiao-Tong University, Shanghai 200030, China*
[2] *Institute of Astronomy and Department of Physics, National Central University, Chung-Li, 32054, Taiwan, China*
[3] *Centre for Astrophysics, Physics Department, Tsinghua University, Beijing, 100084, China*

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ABSTRACT

We present simple analytical formulae for the emission spectrum and total power of a special kind of resonant inverse Compton scattering (RICS) of a relativistic electron in an intense magnetic field. In contrast with the available formulae system, we obtain a markedly simplified one based on the semiclassical quantum theory, which is more understandable for people who are unfamiliar with quantum electrodynamics. We show that the RICS process, under an appropriate 'accommodation condition' derived in this paper, is predominantly much more efficient than the coexistent ordinary inverse Compton scattering, and produces highly beamed high-frequency radiation with moderately good monochromaticity. Our formulae are simple to use – thus offering a lucid physical intuition for the theory – and may find wide applications in hard X-ray and gamma-ray astrophysics.

Key words: radiation mechanisms: non-thermal – scattering – gamma-rays: theory.

1 INTRODUCTION

The magnetic inverse Compton scattering (ICS) of a relativistic electron in the very strong magnetic field (e.g. $B \sim 10^8$–10^{13} G) of a neutron star (NS) or a strange star is divided into two parts: resonant and non-resonant (Herold 1979; Xia, Qiao & Wu 1985; Daugherty & Harding 1986; Dermer 1989, 1990; Harding & Daugherty 1991; You, Chen & Deng 1997; Chen et al. 1997). The non-resonance part of scattering is basically the same as the ordinary field-free inverse Compton scattering because their cross-sections are approximately the same, $\sigma_{\rm s}^{\rm nres} \approx \sigma_{\rm T}$, when the incident frequency $\nu_{\rm i}'$ of scattered photons in the electron rest frame (ERF) is markedly higher than the main resonance frequency, i.e. the Landau frequency, $\nu_{\rm B} \equiv eB/2\pi mc$, $\nu_{\rm i}' > \nu_{\rm B}$. If $\nu_{\rm i}'$ is much lower than $\nu_{\rm B}$, $\nu_{\rm i}' \ll \nu_{\rm B}$, the scattering diminishes because $\sigma_{\rm s}^{\rm nres} \rightarrow 0$. In this paper, we do not distinguish the terminology 'non-resonant inverse Compton scattering' from 'inverse Compton scattering' and simply denote it as 'ICS'. The second part, which we call 'resonant inverse Compton scattering' (RICS), mainly occurring at $\nu_{\rm i}' \simeq \nu_{\rm B}$, is a very special kind of scattering owing to its remarkable properties. RICS arises from the cyclotron-resonant absorption and subsequent instantaneous re-emission of the incident photon by the relativistic electron. In this paper we show that the resonance nature of this process makes the RICS potentially a very important radiation mechanism in the high-frequency band. We derive an 'accommodation condition' under which the RICS process has predominantly a much higher radiation efficiency than that of the coexistent ordinary ICS. The RICS photons have very high frequencies, $\nu \sim \gamma\nu_{\rm B} = \gamma eB/2\pi m_{\rm e}c$. This means that ν would fall in the γ-ray range when $B \sim 10^{9-13}$ G and $\gamma \sim 10^{1-6}$, where γ is the Lorentz factor, $\gamma \equiv 1/\sqrt{1-\beta^2} = mc^2/m_0c^2$, representing the dimensionless energy of the fast electron. We show that the RICS emission has a moderately good monochromaticity (see equation 11 or Fig. 3 in Section 2), concentrating most of radiation energy in the high-frequency band near $\sim\gamma\nu_{\rm B}$. The high-frequency RICS photons are well collimated along the field line, within an extremely narrow angular cone of the order of $1/\gamma$ owing to the relativistic beaming effect. In an intense magnetic field with approximately parallel field lines, the beaming behaviour would effectively suppress the strong absorption of the γ-ray photons by the magnetic or γ–γ annihilations.

Both the advantages of the RICS mechanism mentioned above and the rapid progress over recent years in observations in X-ray and γ-ray astronomy prompt us to re-examine and improve the currently available formulation of the theory. In fact, a detailed theoretical analysis of the magnetic Compton scattering of soft photons by relativistic electrons beamed along the direction of a strong magnetic field has been examined by Dermer (1989, 1990). However, in his papers, both the resonant and the non-resonant scattering were taken into consideration together using the complicated quantum electrodynamics (QED) formula of the scattering cross-section, which is valid over the whole spectral region of scattered photons. Thus the resulting analytical formulae are complicated and inconvenient for astrophysical applications. Sometimes the

⋆E-mail: jhyou@online.sh.cn (JHY); wchen@joule.phy.ncu.edu.tw (WPC); liu_db77@sina.com (DBL)

physical meaning of the formulae is difficult to understand because of the adoption of the dimensionless photon energy $\varepsilon' \equiv h\nu'/m_0c^2$ and $\varepsilon_{\rm B} \equiv h\nu_{\rm B}/m_0c^2$. For example, the delta function approximation of the resonance cross-section equations (A3) and (A4) in his paper (Dermer 1990), which are simplified from equation (3) in Dermer (1989), gives $\sigma_{\rm res} \approx \sigma_{\rm eff}\delta(\varepsilon' - \varepsilon_{\rm B})$ and $\sigma_{\rm eff} \approx 323\sigma_{\rm T}$, which may give the impression that the resonance scattering cross-section is only 323 times higher than that of the ordinary inverse Compton scattering. In fact, our formula of $\sigma_{\rm res}(\nu') \sim \nu'$ shows that if $B \sim 10^{12}$ G, the peak of the resonance cross-section at the line-centre $\nu_{\rm B}$ is $\sigma_{\rm res}(\nu_{\rm B}) \approx 2 \times 10^7\sigma_{\rm T}$ (see equation 1 or Fig. 2 below). Recently, we wrote a short letter to report part of our results (You et al. 2001). In the present paper, we give a deep physical discussion and detailed derivation to help gain an understanding of the meaning of the relevant RICS formulae.

In this paper, we give a treatment that differs from the previous ones (e.g. Herold 1979; Dermer 1990) in that we deal with the coexistent RICS and ICS processes separately, based on the fact that RICS operates only at the harmonic frequencies $\nu' = \nu_{\rm B}, 2\nu_{\rm B}, \ldots,$ particularly at the base frequency $\nu' \simeq \nu_{\rm B}$; while the ICS responds to the continuous spectrum of incident photons. Furthermore, in our theoretical derivations, we use a simple semiclassical cross-section of scattering equation (1) which is exactly the same as the QED formula in the vicinity of the resonance frequency $\nu_{\rm B}$, as shown in the Appendix and Fig. 2 below. This leads to a great simplification of the formulation for the radiation mechanism. In Section 2 we first deal with the scattering near the resonance frequencies, particularly at the base frequency $\nu_{\rm B}$, and derive simple analytical formulae for the spectral and total power of the RICS process, which are very convenient for use in astrophysical applications, and can be used for any kind of ambient low-frequency radiation field, e.g. an isotropic or anisotropic radiation field, blackbody, bremsstrahlung or power-law non-thermal field, etc. In Section 3 we concentrate on the analysis of the radiation efficiency of the RICS mechanism and derive an 'accommodation condition' under which the RICS process becomes dominant over ICS. To illustrate our results, in Section 4 we give a quantitative example to compare between the powers of the RICS and the ICS of a fast electron. Our theoretical approach is straightforward and provides much more simplified formulae compared with those currently available. In Section 5 we give some discussions on possible applications of the RICS in high-energy astrophysics, e.g. γ-ray bursts (GRBs), pulsars, etc.

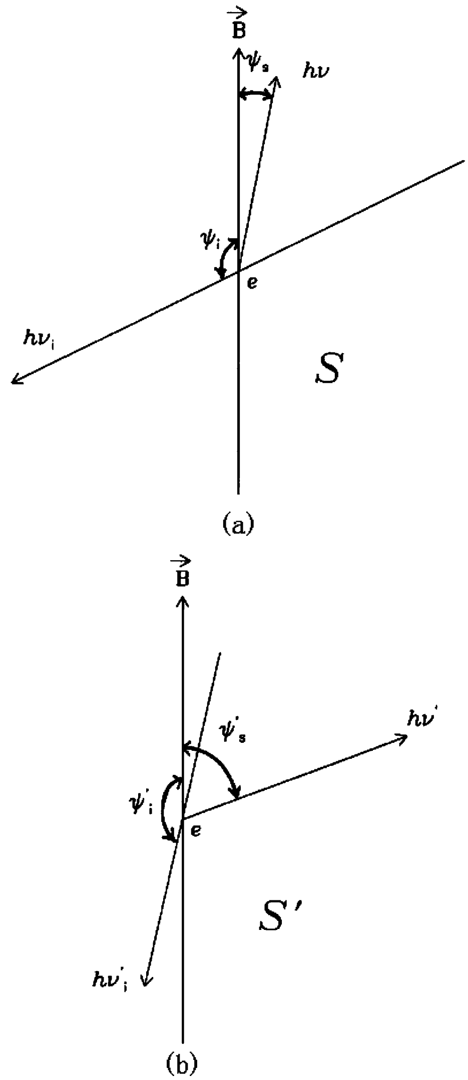

Figure 1. The resonance inverse Compton scattering of a relativistic electron in a strong magnetic field, as observed in (a) the laboratory (S) frame, where the scattering angle $\psi_{\rm s} \sim 0$, i.e. the scattering direction is nearly along the magnetic field line, owing to the relativistic beaming effect, and in (b) the rest frame of the electron S', in which $\psi'_{\rm i} \sim \pi$, i.e. the electron–photon collision is almost head-on, owing to the relativistic aberration effect.

2 SPECTRUM AND TOTAL POWER OF THE RESONANT INVERSE COMPTON SCATTERING

To understand the origin of the RICS mechanism, it seems necessary to restate some aspects of the RICS physics based on the classical quantum theory, which is helpful for people who are unfamiliar with QED theory. We first describe the kinetic behaviour of a relativistic electron in an intense magnetic field. It is known that a fast electron, in a very strong magnetic field, cannot keep a relativistic velocity in directions perpendicular to the magnetic field because of the extremely short synchrotron lifetime ($\tau_{\rm syn}$). For example, with a field strength $B \sim 10^{12}$ G and $\gamma \sim 10^3$, $\tau_{\rm syn} \sim 10^9B^{-2}\gamma^{-1} \sim 10^{-18}$ s. Therefore, the perpendicular component of the electron velocity quickly drops down to $v_\perp \ll c$, and only the relativistic motion along the magnetic field line can be retained for a sufficiently long time. Thus the fast electron will move in a tightened helical orbit along the fieldline. Such a special motion configuration, with $v_\perp \ll c$ and $v_\parallel \lesssim c$, gives the RICS its cyclotron-resonant nature.

The cyclotron resonance can be better understood in the electron rest or comoving frame S' (Fig. 1) in which the velocity components are $v'_\parallel = 0$ and $v'_\perp \ll c$, respectively. Thus in S' one sees a non-relativistic electron moving in a circular orbit with corresponding energy level $(n + \frac{1}{2})h\nu_{\rm B}$ $(n = 0, 1, 2, \ldots)$, where $\nu_{\rm B} = eB/2\pi m_0c = 2.8 \times 10^6B$ Hz is the Landau frequency. The most probable transition occurs at the base frequency $\nu_{\rm B}$ owing to the largest transition probabilities between the neighbouring levels $n \rightleftharpoons n + 1$, particularly, $0 \rightleftharpoons 1$ (You et al. 1997). In the S' frame absorption occurs as long as the frequency of the incident photon equals the base frequency, $\nu'_{\rm i} = \nu_{\rm B}$. The relevant absorption transition is $0 \rightarrow 1$, with the emission transition $1 \rightarrow 0$ following immediately, owing to the extremely high probability of spontaneous transition, e.g. $a_{10} \sim 10^{15}$ s^{-1} for $B \sim 10^{12}$ G (You et al. 1997). Therefore, the combined process of an absorption $0 \rightarrow 1$ and the subsequent, instantaneous re-emission $1 \rightarrow 0$ is equivalent to a 'resonant scattering' of an incident photon with the base frequency $\nu'_{\rm i} = \nu_{\rm B}$.

Transforming to the laboratory frame, $S' \rightarrow S$, we obtain the cyclotron-resonant inverse Compton scattering of a relativistic electron, simply denoted as RICS. In the case where the resonance condition is not satisfied, i.e. if $\nu_i' \neq \nu_B$, or to be exact, $\nu_i' > \nu_B$, we then have the non-resonant ordinary ICS (Herold 1979). In the following discussion, scattering from excited states $n = 1, 2, 3, \ldots$, e.g. $1 \rightarrow 2 \rightarrow 1$, $2 \rightarrow 3 \rightarrow 2$, etc, and the transitions with higher harmonics $2\,\nu_B$, $3\,\nu_B$, ... will all be neglected because the populations $N_0 \gg N_1 \simeq N_2 \simeq N_3 \simeq 0$, and because the probabilities of absorption transition $p_{0\rightarrow1} \gg p_{0\rightarrow2} \gg p_{0\rightarrow3}$ in the case where $B < B_{cr} \simeq 4.4 \times 10^{13}$ G (You et al. 1997).

The RICS radiation spectrum and the total power is calculated via a Lorentz transformation of the reference frames, $S' \rightarrow S$, a technique previously used by Blumenthol & Gould (1970). In order to calculate the scattering spectrum in the S' system, it is convenient to decompose the incident and the scattering waves into monochromatic beams. Denote the intensity of a particular monochromatic, incident beam in the S' system as $I'(\nu_i', \psi_i')$, where ν_i' and ψ_i' are the incident frequency and the incident angle, respectively (Fig. 1). First, we discuss the scattering of this elementary beam. In the S' system, the electron moves in a circular orbit perpendicular to the field B. The number of incident photons passing through a unit area perpendicular to the incident direction, within the element of a solid angle $\mathrm{d}\Omega_i'$ along the incident direction in a time interval $\mathrm{d}t'$ and a frequency range $\nu_i' + \mathrm{d}\nu_i'$, is $[I'(\nu_i', \psi_i')/h\nu_i']\,\mathrm{d}\nu_i'\,\mathrm{d}\Omega_i'\,\mathrm{d}t'$.

The simplified differential resonance scattering cross-section at the base frequency $\nu_i' \simeq \nu_B$ in the S' frame has been derived previously (You et al. 1997; Chen et al. 1997),

$$\sigma^s(\nu_i', \psi_i', \psi_s') = \frac{3}{32} r_0 c \left(1 + \cos^2\psi_i'\right)\left(1 + \cos^2\psi_s'\right)\phi(\nu_i' - \nu_B), \tag{1}$$

where ψ_s' is the scattering angle in S', r_0 is the classical radius of an electron and the Lorentz profile $\phi(\nu_i' - \nu_B) = (\Gamma_{lu}/4\pi^2)[(\nu_i' - \nu_B)^2 + (\Gamma_{lu}/4\pi)^2]^{-1}$, with $\Gamma_{lu} \equiv \Gamma_l + \Gamma_u$ being the total quantum damping constant of the upper (u) and the lower (l) levels. $\Gamma_u = \sum_{k<u} A_{uk}$, $\Gamma_l = \sum_{k<l} A_{lk}$, A_{uk} and A_{lk} are the probabilities of spontaneous transition of $u \rightarrow k$ and $l \rightarrow k$, respectively. Equation (1) shows that σ^s is extremely large when the resonance condition, $\nu_i' = \nu_B$, is satisfied; for example, the total scattering cross-section at the line centre, $\nu_i' = \nu_B$, is $\sigma^s(\nu_B) = \int \sigma^s(\nu_B, \psi_i', \psi_s')\,\mathrm{d}\Omega_s' \lesssim 2 \times 10^7 \sigma_T$ if $B \sim 10^{12}$ G (see Fig. 2), where σ_T is the Thomson cross-section. We show in the Appendix that equation (1) is equivalent to that derived in the QED theory with the **S**-matrix method near the resonance frequency (equation A′ in the Appendix), but (1) is more convenient for analytical studies.

The total number of incident photons scattered into the element of solid angle $\mathrm{d}\Omega_s' = 2\pi \sin\psi_s'\,\mathrm{d}\psi_s'$ in $\mathrm{d}t'$ (in ERF S') is thus

$$\begin{aligned}
&\frac{2\pi I'(\nu_i', \psi_i')}{h\nu_i'} \sigma^s(\nu_i', \psi_i', \psi_s') \sin\psi_s'\,\mathrm{d}\psi_s'\,\mathrm{d}\Omega_i'\,\mathrm{d}\nu_i'\,\mathrm{d}t' \\
&= \frac{3\pi}{16} r_0 c \left[\frac{I'(\nu_i', \psi_i')}{h\nu_i'}\right]\left(1 + \cos^2\psi_i'\right)\left(1 + \cos^2\psi_s'\right) \\
&\times \phi(\nu_i' - \nu_B) \sin\psi_s'\,\mathrm{d}\psi_s'\,\mathrm{d}\Omega_i'\,\mathrm{d}\nu_i'\,\mathrm{d}t'.
\end{aligned} \tag{2}$$

Equation (2) can be simplified further because the Lorentz profile $\phi(\nu_i' - \nu_B)$ is quite similar to the δ-function, a consequence of the resonance nature of RICS. For this reason we first integrate for $\mathrm{d}\nu_i'$, and obtain the number of photons in a narrow frequency band near $\nu_i' \simeq \nu_B \equiv eB/2\pi m_0 c$ scattered into $\psi_s' \rightarrow \psi_s' + \mathrm{d}\psi_s'$ in $\mathrm{d}t'$,

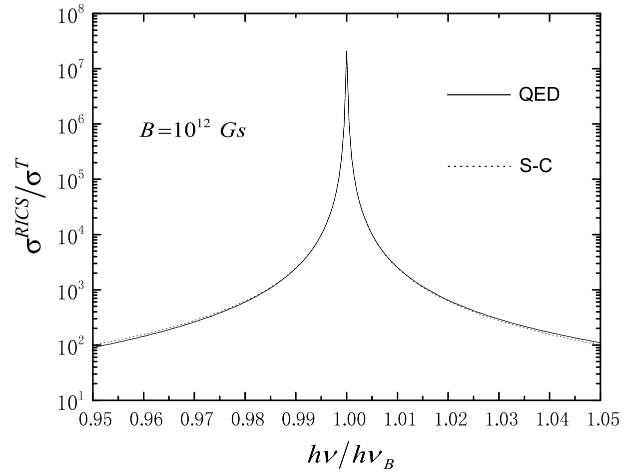

Figure 2. Comparison between the differential scattering cross-section derived in the QED **S**-matrix method, i.e. equation (A′) in the Appendix, and that obtained by our semiclassical quantum method, i.e. equation (1) in text. It is obvious that the classical quantum result is very close to the QED one, especially near the resonance frequency.

$$\begin{aligned}
\mathrm{d}N &= \frac{3\pi}{16} r_0 c \left(1 + \cos^2\psi_i'\right)\left(1 + \cos^2\psi_s'\right) \\
&\times \left[\int_0^\infty \frac{I'(\nu_i', \psi_i')}{h\nu_i'} \phi(\nu_i' - \nu_B)\,\mathrm{d}\nu_i'\right] \sin\psi_s'\,\mathrm{d}\psi_s'\,\mathrm{d}\Omega_i'\,\mathrm{d}t' \\
&\simeq \left(\frac{3\pi}{16} r_0 c\right)\left(1 + \cos^2\psi_i'\right)\left(1 + \cos^2\psi_s'\right) \\
&\times \frac{I'(\nu_B, \psi_i')}{h\nu_B} \sin\psi_s'\,\mathrm{d}\psi_s'\,\mathrm{d}\Omega_i'\,\mathrm{d}t',
\end{aligned} \tag{3}$$

for which the normalization $\int_0^\infty \phi(\nu_i' - \nu_B)\,\mathrm{d}\nu_i' = 1$ has been used.

Returning to the laboratory frame S, the Lorentz transformation gives (taking $\beta \sim 1$) (Tucker 1979):

$$\begin{aligned}
&\nu_s' \equiv \nu' = \gamma\nu(1 - \beta\cos\psi_s) \simeq \gamma\nu(1 - \cos\psi_s) \\
&\cos\psi_i' = (\cos\psi_i - \beta)(1 - \beta\cos\psi_i)^{-1} \\
&\qquad \simeq (\cos\psi_i - 1)(1 - \cos\psi_i)^{-1} = -1 \\
&\mathrm{d}\Omega_i' = \mathrm{d}\Omega_i\gamma^{-2}(1 - \beta\cos\psi_i)^{-2} \simeq \mathrm{d}\Omega_i\gamma^{-2}(1 - \cos\psi_i)^{-2} \\
&I'(\nu_i', \psi_i') = I(\nu_i, \psi_i)\gamma^3(1 - \beta\cos\psi_i)^3 \\
&\qquad \simeq I(\nu_i, \psi_i)\gamma^3(1 - \cos\psi_i)^3 \\
&\mathrm{d}t' = \frac{1}{\gamma}\,\mathrm{d}t.
\end{aligned}$$

Therefore, $I'(\nu_i', \psi_i')$, $\mathrm{d}\Omega_i'$ and $\mathrm{d}t'$ in equation (3) can be replaced by the corresponding quantities in the laboratory system S, i.e. $\mathrm{d}\Omega_i' \rightarrow \mathrm{d}\Omega_i$, $\mathrm{d}t' \rightarrow \mathrm{d}t$ and $I'(\nu_i', \psi_i') \rightarrow I(\nu_i, \psi_i)$. Note that $(1 + \cos^2\psi_i') \simeq 2$ because in the S' system the incident direction $\psi_i' \simeq \pi$ (see the second equation above), i.e. we have approximately a head-on collision for the fast electron with $\beta \simeq 1$. Thus (3) can be rewritten as

$$\begin{aligned}
\mathrm{d}N &= \left(\frac{3\pi}{8} r_0 c\right)\left(\frac{1}{h\nu_B}\right) I(\nu_i, \psi_i)(1 - \cos\psi_i)\left(1 + \cos^2\psi_s'\right) \\
&\times \sin\psi_s'\,\mathrm{d}\psi_s'\,\mathrm{d}\Omega_i\,\mathrm{d}t.
\end{aligned} \tag{4}$$

Equation (4) represents the number of photons scattered into the solid angle $\psi_s' \rightarrow \psi_s' + \mathrm{d}\psi_s'$, in $\mathrm{d}t$ in the S frame. We note that, for a given incident beam $I(\nu_i, \psi_i)$, the value of the incident frequency ν_i cannot be taken as a free variable if the incident direction ψ_i has been fixed. The ν_i-value is determined by the resonance condition of scattering $\nu_i' = \nu_B$ in ERF S' and the Doppler formula, $\nu_i' = \gamma\nu_i(1-\beta\cos\psi_i) \simeq \gamma\nu_i(1-\cos\psi_i)$; that is,

$$\nu_i \simeq \frac{\nu_B}{\gamma(1-\cos\psi_i)}. \tag{5}$$

Equation (5) is the expression of the resonance condition $\nu_i' = \nu_B$ in the laboratory frame S, which indicates that the frequency of the incident photons to be absorbed cannot be taken arbitrarily, but is restricted by the given ψ_i-angle, $\nu_i = \nu_i(\psi_i)$. This is quite different from the conventional non-resonant inverse Compton scattering of a free electron, in which case ν_i and ψ_i are free variables, independent of each other.

For a qualitative insight, we note that in S', the scattering frequency is given by the resonance condition, $\nu_S' \equiv \nu' = \nu_i' = \nu_B$, which shows a strict monochromaticity of scattering in S'. However, returning to the S frame, we obtain the scattering frequency (again taking $\beta \simeq 1$)

$$\nu = \gamma\nu'(1+\beta\cos\psi_s') \simeq \gamma\nu_B(1+\cos\psi_s'). \tag{6}$$

Owing to the approximate isotropy of scattering in S' (see equation 1), the range for the scattering angle ψ_s' is $(0, \pi)$. Therefore, the scattering frequencies in the S system should spread over a wide range $(0, 2\gamma\nu_B)$, with a large high-frequency cut-off, $2\gamma\nu_B$. However, we will show in the following that the RICS radiation still keeps a moderately good monochromaticity.

We now derive the RICS spectral power for an electron with energy γ. Multiplying equation (4) by the photon energy $h\nu$, with ν as given by equation (6), we obtain the elementary power

$$\begin{aligned} \mathrm{d}p^{\mathrm{RICS}} &= \frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}N}{\mathrm{d}t}h\nu \\ &= \frac{3\pi}{8}r_0c\left(\frac{h\nu}{h\nu_B}\right) I(\nu_i,\psi_i)(1-\cos\psi_i)\left(1+\cos^2\psi_s'\right) \\ &\quad\times \sin\psi_s'\,\mathrm{d}\psi_s'\,\mathrm{d}\Omega_i \\ &= \left(\frac{3\pi}{8}r_0c\right)\gamma I(\nu_i,\psi_i)(1-\cos\psi_i)\left(1+\cos^2\psi_s'\right) \\ &\quad\times(1+\cos\psi_s')\sin\psi_s'\,\mathrm{d}\psi_s'\,\mathrm{d}\Omega_i. \end{aligned} \tag{7}$$

In equation (7), there exists a one-to-one correspondence between the scattering frequency ν and the scattering angle ψ_s' (see equation 6). Integrating equation (7) over the whole incident solid angle $\mathrm{d}\Omega_i = 2\pi\sin\psi_i\,\mathrm{d}\psi_i$, we obtain the RICS power at frequency ν (i.e. at angle ψ_s'),

$$\begin{aligned} \mathrm{d}p^{\mathrm{RICS}} = &\ \frac{3\pi}{8}r_0c\gamma(1+\cos\psi_s')\left(1+\cos^2\psi_s'\right)\sin\psi_s'\,\mathrm{d}\psi_s' \\ &\times\left[2\pi\int_0^\pi I(\nu_i,\psi_i)(1-\cos\psi_i)\sin\psi_i\,\mathrm{d}\psi_i\right]. \end{aligned} \tag{8}$$

Note that the intensity I is always a function of ψ_i, even when the radiation field is isotropic, for which the dependence on ψ_i does not seem explicit, but I in fact is still related to ψ_i through $\nu_i(\psi_i)$, i.e. $I = I(\nu_i) = I[\nu_i(\psi_i)]$, where the form of $I(\nu_i)$ is determined by the given radiation field of soft photons, and the function $\nu_i = \nu_i(\psi_i)$ is given by the resonance condition (5). Denoting the integral in equation (8) by η, namely,

$$2\pi\int_0^\pi I[\nu_i(\psi_i)](1-\cos\psi_i)\sin\psi_i\,\mathrm{d}\psi_i \equiv \eta(\gamma, B). \tag{9}$$

The quantity η in unit of erg cm^{-2}, a function of γ and B, can be regarded as a measure of the resonance scattering efficiency, in the sense that it includes all the eligible low-frequency photons incident from all directions ψ_i, which satisfy the resonance condition equation (5), thus it can be resonantly absorbed (scattered) by an electron with energy γ. Hereafter we simply call $\eta(\gamma, B)$ the 'efficiency of the RICS'. It can be seen that, the stronger the intensity $I[\nu_i(\psi_i)]$ at the resonance frequency $\nu_i = \nu_i(\psi_i)$, the higher the 'resonance scattering efficiency' η.

The dependence of the spectral power on the frequency can be derived by replacing the scattering angle ψ_s' in equation (8) by the scattering frequency ν. Using equation (6), the quantity $\cos\psi_s'$ can be rewritten as $\cos\psi_s' = \gamma^{-1}\nu_B^{-1}\nu - 1 = 2x - 1$, where $x \equiv \nu/2\gamma\nu_B = \nu/\nu_{\max}$ is a dimensionless scattering frequency in units of the maximum scattering frequency $2\gamma\nu_B$.

The spectral power hence becomes

$$\frac{\mathrm{d}p^{\mathrm{RICS}}}{\mathrm{d}x} = (3\pi r_0 c)\eta(\gamma,\nu_B)\gamma f(x) \quad \left(x \equiv \frac{\nu}{2\gamma\nu_B} = \frac{\nu}{\nu_{\max}}\right), \tag{10}$$

where the function $f(x)$ is

$$f(x) = \begin{cases} 2x^3 - 2x^2 + x & (\text{if } 0 \leqslant x \leqslant 1) \\ 0 & (\text{if } x > 1). \end{cases} \tag{11}$$

The dimensionless function $f(x)$, depicted in Fig. 3, specifies the spectral shape of the RICS emission of a single electron. In the same figure we also show the results of our Monte Carlo simulations for different values of γ. These simulations do not include any simplifications we used in obtaining the analytical formula (11) and the results show clearly that equation (11) is sufficiently accurate when $\gamma > 5$.

We see in Fig. 3 that the RICS spectrum still retains a moderately good monochromaticity despite the very wide frequency band – a sharp peak near the maximum frequency $2\gamma\nu_B$ with a weak low-frequency tail extending to $\nu \simeq 0$. In a semiquantitative discussion, one can thereby adopt a quasi-monochromatic approximation by assuming a one-to-one correspondence between the

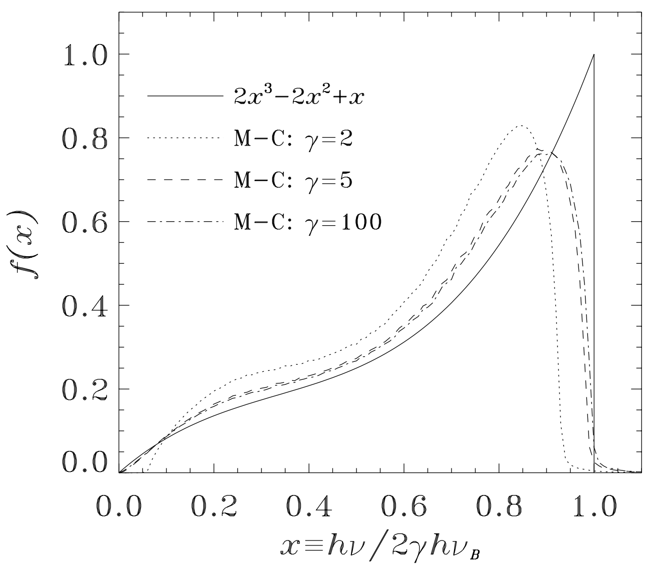

Figure 3. The radiation spectrum of the RICS of a single electron with energy γ shows good monochromaticity, which is quite different from the ordinary inverse Compton scattering. Note the sharp peak at the maximum frequency $2\gamma\nu_B$, and the weak low-frequency tail extending to $\nu \to 0$. The solid curve is from the analytical equation (11); the other curves are from Monte Carlo simulations without any simplification or approximation. Note that when $\gamma > 5$, the simulation results are very close to the analytical one.

electron energy γ and the emission frequency ν, i.e. an electron with energy γ produces a monochromatic line radiation with frequency $2\gamma\nu_B$.

With $dp^{RICS}/dx = (2\gamma\nu_B)\,(dp^{RICS}/d\nu)$, the spectral power (10) becomes

$$\frac{dp^{RICS}}{d\nu} = \left(\frac{3\pi r_0 c}{2\nu_B}\right)\eta(\gamma, \nu_B) f(\nu/2\gamma\nu_B). \tag{12}$$

The total power of the RICS process is thus

$$P^{RICS} = \int_0^1 \frac{dp^{RICS}}{dx}\,dx = (\pi r_0 c)\eta(\gamma, \nu_B)\gamma. \tag{13}$$

Thus the power P^{RICS} is proportional to the product of the 'efficiency' A and the energy γ of the electron.

3 THE ACCOMMODATION CONDITION

As can be seen in equation (10) or (13), the RICS radiation power is proportional to the 'resonance scattering efficiency' $\eta(\gamma, B)$, which depends on the incident intensity $I[\nu_i(\psi_i)]$ at the resonance frequency $\nu_i(\psi_i)$ (see equations 5 and 9). The RICS mechanism is important only if η is large. When $\eta \approx 0$, RICS emission becomes negligibly weak.

Note that, in general, the fractional number of photons with small incident angles is very small in a general isotropic radiation field. Furthermore, equation (5) shows that $\psi_i \simeq 0$ corresponds to a high incident frequency ν_i. Since the number of photons with very high ν_i is nominally very small in a low-frequency radiation field, neglecting scattering with small incident angles, equation (5) hence becomes approximately

$$\gamma h\nu_i \simeq h\nu_B \quad \text{or} \quad \gamma\nu_i \simeq \nu_B. \tag{14}$$

Equation (14) gives the approximate position of the resonance frequency ν_i, and serves not only as an approximate resonance condition for a single electron with energy γ, but also provides a useful criterion to estimate the efficiency of the RICS emission. According to equation (14), the approximate resonance frequency $\nu_i \simeq \nu_B/\gamma$ is no longer dependent on the incident angle ψ_i, hence the approximate resonance intensity $I[\nu_i(\psi_i)] \simeq I(\nu_B/\gamma)$ is also ψ_i independent. Therefore, equation (9) becomes approximately

$$\eta(\gamma, B) \simeq 2\pi I(\nu_B/\gamma)\int(1-\cos\psi_i)\sin\psi_i\,d\psi_i \propto I(\nu_B/\gamma). \tag{15}$$

Thus, the stronger the intensity at the approximate resonance frequency is, the higher the scattering efficiency. For any given ambient low-frequency field $I(\nu_i)$, if its average frequency $\bar{\nu}_i = \int \nu_i I(\nu_i)\,d\nu_i / \int I(\nu_i)\,d\nu_i$ is near the approximate resonance frequency, i.e. $\bar{\nu}_i \simeq \nu_B/\gamma$, then the resonance intensity $I(\nu_B/\gamma) \simeq I(\bar{\nu}_i)$ should be large, hence a large η. On the other hand, if ν_B/γ is far from $\bar{\nu}_i$, then $I(\nu_B/\gamma) \to 0$, and $\eta(\gamma, B) \to 0$, the RICS emission diminishes.

In brief, when the mean frequency $\bar{\nu}_i$ satisfies the approximate resonance condition (14), i.e.

$$\gamma h\bar{\nu}_i \simeq h\nu_B \tag{16}$$

the RICS is important. Equation (16) can be regarded as an 'accommodation condition' or an 'accommodation relation', which links the field strength B, the electron energy γ, and the average frequency $\bar{\nu}_i$ of the ambient radiation field. The RICS emission is significant only when the electron energy γ times the average ambient photon energy $(h\bar{\nu}_i)$ is comparable to $h\nu_B$.

Likewise, for the assembly of relativistic electrons with average energy $\bar{\gamma}$, the collective RICS emission is significant when $\bar{\gamma}h\bar{\nu}_i$ is comparable to $h\nu_B$,

$$\bar{\gamma}h\bar{\nu}_i \simeq h\nu_B. \tag{17}$$

Therefore, equations (5), (14), (16) and (17) are the resonance condition, the approximate resonance condition, the 'accommodation condition for a single electron with energy γ' and the 'accommodation condition for assembly of fast electrons with average energy $\bar{\gamma}$', respectively.

4 COMPARISON OF RICS WITH THE ORDINARY INVERSE COMPTON SCATTERING ICS

To illustrate the conclusion of equation (16), we now give a numerical example to compare quantitatively between the powers of the RICS and of the ICS of a fast electron in a given scattered low-frequency field that assumes a power-law form,

$$I(\nu_i) = I_0\nu_i^{-p} \quad (\nu_1 < \nu_i < \nu_2). \tag{18}$$

The total power of the field-free ICS of an electron is well known (Tucker 1979),

$$P^{ICS} = \frac{32\pi}{9} r_0^2 c U_{ph}\gamma^2 \simeq 2.6\times10^{-14} U_{ph}\gamma^2\ \mathrm{erg\ s^{-1}}, \tag{19}$$

where U_{ph} is the energy density of the low-frequency field. However, in the case of an intense magnetic field, the power of ordinary non-resonant inverse Compton scattering is much smaller than that given by equation (19), owing to the fact that the non-resonant magnetic ICS is equivalent to the field-free ICS only for the scattered soft photons with frequency $\nu_i' > \nu_B$, equivalently, $\nu_i > \nu_B/\gamma$ in the observer frame S. The soft photons with $\nu_i < \nu_B/\gamma$ ($\nu_i' < \nu_B$) are useless for ordinary ICS. However, in equation (19), the energy density of the low-frequency field U_{ph} contains all the soft photons, including both the $\nu_i > \nu_B/\gamma$ and $\nu_i < \nu_B/\gamma$. Therefore, in the following comparison, we take the power given by equation (19) as an upper limit of that for non-resonant inverse Compton scattering.[1]

The RICS power P^{RICS} is given by (13). With an ambient field of (18), from equation (5) we find $I(\nu_i) = I_0\nu_B^{-p}\gamma^p(1-\cos\psi_i)^p$. We note that the upper and lower limits of the integral in (9) are (ψ_1, ψ_2) rather than $(0, \pi)$, owing to the existence of the upper- and the lower-frequency cutoffs ν_1 and ν_2, i.e. $I(\nu_i) = 0$ for $\nu_i < \nu_1$ and $\nu_i > \nu_2$. According to equation (5), we obtain $(1-\cos\psi_1) = \nu_B/\gamma\nu_1$ and $(1-\cos\psi_2) = \nu_B/\gamma\nu_2$, which determine the limits (ψ_1, ψ_2). Thus we obtain

$$\begin{aligned}\eta(\gamma, \nu_B) &= 2\pi I_0\nu_B^{-p}\gamma^p\int_{\psi_1}^{\psi_2}(1-\cos\psi_i)^{p+1}\sin\psi_i\,d\psi_i\\ &= 2\pi I_0\nu_B^{-p}\gamma^p\int_{\nu_B/\gamma\nu_2}^{\nu_B/\gamma\nu_1}\chi^{p+1}\,d\chi\\ &\simeq 2\pi I_0\nu_B^2\left(\frac{1}{p+2}\right)\nu_1^{-(p+2)}\gamma^{-2},\end{aligned} \tag{21}$$

where the term $\nu_2^{-(p+2)}$ has been neglected for $\nu_2^{-(p+2)} \ll \nu_1^{-(p+2)}$.

[1] Strictly speaking, the power of non-resonance inverse Compton scattering in a magnetic field is given by

$$P^{\mathrm{nres\,ICS}} \approx \frac{32}{9}\pi r_0^2 c U_{ph}(\nu_i' > \nu_B)\gamma^2. \tag{20}$$

We emphasize again that for the RICS mechanism to work the low-frequency spectrum in the band ($\nu_1 < \nu_i < \nu_2$) has to meet the 'accommodation condition' (16) $\gamma\bar{\nu}_i \simeq \nu_B$ or $\gamma\nu_1 \lesssim \nu_B$ (for power-law field, $\bar{\nu}_i \gtrsim \nu_1$), as otherwise (e.g. $\nu_B \ll \gamma\nu_1 \ll \gamma\nu_2$, or $\gamma\nu_1 \ll \gamma\nu_2 \ll \nu_B$) the integral vanishes and the 'scattering efficiency' $\eta(\gamma, B) \to 0$.

Inserting (21) into (13) we obtain

$$P^{\rm RICS} = \frac{2\pi^2 r_0 c}{p+2} I_0 \nu_B^2 \nu_1^{-(p+2)} \gamma^{-1}$$
$$\simeq 1.31 \times 10^{12} \frac{1}{p+2} I_0 B^2 \nu_1^{-(p+2)} \gamma^{-1}\ {\rm erg\,s^{-1}}. \quad (22)$$

On the other hand, from the relation $U_{\rm ph}(\nu_i) = 4\pi I(\nu_i)/c$ and (18), we obtain the energy density of the low-frequency field,

$$U_{\rm ph} = \int_{\nu_1}^{\nu_2} U_{\rm ph}(\nu_i)\,{\rm d}\nu_i = \frac{4\pi I_0}{c}\int_{\nu_1}^{\nu_2} \nu_i^{-p}\,{\rm d}\nu_i$$
$$\simeq \frac{4\pi I_0}{c}\frac{\nu_2^{1-p}}{1-p}, \quad (23)$$

where the term ν_1^{1-p} has been neglected because $\nu_1^{1-p} \ll \nu_2^{1-p}$ if $p < 1$. Inserting (23) into (19), we obtain

$$P^{\rm ICS} = 1.09 \times 10^{-23} I_0 \frac{\nu_2^{1-p}}{1-p}\gamma^2. \quad (24)$$

By comparing (22) and (24), it is evident that $P^{\rm RICS} \gg P^{\rm ICS}$ holds as long as (16) is satisfied (i.e. $\gamma\bar{\nu}_i \simeq \nu_B$). If the values of ν_1, ν_2, p, B and γ are in physically reasonable ranges, for example, taking $p \sim 0.5$–0.9, $B \sim 10^{12}$ G, $\gamma \sim 10^3$, $\nu_1 \sim 10^{15}$ Hz and $\nu_2 \sim 10^{16}$ Hz, we find that $P^{\rm RICS}$ is 10^3–10^4 times stronger than $P^{\rm ICS}$.

As illustrated above one can prove in a similar way that, under the 'accommodation condition' $\gamma\bar{\nu}_i \simeq \nu_B$, the total radiation power of the RICS is always much greater than that of the coexistent ICS, no matter what kind of low-frequency radiation field is present (e.g. blackbody, bremsstrahlung, etc.).

5 CONCLUSIONS AND DISCUSSIONS

In summary, when the relativistic electrons, beamed along the direction of strong magnetic field of the magnetized neutron star or strange star, pass through an ambient soft photon field, the resonant inverse Compton scattering (RICS) will be produced. It may become the dominant γ-ray radiation mechanism if the 'accommodation condition' $\bar{\gamma} h\bar{\nu}_i \approx h\nu_B$ is satisfied. The 'accommodation condition' may be easily satisfied at some distance from the neutron star because the magnetic field is markedly non-uniform along the magnetic field line, dropping off with distance r drastically, $B \propto (r/R_*)^{-3}$. In addition to the high efficiency, the high-frequency and the high-beaming behaviour, another prominent advantage of the RICS radiation is the relatively good monochromaticity, compared with the coexistent ICS. The quasi-monochromatic approximation gives a one-to-one correspondence between the energy γ of the fast electron and the RICS frequency $2\gamma\nu_B$, $\gamma \to 2\gamma\nu_B$, which implies that most of the RICS photons concentrate in the high-frequency band, $\nu \sim \gamma\nu_B$. Therefore, the resonant inverse Compton scattering in the intense magnetic field is significantly different from the field-free case, and may be a very important radiation mechanism in the hard X-ray and γ-ray astronomy. The formulae we derived for the emission spectrum and the total power are simple to use and may find wide applications in high-energy physics and astrophysics.

As subsequent work, we plan to calculate the collective RICS spectrum of the assembly of relativistic electrons, i.e. the RICS emissivity $J^{\rm RICS}(\nu)$, based on the spectrum for a single electron given by equations (10) and (11). We will consider various forms of ambient soft photon fields around the central neutron star (power-law form, blackbody, bremsstrahlung, etc.). However, without any calculation, we can infer qualitatively the basic characteristics of the collective RICS spectrum based on the quasi-monochromatic approximation. If the relativistic electrons, moving along the field line, have a power-law energy spectrum, $N(\gamma) \propto \gamma^{-n}$ ($\gamma_1 < \gamma < \gamma_2$), according to the one-to-one correspondence $\gamma \to 2\gamma\nu_B$, the collective RICS spectrum $J^{\rm RICS}(\nu) \sim \nu$ in the main frequency band also has an approximate power-law form, and the upper and lower cut-offs are $2\gamma_2\nu_B$ and $\sim 2\gamma_1\nu_B$, respectively, i.e. $2\gamma_1\nu_B \lesssim \nu \leqslant 2\gamma_2\nu_B$. However, the quasi-monochromaticity is a rough approximation. For an electron with energy γ, except for the main radiation near the maximum frequency $2\gamma\nu_B$, there still exists weak radiation extending to $\nu \approx 0$ (see equation 11 and Fig. 3). Therefore, electrons in the range $\gamma_1 < \gamma < \gamma_2$ all make a contribution to the low-frequency band $\nu < 2\gamma_1\nu_B$, down to $\nu \simeq 0$. Therefore, the final collective RICS spectrum should have a broken power-law form, with a break point at $\sim 2\gamma_1\nu_B$. The detailed quantitative calculation confirms this conclusion. Such a broken power-law spectrum is a basic property of the collective RICS spectrum, which could be used to explain the observed broken power-law spectra of some astrophysical sources.

There exist various classes of high-energy hard X-ray and/or γ-ray objects with strong magnetic fields, e.g. the γ-ray pulsars, magnetars, GRBs, etc. Most known pulsars are radio sources; the radio photons are believed to be produced by synchrotron and/or curvature radiation of relativistic electrons. Though viable models exist to explain the X-ray and gamma-ray pulses from some of the radio pulsars, here we suggest that the RICS mechanism with its high-efficiency, high-frequency and beaming behaviour may also contribute to the observed high-energy γ-ray radiation. However, when using the new RICS mechanism to construct a reasonable model for a γ-ray pulsar, one should be careful because the necessary low-frequency photons for the RICS process cannot originate from the observed radio pulses themselves. For these radio photons, the incident angle is too small ($\psi_i \simeq 0$, the tail-up collision between the photon and the electron) to make the incident frequency ν_i satisfy the resonance condition, equation (5). Therefore, we are inclined to consider another source of radio photons that is connected with the gamma-ray pulsar models suggested by Cheng, Ho & Ruderman (1986, see also Zhang & Cheng 1999) in which the pairs $e^{\pm}$ mainly originate from the outer gap, far from the region of strong magnetic field of the central neutron star. In this case, the synchrotron radio photons produced by the position e^+ enter the strong magnetic funnel and have a head-on collision with the outgoing relativistic electrons from the inner gap. Further detailed calculations are needed to investigate the relative efficiency of the RICS mechanism, compared with other available radiation mechanisms.

Another possible application of the RICS mechanism might be the γ-ray bursts, which may contain one or two magnetized neutron stars in the centre of the fireball (Piran 1999). If the NS–NS mergers or NS–black hole mergers are indeed the 'inner engines' of the fireball during the process of release of gravitation energy, the operation of the RICS mechanism seems to be inevitable. The production of an enormous number of relativistic electrons seems likely because of the tremendous release of gravitation energy. Therefore, strong gamma-rays might be produced during the merging process through the RICS mechanism. Here we emphasize the special advantage of the direction of the magnetic axis, along which the highly beamed RICS photons may easily escape from the neutron star with virtually no absorption by the magnetic and/or γ–γ annihilations. In other words, the magnetic axis can be regarded as an optically thin 'transparent channel' for the high-energy RICS photons with $h\nu \geqslant 1$ MeV. Consequently, the serious 'compactness problem' encountered in current GRBs models could be resolved in the RICS model. Because the RICS mechanism operates only when the orderly magnetic field is not destroyed completely, it may only be possible to account for the initial gamma-ray emission of GRBs. Further detailed calculations are needed to investigate whether the RICS mechanism can account for the observed complexity of high-energy radiation.

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APPENDIX: COMPARISON WITH THE SCATTERING CROSS-SECTION FROM QED

We demonstrate here that the differential scattering cross-section in the S' (ERF) given in equation (1) is precisely the same as that derived in the QED theory with the **S**-matrix method, if applied in the vicinity of the resonance frequency $\nu_i' = \nu_\mathrm{B}$. However, equation (1) is much more convenient for analytical studies of the resonant inverse Compton scattering, as carried out in this paper.

The QED formula is (e.g. equation 2 in Dermer 1990)

$$\frac{\mathrm{d}\sigma'}{\mathrm{d}\epsilon_\mathrm{s}'\,\mathrm{d}\mu_\mathrm{s}'} = \frac{3\sigma_\mathrm{T}}{8}\delta(\epsilon_\mathrm{s}' - \epsilon_\mathrm{i}') \times \left[\left(1-\mu_\mathrm{i}'^2\right)\left(1-\mu_\mathrm{s}'^2\right) + \frac{1}{4}\left(1+\mu_\mathrm{i}'^2\right)\left(1+\mu_\mathrm{s}'^2\right)(g_1+g_2)\right],$$

where ϵ_i' and ϵ_s' are the energy of the incident and of the scattered photons, respectively. Integrating over ϵ_s', then equation

$$\int \frac{\mathrm{d}\sigma'}{\mathrm{d}\epsilon_\mathrm{s}'\,\mathrm{d}\mu_\mathrm{s}'}\mathrm{d}\epsilon_\mathrm{s}' \equiv \frac{\mathrm{d}\sigma'}{\mathrm{d}\mu_\mathrm{s}'} = \frac{3\sigma_\mathrm{T}}{8}\Big[\left(1-\mu_\mathrm{i}'^2\right)\left(1-\mu_\mathrm{s}'^2\right) + \frac{1}{4}\left(1+\mu_\mathrm{i}'^2\right)\left(1+\mu_\mathrm{s}'^2\right)(g_1+g_2)\Big], \tag{A}$$

where $\mu_\mathrm{i}' \equiv \cos\psi_\mathrm{i}'$, $\mu_\mathrm{s}' \equiv \cos\psi_\mathrm{s}'$, $g_1(\nu) = u^2/[(1+u)^2]$, $g_2(\nu) = u^2/[(u-1)^2 + a^2]$, $u \equiv \epsilon_\mathrm{i}'/\epsilon_\mathrm{B} = \nu_\mathrm{i}'/\nu_\mathrm{B}$, $a \equiv \epsilon_\Gamma/2\epsilon_\mathrm{B}$, $\epsilon_\Gamma = 4/3\alpha_f\epsilon_\mathrm{B}^2$ and $\alpha_f \approx 1/137$ is the fine-structure constant. Since the dimensionless Landau frequency $\epsilon_\mathrm{B} \equiv h\nu_\mathrm{B}/m_0\,c^2 \ll 1$ if $B < 10^{13}$ G, then $a \ll 1$.

Equation (A) thus describes the differential scattering cross-section in QED, for photons incident along the direction $\mu_\mathrm{i}' \equiv \cos\psi_\mathrm{i}'$ and scattered along the direction of $\mu_\mathrm{s}' \equiv \cos\psi_\mathrm{s}'$. In the vicinity of the resonance frequency in the S', i.e. $\nu_\mathrm{i}' \approx \nu_\mathrm{B}$, we have $u \approx 1$, $g_1 \approx 1/4$ and $g_2 \gg 1$. Therefore, in equation (A), $g_1 + g_2 \approx g_2 \gg 1$. Consequently, equation (A) may be simplified as

$$\frac{\mathrm{d}\sigma'}{\mathrm{d}\mu_\mathrm{s}'} = \frac{3\sigma_\mathrm{T}}{8}\left[\frac{1}{4}g_2(\nu)\left(1+\mu_\mathrm{i}'^2\right)\left(1+\mu_\mathrm{s}'^2\right)\right]. \tag{A$'$}$$

Since $u \approx 1$, then

$$\begin{aligned} g_2 &\equiv \frac{u^2}{(u-1)^2 + a^2} \\ &\approx \frac{1}{(u-1)^2 + a^2} \\ &= \frac{\nu_\mathrm{B}^2}{(\nu_\mathrm{i}' - \nu_\mathrm{B})^2 + a^2\nu_\mathrm{B}^2} \\ &= \frac{\omega_\mathrm{B}^2/4\pi^2}{(\nu_\mathrm{i}' - \nu_\mathrm{B})^2 + (\Gamma_{10}/4\pi)^2}, \end{aligned}$$

where $\Gamma_{10} = A_{10} = 4e^2\omega_\mathrm{B}^2/3m_0c^3$ is the spontaneous transition probability from the first excited to the ground Landau energy levels (You et al. 1997). Since $\sigma_\mathrm{T} \equiv 8/3\pi r_0^2 = 8/3\pi r_0 e^2/m_0c^2$, where $r_0 = e^2/m_0\,c^2$ is the classical radius of an electron, equation (A$'$) may be rewritten as

$$\frac{\mathrm{d}\sigma'}{\mathrm{d}\mu_\mathrm{s}'} = \frac{3\pi}{16}r_0c\left(1+\cos^2\psi_\mathrm{i}'\right)\left(1+\cos^2\psi_\mathrm{s}'\right)\phi(\nu_\mathrm{i}' - \nu_\mathrm{B}),$$

where $\phi(\nu_\mathrm{i}' - \nu_\mathrm{B}) = (\Gamma_l u/4\pi^2)/[(\nu_\mathrm{i}' - \nu_\mathrm{B})^2 + (\Gamma_l u/4\pi)^2]$.

Since the definition of $(\mathrm{d}\sigma'/\mathrm{d}\mu_\mathrm{s}')\mathrm{d}\mu_\mathrm{s}'$ is equivalent to that of $\sigma^\mathrm{s}\,\mathrm{d}\Omega_\mathrm{s}' = \sigma^\mathrm{s} 2\pi\sin\psi_\mathrm{s}'\,\mathrm{d}\psi_\mathrm{s}' = \sigma^\mathrm{s}2\pi\,\mathrm{d}\mu_\mathrm{s}'$, we then finally have

$$\frac{\mathrm{d}\sigma'}{\mathrm{d}\mu_\mathrm{s}'} = 2\pi\sigma^\mathrm{s}(\nu_\mathrm{i}', \psi_\mathrm{i}', \psi_\mathrm{s}').$$

Therefore, equation (1), i.e. the differential scattering cross-section we have used in this paper, is precisely the same as the QED result in the vicinity of the resonance frequency of the magnetic field. To further corroborate this result, we show in Fig. 2 the ratio between the differential scattering cross-section derived in the QED **S**-matrix method, i.e. equation (A$'$), and that obtained by our semiclassical method, i.e. equation (1). It is evident that the ratio is very close to unity in the vicinity of the resonance frequency.

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